

## Fundamentals of Linear Control

### A Concise Approach

Taking a different approach from standard thousand-page reference-style control textbooks, *Fundamentals of Linear Control* provides a concise yet comprehensive introduction to the analysis and design of feedback control systems in fewer than 300 pages.

The text focuses on classical methods for dynamic linear systems in the frequency domain. The treatment is, however, modern and the reader is kept aware of contemporary tools and techniques, such as state-space methods and robust and nonlinear control.

Featuring fully worked design examples, richly illustrated chapters, and an extensive set of homework problems and examples spanning across the text for gradual challenge and perspective, this textbook is an excellent choice for senior-level courses in systems and control or as a complementary reference in introductory graduate-level courses. The text is designed to appeal to a broad audience of engineers and scientists interested in learning the main ideas behind feedback control theory.

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**To Beatriz and Victor**

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## Preface

The book you have in your hands grew out of a set of lecture notes scribbled down for MAE 143B, the senior-level undergraduate *Linear Control* class offered by the Department of Mechanical and Aerospace Engineering at the University of California, San Diego.

The focus of the book is on classical methods for analysis and design of feedback systems that take advantage of the powerful and insightful representation of dynamic linear systems in the frequency domain. The required mathematics is introduced or revisited as needed. In this way the text is made mostly self-contained, with accessory work shifted occasionally to homework problems.

Key concepts such as tracking, disturbance rejection, stability, and robustness are introduced early on and revisited throughout the text as the mathematical tools become more sophisticated. Examples illustrate graphical design methods based on the root-locus, Bode, and Nyquist diagrams. Whenever possible, without straying too much from the classical narrative, the reader is made aware of contemporary tools and techniques such as state-space methods, robust control, and nonlinear systems theory.

With so much to cover in the way of insightful engineering *and* relevant mathematics, I tried to steer clear of the curse of the engineering systems and control textbook: becoming a treatise with 1000 pages. The depth of the content exposed in fewer than 300 pages is the result of a compromise between my utopian goal of *at most* 100 pages on the one hand and the usefulness of the work as a reference and, I hope, inspirational textbook on the other. Let me know if you think I failed to deliver on this promise.

I shall be forever indebted to the many students, teaching assistants, and colleagues whose exposure to earlier versions of this work helped shape what I am finally not afraid of calling the *first* edition. Special thanks are due to Professor Reinaldo Palhares, who diligently read the original text and delighted me with an abundance of helpful comments.

I would like to thank Sara Torenson from the UCSD Bookstore, who patiently worked with me to make sure earlier versions were available as readers for UCSD students, and Steven Elliot from Cambridge University Press for his support in getting this work to a larger audience.

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## Overview

This book is designed to be used in a quarter- or semester-long senior-level undergraduate linear control systems class. Readers are assumed to have had some exposure to differential equations and complex numbers (good references are [BD12] and [BC14]), and to have some familiarity with the engineering notion of signals and systems (a standard reference is [Lat04]). It is also assumed that the reader has access to a high-level software program, such as MATLAB, to perform calculations in many of the homework problems. In order to keep the focus on the content, examples in the book do not discuss MATLAB syntax or features. Instead, we provide supplementary MATLAB files which can produce all calculations and figures appearing in the book. These files can be downloaded from <http://www.cambridge.org/deOliveira>.

Chapters 1 and 2 provide a quick overview of the basic concepts in control, such as feedback, tracking, dynamics, disturbance rejection, integral action, etc. Math is kept at a very basic level and the topics are introduced with the help of familiar examples, such as a simplistic model of a car and a toilet bowl.

Chapter 3 formalizes the concept of a transfer-function for dynamic linear system models. Its first part is a review of the Laplace transform and its application to linear ordinary differential equations. The second part introduces systems concepts such as stability, transient and steady-state response, and the frequency response method. Some topics, e.g. complex integration, the calculus of residues, and norms of signals and systems, are covered in more depth than is usually found in typical introductory courses, and can be safely skipped at first read.

Equipped with the concept of a transfer-function, Chapter 4 formalizes fundamental concepts in feedback analysis, such as tracking, sensitivity, asymptotic and internal stability, disturbance rejection, measurement noise, etc. Homework problems in this chapter expose readers to these concepts and anticipate the more sophisticated analytic methods to be introduced in the following chapters.

Chapter 5 takes a slight detour from classic methods to introduce the reader to state-space models. The focus is on practical questions, such as realization of dynamic systems and controllers, linearization of nonlinear systems, and basic issues that arise when using linear controllers with nonlinear systems. It is from this vantage point that slightly more complex dynamic systems models are introduced, such as a simple pendulum and a pendulum in a cart, as well as a simplified model of a steering car. The simple pendulum model is used in subsequent chapters as the main illustrative example.

Table I.1 Homework problems classified by theme per chapter

Problem theme	Ch. 1	Ch. 2	Ch. 3	Ch. 4	Ch. 5	Ch. 6
DC motor		2.41–2.46	3.95–3.99	4.29–4.37	5.30–5.33	6.30–6.33
Elevator		2.18–2.26	3.62–3.70	4.23–4.28	5.19–5.22	6.14–6.17
Free-fall		2.4–2.7	3.53		5.8–5.13	
Inclined plane	1.10					
Insulin homeostasis					5.49–5.52	6.38–6.41
Mass–spring–damper		2.27–2.33	3.71–3.79		5.23–5.27	6.16–6.19
Modulator			3.50			
One-eighth-car model						6.19–6.22
One-quarter-car model						6.24–6.27
OpAmp circuit		2.38–2.40	3.90–3.94		5.28–5.29	
Orbiting satellite					5.41–5.45	6.37
Pendulum in a cart					5.6	
Population dynamics					5.46–5.48	
RC circuit		2.34–2.35	3.80–3.84			
Rigid body					5.39–5.40	
RLC circuit		2.36–2.37	3.85–3.89			
Rotating machine		2.10–2.17	3.54–3.61	4.20–4.22	5.15–5.18	6.11–6.14
Sample-and-hold			3.51–3.52			
Simple pendulum						6.9–6.10
Sky-diver		2.8–2.9			5.14	
Smith predictor				4.11–4.14		
Steering car					5.7	
Water heater		2.49–2.56	3.100–3.104	4.38–4.43	5.36–5.38	6.34–6.37
Water tank					5.34–5.35	

Chapter 6 takes the reader back to the classic path with an emphasis on control design. Having flirted with second-order systems many times before in the book, the chapter starts by taking a closer look at the time-response of second-order systems and associated performance metrics, followed by a brief discussion on derivative action and the popular proportional–integral–derivative control. It then introduces the root-locus method and applies it to the design of a controller with integral action to the simple pendulum model introduced in the previous chapter.

Chapter 7 brings a counterpoint to the mostly time-domain point of view of Chapter 6 by focusing on frequency-domain methods for control design. After introducing Bode and polar plots, the central issue of closed-loop stability is addressed with the help of the Nyquist stability criterion. The same controller design problem for the simple pendulum is revisited, this time using frequency-domain tools.

An introductory discussion on performance and robustness is the subject of the final chapter, Chapter 8. Topics include Bode's sensitivity integral, robustness analysis using small gain and the circle criterion, and feedforward control and filtering. Application of some of these more advanced tools is illustrated by certifying the performance of the controllers designed for the simple pendulum beyond the guarantees offered by local linearization.

In a typical quarter schedule, with 20 or 30 lectures, the lightweight Chapters 1 and 2 can be covered rather quickly, serving both as a way to review background material and as a means to motivate the reader for the more demanding content to come. Instructors can choose to spend more or less time on Chapter 3 depending on the prior level of comfort with transfer-functions and frequency response and the desired depth of coverage.

Homework problems at the end of Chapters 1 through 3 introduce a variety of examples from various engineering disciplines that will appear again in the following chapters and can be used as effective tools to review background material.

Chapters 4 through 7 constitute the core material of the book. Chapters 5 and 7, especially, offer many opportunities for instructors to select additional topics for coverage in class or relegate to reading, such as discussions on nonlinear analysis and control, a detailed presentation of the argument principle, and more unorthodox topics such as non-minimum-phase systems and stability analysis of systems with delays.

The more advanced material in Chapter 8 can be covered, time permitting, or may be left just for the more interested reader without compromising a typical undergraduate curriculum.

This book contains a total of almost 400 homework problems that appear at the end of each chapter, with many problems spanning across chapters. Table I.1 on page xiv provides an overview of select problems grouped by their motivating theme. Instructors may choose to follow a few of these problems throughout the class. As mentioned previously, many of the problems require students to use MATLAB or a similar computer program. The supplementary MATLAB files provided with this book are a great resource for readers who need to develop their programming skills to tackle these problems.