Cambridge University Press
978-1-107-18619-4 — Geophysical Waves and Flows
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Part I

Introductory Material

In this first part, we get oriented to the general subject matter of the book and the style of approach. The orientation in Chapter 1 includes a discussion of broader aspects, such as mathematical modeling, continuum mechanics, the role of energy in the universe and the apparent need of astronomical bodies to cool as rapidly as possible. Central to our study is the concept of a continuous body, as discussed in § 1.2.

The second chapter provides a bit more introductory detail, including the reference coordinate system, a definition of waves and flows, a summary of the scope of the book and brief introductions to a number of related issues.

1
Introduction

This monograph is a survey of – and introduction to – the ways in which the atmosphere, oceans and various parts of the so-called solid Earth can deform and move. The most common type of deformation is a *wave* and the most common type of motion is *flow*. Our goals are to quantify and understand these deformations and motions. In order to achieve these goals, we will need to develop an appropriate set of procedures and tools.

An appropriate procedure is *mathematical modeling*, which consists of three steps:

- *Formulation* of a mathematical model of a physical system. This is the crucial element in the procedure. If the model does not mimic the relevant physical processes, it will fail to yield useful results.
- *Solution* of the mathematical problem. This is the part stressed in most mathematics courses, but in fact is the most routine aspect of the procedure.
- *Interpretation* of the solution. This is the payoff; solution of a well-conceived model should yield new physical insight and make testable predictions.

We won't be following these steps formally, instead they will come naturally as we investigate each type of wave and flow.

Our mathematical models will treat the physical system (that is, the portion of Earth under consideration) as a continuous body, using the mathematical apparatus of *continuum mechanics*. The concept of a continuous body is discussed in § 1.2. Continuum mechanics consists of three fundamental elements:

- *Kinematics* describes how bodies can move and deform. For example, it limits the types of deformation which leave a body competent (i.e., unfractured).
- *Dynamics* explains why bodies move and deform. They do so in response to external and internal forces, within the context of Newton's second law.
- *Rheology* quantifies the kinematic response to dynamic forces, taking into account the material properties of the body under investigation. Two identically shaped bodies, composed of water and of steel, respond quite differently to the same applied forces. Rheology encompasses the change of volume of a body induced by a change in pressure (*compressive rheology*) and the change of shape caused by a change in deviatoric stresses (*shear rheology*).

The nature of the physical system under consideration is determined primarily by its shear rheology. While continuous bodies can exhibit a wide range of possible rheological behaviors, we will consider only a few relatively simple types: linear elastic solid and three types of fluid:[1] inviscid, Newtonian and power-law.[2] Other exotic states of matter, such as plasmas (found in the Van Allen belts) and thixotropic solids (such as clays) having history-dependent rheology, will not be included in our survey.

The emphasis throughout this monograph will be on the fundamentals of the processes being modeled with mathematical complexity minimized where possible. In that spirit, although we live in a three-dimensional world, many physical processes are effectively two or even one dimensional. The fundamental concepts will be introduced in general form, but will be applied only in simple situations.

At various points in the text concepts are introduced which may be unfamiliar to some readers. As an aid to these readers, many of these concepts are explained in various fundaments, found in the appendices.

1.1 A Broader Perspective

Before delving into details of geophysical waves and flows in the following chapters, it may be helpful to place them within a broader cosmological context by briefly discussing the related concepts of energy and cooling.

1.1.1 Energy

Energy is the essence of the universe, with much of it in a frozen phase called mass. It is possible that the total energy of the universe is zero, with the (negative) gravitational energy equaling all other (positive) forms of energy. The study of energy is concerned with both its storage within a body and its transfer between bodies. Broadly speaking, there are four forms of energy:

- *Energy associated with the short-range nuclear forces*. This energy is quantified by the rest mass, M, which is related to the energy E by Einstein's celebrated equation: $M = E/c^2$, where c is the speed of light. Barring nuclear reactions, this energy is in a "frozen" state and is considered separately from all other forms of energy. Kinematics is the study of the behavior of mass within a continuous body; see Chapter 3 and Appendix C. The deformation of a continuous body is limited by the equation of conservation of mass, also known as the continuity equation; see § 3.3.1 and § 3.4.1.
- *Electromagnetic energy*. The study of electromagnetism is called *electrodynamics*. Relatively little energy is stored as electromagnetic energy, but electromagnetic photons very efficiently transfer energy between bodies separated in space, a process referred to

[1] We learned in grade school that there are three types of matter: solid, liquid and gas. However, liquids and gases are both fluids, distinguished primarily by their compressibility.
[2] The rate of flow of power-law fluid is proportional to the deviatoric stress raised to an exponent or power.

as *radiative transfer*. However, we will be concerned with this mode only peripherally, with Earth receiving heat in the form of solar electromagnetic radiation, primarily in and near the visible range of frequencies, and casting heat off to outer space, primarily in the infrared. Electromagnetic forces affect waves and flows within Earth's core and in the ionosphere, but these regions are not included in the scope of this book.

- *Energy stored in disorganized motions of the atoms comprising a body*. *Thermodynamics* is the study of this form of energy; see Appendix E. Commonly the term "energy" is taken to mean this form, but properly it should be called thermodynamic energy.
- *Energy stored in the organized positions and motions of macroscopic bodies*. This includes kinetic and potential energy (including gravitational potential energy). These energies are relative quantities, dependent on the position and velocity of the observer. Since they are not intrinsic quantities, kinetic and potential energy are studied separately from thermodynamics, in a subject area called *dynamics*; see Chapter 4. The equation of conservation of momentum[3] is a primary focus of dynamics; see § 4.6.

1.1.2 Cooling

The visible Universe is composed of stars, planets and a myriad of smaller objects each of which appears to be attempting to *cool as rapidly as possible*. The predominant mode of cooling for celestial bodies is electromagnetic radiation, and Earth is no exception; it cools by radiation to outer space. A small fraction of the radiation emitted by our Sun is absorbed by Earth. This solar insolation is a dominant factor in Earth's heat balance. Earth in turn re-radiates this to space, along with a much smaller amount of heat from the interior, as the bulk of Earth slowly cools.

Portions of Earth are in motion because these flows aid in the cooling of Earth. For example, most of the solar insolation strikes the tropics and this heat is re-radiated to space more efficiently by the poleward transport of heat via atmospheric and oceanic circulations. In addition, the bulk of Earth (that is, the mantle and core) cool more rapidly via convection within the mantle than by conduction. All the wave motions and flows on Earth are caused either primarily or incidentally by the need for Earth to cool as rapidly as possible.

1.2 The Concept of a Continuous Body

Perhaps the first step in defining a continuous body is to define what is meant by a body. A physical body is a group of atoms or molecules held together by inter-atomic forces, typically electro-chemical in origin, or confined by impenetrable walls. In considering the collection of atoms as a body, we abandon the possibility of describing the behavior of each individual atom, and consider only the collective behavior of relatively large numbers of atoms. Given that the number of atoms in a body of hand-specimen size is on the order of 10^{26}, whereas the largest computers currently available (in 2017) can model fewer than

[3] Momentum is the gradient of energy.

10^{12} particles, this is an eminently reasonable approach. Even if we could model the behavior of each atom, we must question the motivation for doing so: what sense could we make of that large amount of information? As the old saying goes, in such a case, "we couldn't see the forest for the trees".

We wish to describe and predict the behavior of bodies as they move and deform at a level of accuracy sufficient for scientific curiosity and practical need. With this in mind, we shall treat the body as a macroscopic continuum, ignoring its microscopic atomic nature. In this approach, the smallest volume we shall consider, called a *particle*, contains a large number of atoms.[4] This number is sufficiently large that atomic fluctuations in position and velocity are not evident. Given the large value of Avogadro's number (the number of atoms in a mole of substance[5]), we can do this with some confidence. For example, suppose we wish to model the large-scale dynamics of the oceans. If we chose our particle to be the size of a drinking glass (which is far smaller than necessary) the number of molecules of water in each particle would be greater than the number of such particles in all the oceans.

In the context of macroscopic continuum mechanics, the particles that comprise a continuous body are of infinitesimal size and the locations of the particles are quantified by the positions of a continuum of points as a function of time, e.g., $\mathbf{x}(t)$, where $\mathbf{x}$ is the position vector, measured using some reference coordinate system, and t is time; see §2.1.

In order to quantify deformation, we must consider localized groups of particles, called *parcels*. A parcel consists of all the particles within the neighborhood[6] of a given particle. Each particle is a member of a large number of parcels. Particles have no discernible size or shape, while parcels and bodies do.

To summarize:

- a *continuous body* is composed of a vast number of particles;
- a *particle* is composed of a vast number of atoms or molecules and is identified by its reference position or current location;
- a *point* is a position in three-dimensional space used to identify or locate particles;[7] and
- a *parcel* is a tiny body composed of all particles in the neighborhood of a given point or particle.

Displacements of the particles comprising a body relative to a reference state are constrained by kinematics. The displacement of a body consists of *rigid-body motion* plus *deformation* and *flow*. Rigid body motion consists of *translation* and *rotation*, while deformation and flow consist of change of volume and change of shape, as illustrated in Figure 1.1.

[4] Except when considering the behavior of an ideal gas, when a particle is an atom or a molecule; see Appendix E.8.4.
[5] See Appendix E.2.
[6] A *neighborhood* is a small volume surrounding a specified point.
[7] See § 2.1.

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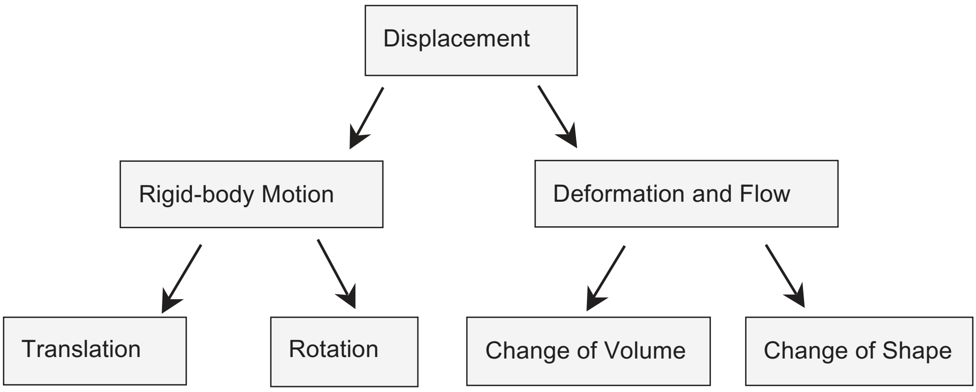

Figure 1.1 Components of displacement of a continuous body.

The motions of bodies are quantified by use of a reference coordinate system. We will employ a Cartesian reference system.[8] Large-scale translation and rotation (that is, rigid-body motion) may be removed from the kinematic description by using a coordinate system which moves with the body under investigation. While this simplifies kinematics, it complicates dynamics. In particular, the use of a non-inertial coordinate system introduces so-called virtual forces into the force balance.[9]

1.2.1 Composition of a Continuous Body

A particle may consist of various types of atoms and molecules in differing phases, with each type or phase being called a *constituent*.[10] The relative magnitudes of the constituents will be represented as mass fractions, treated as functions of position and time. Barring change of phase, the composition of a particle can change only by diffusion. Diffusion in solid bodies is an exceedingly slow process (as can be verified by the banding seen in geodes) and can be neglected. If the body is a fluid (a liquid or gas), adjoining particles can exchange atoms and molecules much more readily and diffusion can be an important process.

A body may be characterized by the chemical composition and state of matter at a point and by the physical structure in the neighborhood of a point. If the composition and state are the same at all points in a body, it is said to be *homogeneous*. If the structure of the body is the same in all directions, the body is said to be *isotropic*.

Transition

In this first chapter, we have discussed the broad goals of this book, introduced the concept of a continuous body and discussed the possible displacements of its constituent particles.

[8] Other possible reference systems are briefly discussed in Appendix A.3.
[9] See Appendix C.1.
[10] The chemically distinct materials comprising a body are called *components*. The phases of a component are differing constituents.

These displacements will be analyzed in Chapter 3, which deals with kinematics – how a continuous body can move and deform. Before delving into the study of kinematics, we need to establish a basis for quantification of deformation and flow, using a *reference coordinate system*. This is described in Chapter 2. This chapter also provides an orientation to the main topic of this book, waves and flows, explains the format of the book and briefly introduces a number of ancillary topics.