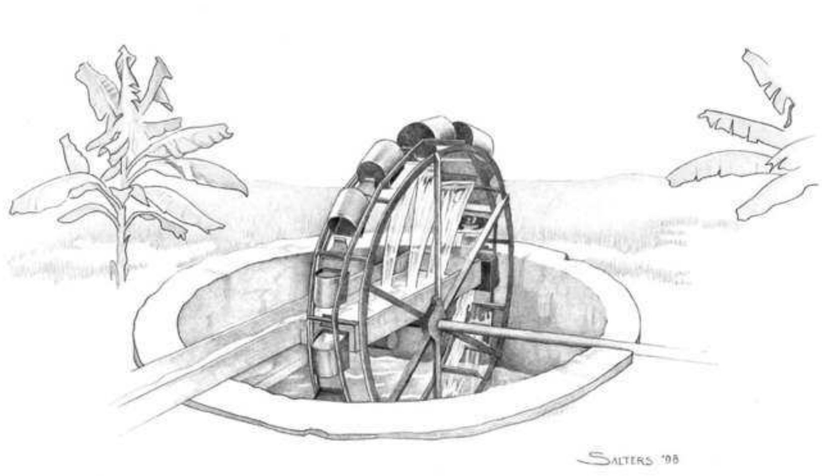
Network Flow Algorithms

Network flow theory has been used across a number of disciplines, including theoretical computer science, operations research, and discrete math, to model not only problems in the transportation of goods and information but also a wide range of applications from image segmentation problems in computer vision to deciding when a baseball team has been eliminated from contention.

This graduate text and reference presents a succinct, unified view of a wide variety of efficient combinatorial algorithms for network flow problems, including many results not found in other books. It covers maximum flows, minimum-cost flows, generalized flows, multicommodity flows, and global minimum cuts and also presents recent work on computing electrical flows along with recent applications of these flows to classical problems in network flow theory.

DAVID P. WILLIAMSON is a professor at Cornell University in the School of Operations Research and Information Engineering. He has won several awards for his work in discrete optimization, including the 2000 Fulkerson Prize, sponsored by the American Mathematical Society and the Mathematical Programming Society. His previous book, *The Design of Approximation Algorithms*, coauthored with David B. Shmoys, won the 2013 INFORMS Lanchester Prize. He has served on several editorial boards, and was editor-in-chief of the *SIAM Journal on Discrete Mathematics*. He is a Fellow of the ACM and of SIAM.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Preface

> I have gathered a posy of other men's flowers, and nothing but the thread that binds them is mine own.
>
> – Montaigne

Any new book on network flow would seem to need to justify its existence, since the definitive book on the topic has perhaps already been written. I am referring to the magisterial *Network Flows: Theory, Algorithms, and Applications*, by Ahuja, Magnanti, and Orlin [4], written by some of the premier researchers in the theory and practice of efficient network flow algorithms, and published in 1993; I will refer to the book as AMO, using the initials of its authors. The late 1980s and early 1990s were a golden era for research in combinatorial, polynomial-time algorithms for network flow problems, and not only does AMO discuss most of the work done during this period, it also gives an extensive overview of the entire area of network flows and is full of applications of network flow theory to practical problems. So why another book on the topic? I offer three reasons.

The first is a matter of focus. It is hard to be both definitive and succinct, as I know from having tried to write a definitive book on another topic [206]. AMO is certainly the former; it is my aim here to be the latter. In this volume, I am concerned primarily with combinatorial, polynomial-time algorithms for network flow problems and their analyses. The material for this book comes from having taught several iterations of a graduate-level course in network flow algorithms at Cornell University's School of Operations Research and Information Engineering. The students were primarily in operations research and computer science, but there were also some in electrical and civil engineering. Thus I know from experience that with a bit of selection, the bulk of the material in this book can be taught in a semester-long course. Additionally, because the book is a result of a course, the results covered are ones that I was able to teach successfully in a single lecture. As a consequence, results that are either too long or too complex to cover in a single lecture are not included here. I do not say much concerning other parts of network flow theory, such as applications or algorithms without polynomially bounded running times. Here the existence of AMO is a boon; the interested reader is welcome to refer to that volume for the parts of network flow theory that are not covered in this book.

The second reason is to provide some coverage that AMO does not. Although several of the algorithms covered here for the maximum flow problems and minimum-cost circulation problems are also covered by AMO, there are some important exceptions. As mentioned above, although the late 1980s and early 1990s were a golden era for research in network flow algorithms, there has continued to be work in the area in the last 25 years, which AMO does not cover. One notable example is the 1998 paper of Goldberg and Rao [90], giving what until recently was the theoretically fastest known algorithm for the maximum flow problem. Another is the 1991 algorithm of Wallacher [201] for the minimum-cost circulation problem; the algorithm has a relatively simple analysis. Furthermore, interesting polynomial-time combinatorial algorithms for several types of flow problems were emerging just as AMO went to press, and are not covered there; I am thinking primarily of algorithms for global minimum cut, generalized maximum flow, and multicommodity flow problems. In recent years, specializations of interior-point methods to network flow problems have resulted in still faster algorithms; while these algorithms are not combinatorial and thus do not fall in the scope of this book, I include a few of these results connected to the classical topic of electrical flows.

The third reason is that in the end, I could not really help myself. My main area of research interest is combinatorial, polynomial-time algorithms, but with one exception [173], none of my work has been on network flow problems. So I can say as an unbiased outside observer that the area is one with truly beautiful and useful algorithmic ideas that build on each other in a very aesthetically pleasing way. Following the Montaigne quotation above, my goal in writing this book has been one of selection and arrangement to try to bring out as best I can the beauty that is already inherent in the algorithms and analysis of others; I hope the reader enjoys the resulting bouquet as much as I do.

David P. Williamson
Ithaca, New York
January 2019

Acknowledgments

> Beggar that I am, I am even poor in thanks, but I thank you; and sure, dear friends, my thanks are too dear a halfpenny.
>
> – William Shakespeare, *Hamlet*, Act II, Scene II

This book had its genesis in an advanced algorithms class I taught at Stanford University in Spring 2003 (CS 361B). The section on network flow algorithms from that class was expanded into a full-semester course in Spring 2004 when I moved to Cornell University (ORIE 633). Since then I've taught several iterations of the class (Spring 2004, Fall 2007, Fall 2012, and Fall 2015), and tried to make the material into a more cohesive whole. I became familiar with the material on electrical flows when I taught a spectral graph theory and algorithms course in Fall 2016. I owe many thanks to my students from these courses for asking questions and forcing me to clarify my presentation of the material and the exercises that were part of their problem sets.

My first exposure to this subject came when I was a student at MIT via courses from Ron Rivest, David Shmoys, and Michel Goemans. Some of the material they presented, such as the Goldberg-Tarjan minimum-mean cycle canceling algorithm, was brand new at the time. I am grateful for their clear and exciting presentations that started my interest in this area.

Over the years I have learned a good deal from the researchers who developed the material presented in this book, including András Benczúr, Joseph Cheriyan, Lisa Fleischer, Hal Gabow, Andrew Goldberg, Don Goldfarb, Nick Harvey, Alan Hoffman, David Karger, Matt Levine, Tom McCormick, Aleksandr Mądry, Kurt Mehlhorn, Jim Orlin, Satish Rao, David Shmoys, Martin Skutella, Dan Spielman, Cliff Stein, Éva Tardos, Bob Tarjan, Laci Vegh, and Kevin Wayne. I am grateful to them all for their development of this beautiful area of work, and their willingness to share it with me. I apologize to those I will have inevitably left off the list via oversight.

I am indebted to those who wrote excellent books in this area before me that served as references for me, especially those of Ahuja, Magnanti, and Orlin [4], Ford and Fulkerson [66], and Tarjan [192], as well as more general references in algorithms and combinatorial optimization, such as those by Cook, Cunningham, Pulleyblank, and Schrijver [44], Cormen, Leiserson, Rivest, and Stein [45], Kleinberg and Tardos [134], Korte and Vygen [135], and Schrijver [177].

Several people took the time to look at my manuscript and pointed out various errors and made useful suggestions. I wish to thank Joseph Cheriyan, Jakob Degen, Daniel Fleischman, Daniel Freund, Agustin Garcia, Sam Gutekunst, Harsh Parekh, Glenn Sun, and Jessica Xu. Rajiv Gandhi helped me by finding several students willing to read through a draft of the manuscript.

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This book was written at Cornell University and while I was on sabbatical at the Simons Institute on the Theory of Computing at the University of California, Berkeley. I am grateful to both institutions for their support.

Though I acknowledge the help of so very many people, all mistakes and misunderstandings that remain in this volume are mine alone.

Additional materials related to the book (such as contact information and errata) can be found at the website www.networkflowalgs.com.

Finally, I wish to thank my children, Abigail, Daniel, and Ruth, and my wife Ann especially: without her encouragement to finish this book, it would not have been completed.

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