

## Contents

---

|  |                |
|--|----------------|
| <i>Preface</i>                         | <i>page xi</i> |
| <b>1 Statistical Preliminary</b>       | 1              |
| 1.1 General Linear Models              | 1              |
| 1.2 Logistic Regression                | 6              |
| 1.3 Random Fields                      | 9              |
| 1.4 Statistical Inference on Fields    | 16             |
| <b>2 Brain Network Nodes and Edges</b> | 27             |
| 2.1 Brain Templates                    | 27             |
| 2.2 Brain Parcellations                | 28             |
| 2.3 Deterministic Connectivity         | 34             |
| 2.4 Probabilistic Connectivity         | 46             |
| 2.5 Parcellation-Free Brain Network    | 50             |
| 2.6 Structural Covariates              | 55             |
| <b>3 Graph Theory</b>                  | 61             |
| 3.1 Trees and Graphs                   | 61             |
| 3.2 Minimum Spanning Trees             | 62             |
| 3.3 Node Degree                        | 65             |
| 3.4 Shortest Path Length               | 70             |
| 3.5 Clustering Coefficient             | 71             |
| 3.6 Small-Worldness                    | 72             |
| 3.7 Fractal Dimension                  | 73             |
| <b>4 Correlation Networks</b>          | 76             |
| 4.1 Pearson Correlations               | 76             |
| 4.2 Partial Correlations               | 78             |
| 4.3 Averaging Correlations             | 79             |

|           |                                       |     |
|-----------|---------------------------------------|-----|
| 4.4       | Correlation as Metric                 | 85  |
| 4.5       | Statistical Inference on Correlations | 87  |
| 4.6       | Cosine Series Representation          | 89  |
| 4.7       | Correlating Functional Signals        | 103 |
| 4.8       | Thresholding Correlation Networks     | 106 |
| <b>5</b>  | <b>Big Brain Network Data</b>         | 108 |
| 5.1       | Big Data                              | 108 |
| 5.2       | Sparsity                              | 112 |
| 5.3       | Hierarchy                             | 114 |
| 5.4       | Computing Large Correlation Matrices  | 120 |
| 5.5       | Online Algorithms                     | 123 |
| <b>6</b>  | <b>Network Simulations</b>            | 129 |
| 6.1       | Multivariate Normal Distributions     | 129 |
| 6.2       | Multivariate Linear Models            | 136 |
| 6.3       | Mixed Effects Models                  | 143 |
| 6.4       | Simulating Dependent Images           | 149 |
| 6.5       | Dependent Correlation Networks        | 152 |
| <b>7</b>  | <b>Persistent Homology</b>            | 156 |
| 7.1       | Simplicial Homology                   | 157 |
| 7.2       | Morse Filtrations                     | 163 |
| 7.3       | Graph Filtrations                     | 168 |
| 7.4       | Betti Plots                           | 173 |
| <b>8</b>  | <b>Diffusions on Graphs</b>           | 180 |
| 8.1       | Diffusion as a Cauchy Problem         | 180 |
| 8.2       | Finite Difference Method              | 184 |
| 8.3       | Laplacian on Planner Graphs           | 188 |
| 8.4       | Graph Laplacian                       | 189 |
| 8.5       | Fiedler Vectors                       | 193 |
| 8.6       | Heat Kernel Smoothing on Graphs       | 196 |
| 8.7       | Laplace Equation                      | 204 |
| <b>9</b>  | <b>Sparse Networks</b>                | 207 |
| 9.1       | Why Sparse Models?                    | 207 |
| 9.2       | Sparse Likelihood                     | 210 |
| 9.3       | Sparse Correlation Network            | 213 |
| 9.4       | Partial Correlation Network           | 222 |
| <b>10</b> | <b>Brain Network Distances</b>        | 226 |
| 10.1      | Matrix Norms                          | 227 |

*Contents*

ix

|  |     |
|--|-----|
| 10.2 Bottleneck Distance                               | 229 |
| 10.3 Gromov–Hausdorff Distance                         | 231 |
| 10.4 Kolmogorov–Smirnov Distance                       | 233 |
| 10.5 Performance Analysis                              | 236 |
| 10.6 Comparisons on Modules                            | 238 |
| 10.7 Hypernetworks                                     | 241 |
| <b>11 Combinatorial Inferences for Networks</b>        | 246 |
| 11.1 Permutation Test                                  | 246 |
| 11.2 Exact Combinatorial Inference                     | 253 |
| 11.3 Bootstrap   | 263 |
| <b>12 Series Expansion of Connectivity Matrices</b>    | 269 |
| 12.1 Spectral Decomposition                            | 269 |
| 12.2 Iterative Residual Fitting                        | 271 |
| 12.3 Spectral Decomposition with Different Bases       | 278 |
| 12.4 Spectral Permutation                              | 279 |
| 12.5 Karhunen–Loëve Expansion                          | 280 |
| 12.6 Vandermonde Matrix Expansion                      | 283 |
| 12.7 The Space of Positive Definite Symmetric Matrices | 287 |
| <b>13 Dynamic Network Models</b>                       | 292 |
| 13.1 Dynamic Causal Model                              | 293 |
| 13.2 Dynamic Time Series Models                        | 295 |
| 13.3 Persistent Homological Dynamic Network Model      | 298 |
| <i>Bibliography</i>                                    | 302 |
| <i>Index</i>   | 326 |