

## Brain Network Analysis

This tutorial reference serves as a coherent overview of various statistical and mathematical approaches used in brain network analysis, where modeling the complex structures and functions of the human brain often poses many unique computational and statistical challenges. This book fills a gap as a textbook for graduate students while simultaneously articulating important and technically challenging topics. Whereas most available books are graph theory centric, this text introduces techniques arising from graph theory and expands to include other advanced models in its discussion on network science, regression, and algebraic topology. Links are included to the sample data and codes used in generating the book's results and figures, helping empower methodological understanding in a manner immediately usable to both researchers and students.

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To my parents  
for their endless love



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## Preface

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Brain network analysis is an emerging field that utilizes various noninvasive brain imaging modalities such as magnetic resonance imaging (MRI), functional MRI (fMRI), positron emission tomography (PET), diffusion tensor imaging (DTI), and electroencephalography (EEG) in mapping out the four-dimensional (4D) spatiotemporal dynamics of the human brain networks in both normal and clinical populations at the macroscopic level. There has been substantial progress in the past decade on this topic. A major challenge in the field is caused by the massive amount of nonstandard high-dimensional network data that are difficult to analyze using available standard techniques. This requires new computational approaches and solutions.

The main goals of this book are to provide a coherent overview of various statistical and mathematical approaches used in brain network analysis to a wide range of researchers and students, and to articulate important yet technically challenging topics further. It is hoped that the book presents the coherent mathematical treatment of underlying methods. The book is mainly focused on methodological issues beyond widely used graph theory-based approaches. We wish to provide methodological understanding in a manner immediately usable to researchers and students. Concepts and methods are illustrated with brain imaging applications and examples. Some of the brain network data sets along with MATLAB and R codes used in the book can be downloaded from the author's website. The web links are provided in appropriate places. By making some of the data and codes available, we tried to make the book more accessible to a wide range of readers.

Although I am indebted to many colleagues and students in writing this book, I would particularly like to thank the following individuals, in no particular order. Richard Davidson, Andrew Alexander, Seth Pollak, Hill Goldsmith of the University of Wisconsin–Madison; David Zald of Vanderbilt University; and Benjamin Lahey of the University of Chicago provided various

brain imaging data used in illustrating the methods. Hyekyoung Lee of Seoul National University and Yuan Wang of University of South Carolina helped me write chapters related to persistent homology and topological distances. Hernando Ombao of the King Abdullah University of Science and Technology and Dustin Pluta of the University of California–Irvine helped me write chapters related to the dynamic network models. Andrey Gritsenko of the University of Wisconsin–Madison performed some of basic image processing on the resting-state fMRI from Human Connectome Project data and helped compile the list of Automatic Anatomical Labeling (AAL) parcellation. Although most figures are produced by myself using MATLAB, some figures are generated by my current and former students, postdocs, and colleagues. Such figures are identified in figure captions and the proper credits are given. I am also indebted to Fred Boehm of the University of Wisconsin–Madison and Feng Liu of Harvard University for proofreading a few chapters.