

Index of Symbols

Σ, Γ, Λ	Typical names for alphabets (finite and nonempty sets whose elements are viewed as symbols).	1
\mathbb{C}^Σ	The complex Euclidean space of functions from an alphabet Σ to the complex numbers. (Equivalently, the complex Euclidean space of vectors having entries indexed by Σ .)	2
$\mathcal{W}, \mathcal{X}, \mathcal{Y}, \mathcal{Z}$	Typical names for complex Euclidean spaces.	2
$\langle u, v \rangle$	The inner product between vectors u and v .	3
$\ u\ $	The Euclidean norm of a vector u .	4
$\mathcal{S}(\mathcal{X})$	The unit sphere in a complex Euclidean space \mathcal{X} .	4
$\ u\ _p$	The p -norm of a vector u .	4
$\ u\ _\infty$	The ∞ -norm of a vector u .	4
$u \perp v, u \perp \mathcal{A}$	Indicates that a vector u is orthogonal to a vector v , or to every element of a set of vectors \mathcal{A} .	4
e_a	An element of the vector standard basis, corresponding to a symbol (or index) a .	5

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$\Sigma_1 \sqcup \cdots \sqcup \Sigma_n$	The disjoint union of alphabets $\Sigma_1, \dots, \Sigma_n$.	5
$\mathcal{X}_1 \oplus \cdots \oplus \mathcal{X}_n$	The direct sum of complex Euclidean spaces $\mathcal{X}_1, \dots, \mathcal{X}_n$.	5
$u_1 \oplus \cdots \oplus u_n$	The direct sum of vectors u_1, \dots, u_n .	5
$\mathcal{X}_1 \otimes \cdots \otimes \mathcal{X}_n$	The tensor product of complex Euclidean spaces $\mathcal{X}_1, \dots, \mathcal{X}_n$.	6
$u_1 \otimes \cdots \otimes u_n$	The tensor product of vectors u_1, \dots, u_n .	6
$\mathcal{X}^{\otimes n}$	The n -fold tensor product of a complex Euclidean space \mathcal{X} with itself.	7
$u^{\otimes n}$	The n -fold tensor product of a vector u with itself.	7
\mathbb{R}^Σ	The real Euclidean space of functions from an alphabet Σ to the real numbers. (Equivalently, the real Euclidean space of vectors having entries indexed by Σ .)	7
$L(\mathcal{X}, \mathcal{Y})$	Space of all linear operators mapping a complex Euclidean space \mathcal{X} to a complex Euclidean space \mathcal{Y} .	8
$E_{a,b}$	An element of the operator standard basis, corresponding to symbols (or indices) a and b .	10
\bar{A}, \bar{u}	The entry-wise complex conjugate of an operator A or a vector u .	10
A^\top, u^\top	The transpose of an operator A or a vector u .	10
A^*, u^*	The adjoint of an operator A or a vector u .	11
$\ker(A)$	The kernel of an operator A .	11
$\text{im}(A)$	The image of an operator A .	11
$\text{rank}(A)$	The rank of an operator A .	12

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$A_1 \otimes \cdots \otimes A_n$	The tensor product of operators A_1, \dots, A_n .	13
$A^{\otimes n}$	The n -fold tensor product of an operator A with itself.	14
$L(\mathcal{X})$	Space of linear operators mapping a complex Euclidean space \mathcal{X} to itself.	14
$\mathbb{1}$	The identity operator; denoted $\mathbb{1}_{\mathcal{X}}$ when it is helpful to indicate that it acts on a complex Euclidean space \mathcal{X} .	14
X^{-1}	The inverse of an invertible square operator X .	14
$\text{Tr}(X)$	The trace of a square operator X .	15
$\langle A, B \rangle$	The inner product of operators A and B .	15
$\text{Det}(X)$	The determinant of a square operator X .	15
$\text{Sym}(\Sigma)$	The set of permutations, or bijective functions, of the form $\pi : \Sigma \rightarrow \Sigma$.	15
$\text{sign}(\pi)$	The sign, or parity, of a permutation π .	15
$\text{spec}(X)$	The spectrum of a square operator X .	16
$[X, Y]$	The Lie bracket of square operators X and Y .	17
$\text{comm}(\mathcal{A})$	The commutant of a set \mathcal{A} of square operators.	17
$\text{Herm}(\mathcal{X})$	The set of Hermitian operators acting on a complex Euclidean space \mathcal{X} .	17
$\text{Pos}(\mathcal{X})$	The set of positive semidefinite operators acting on a complex Euclidean space \mathcal{X} .	17
$\text{Pd}(\mathcal{X})$	The set of positive definite operators acting on a complex Euclidean space \mathcal{X} .	18

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$D(\mathcal{X})$	The set of density operators acting on a complex Euclidean space \mathcal{X} .	18
$\text{Proj}(\mathcal{X})$	The set of projection operators acting on a complex Euclidean space \mathcal{X} .	18
$\Pi_{\mathcal{V}}$	The projection operator whose image is \mathcal{V} .	18
$U(\mathcal{X}, \mathcal{Y})$	The set of isometries mapping a complex Euclidean space \mathcal{X} to a complex Euclidean space \mathcal{Y} .	18
$U(\mathcal{X})$	The set of unitary operators acting on a complex Euclidean space \mathcal{X} .	18
$\text{Diag}(u)$	The diagonal square operator whose diagonal entries are described by the vector u .	19
$\lambda(H)$	The vector of eigenvalues of a Hermitian operator H .	20
$\lambda_k(H)$	The k -th largest eigenvalue of a Hermitian operator H .	20
$X \geq Y$ or $Y \leq X$	Indicates that $X - Y$ is positive semidefinite, for Hermitian operators X and Y .	21
$X > Y$ or $Y < X$	Indicates that $X - Y$ is positive definite, for Hermitian operators X and Y .	21
$T(\mathcal{X}, \mathcal{Y})$	The space of linear maps from $L(\mathcal{X})$ to $L(\mathcal{Y})$, for complex Euclidean spaces \mathcal{X} and \mathcal{Y} .	21
Φ^*	The adjoint of a map $\Phi \in T(\mathcal{X}, \mathcal{Y})$.	21
$\Phi_1 \otimes \cdots \otimes \Phi_n$	The tensor product of maps Φ_1, \dots, Φ_n .	22
$\Phi^{\otimes n}$	The n -fold tensor product of a map Φ with itself.	22
$\mathbb{1}_{L(\mathcal{X})}$	The identity map acting on $L(\mathcal{X})$.	22

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$\text{Tr}_{\mathcal{X}}$	The partial trace over a complex Euclidean space \mathcal{X} .	22
$\text{CP}(\mathcal{X}, \mathcal{Y})$	The set of completely positive maps of the form $\Phi \in \mathbb{T}(\mathcal{X}, \mathcal{Y})$.	23
$\text{vec}(A)$	The vec mapping applied to an operator A .	23
\sqrt{P}	The square root of a positive semidefinite operator P .	27
$s(A)$	The vector of singular values of an operator A .	28
$s_k(A)$	The k -th largest singular value of an operator A .	28
A^+	The Moore–Penrose pseudo-inverse of an operator A .	30
$\ A\ _p, \ A\ _\infty$	The Schatten p -norm or ∞ -norm of an operator A .	32
$\ A\ $	The spectral norm of an operator A . Equivalent to the Schatten ∞ -norm of A .	33
$\ A\ _2$	The Frobenius norm of an operator A . Equivalent to the Schatten 2-norm of A .	33
$\ A\ _1$	The trace norm of an operator A . Equivalent to the Schatten 1-norm of A .	34
$\nabla f(x)$	The gradient vector of a function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at a vector $x \in \mathbb{R}^n$.	37
$(Df)(x)$	The derivative of a (differentiable) function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ at a vector $x \in \mathbb{R}^n$.	37
$\mathcal{B}(\mathcal{X})$	The unit ball in a complex Euclidean space \mathcal{X} .	38
$\text{Borel}(\mathcal{A})$	The collection of all Borel subsets of a subset \mathcal{A} of a real or complex vector space.	38

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$\int f(x) \, d\mu(x)$	The integral of a function f with respect to a Borel measure μ .	40
$\text{cone}(\mathcal{A})$	The cone generated by a subset \mathcal{A} of a real or complex vector space.	43
$\mathcal{P}(\Sigma)$	The set of probability vectors with entries indexed by an alphabet Σ .	44
$\text{conv}(\mathcal{A})$	The convex hull of a subset \mathcal{A} of a real or complex vector space.	44
$E(X)$	The expected value of a random variable X .	48
$\Gamma(\alpha)$	The value of the Γ -function at α .	52
γ_n	The standard Gaussian measure on \mathbb{R}^n .	52
X, Y, Z	Typical names for registers.	59
(X_1, \dots, X_n)	The compound register formed from registers X_1, \dots, X_n .	59
$\omega_{\mathcal{V}}$	The flat state proportional to the projection onto the subspace \mathcal{V} .	64
$\rho[X_1, \dots, X_n]$	The reduction of a state ρ to registers X_1, \dots, X_n .	69
$C(\mathcal{X}, \mathcal{Y})$	The set of all channels mapping $L(\mathcal{X})$ to $L(\mathcal{Y})$.	73
$C(\mathcal{X})$	The set of channels mapping $L(\mathcal{X})$ to itself.	73
$K(\Phi)$	The natural representation of a map Φ .	77
$J(\Phi)$	The Choi representation of a map Φ .	78
Ω or $\Omega_{\mathcal{X}}$	Typical name for the completely depolarizing channel acting on $L(\mathcal{X})$.	93
Δ or $\Delta_{\mathcal{X}}$	Typical name for the completely dephasing channel acting on $L(\mathcal{X})$.	94

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$F(P, Q)$	The fidelity between positive semidefinite operators P and Q .	139
$B(P, Q \mu)$	The Bhattacharyya coefficient of the nonnegative real vectors obtained by applying a measurement μ to positive semidefinite operators P and Q .	153
$F(\Phi, P)$	The mapping fidelity of a map Φ with respect to a positive semidefinite operator P .	160
W or $W_{\mathcal{X}}$	Typical name used to refer to the swap operator acting on a bipartite tensor product space $\mathcal{X} \otimes \mathcal{X}$.	165
$\ \Phi\ _1$	The induced trace norm of a map Φ .	167
$\ \ \Phi\ \ _1$	The completely bounded trace norm of a map Φ .	170
$\mathcal{N}(X)$	The numerical range of a square operator X .	180
$F_{\max}(\Psi_0, \Psi_1)$	The maximum output fidelity of positive maps Ψ_0 and Ψ_1 .	185
\mathbb{Z}_n	The ring of integers modulo n .	212
$W_{a,b}$	A discrete Weyl operator acting on $\mathbb{C}^{\mathbb{Z}_n}$, for $a, b \in \mathbb{Z}_n$.	212
$\sigma_x, \sigma_y,$ and σ_z	The Pauli operators.	213
$A \odot B$	The entry-wise product of operators A and B .	219
V_π	Permutation operator corresponding to the permutation π .	234
$v \prec u$	Indicates that u majorizes v , for real vectors u and v .	235

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$r(u)$	The vector obtained by sorting the entries of a real vector u from largest to smallest.	236
$r_k(u)$	The k -th largest entry of a real vector u .	236
$Y \prec X$	Indicates that X majorizes Y , for Hermitian operators X and Y .	241
S_n	The symmetric group on n symbols. Equivalent to $\text{Sym}(\{1, \dots, n\})$.	243
$H(u)$	The Shannon entropy of a vector u with nonnegative real number entries.	251
$H(\mathbf{X})$	The Shannon entropy of the probabilistic state of a classical register \mathbf{X} , or the von Neumann entropy of the quantum state of a register \mathbf{X} .	252, 266
$H(\mathbf{X}_1, \dots, \mathbf{X}_n)$	Refers to the Shannon entropy or von Neumann entropy of the compound register $(\mathbf{X}_1, \dots, \mathbf{X}_n)$.	252, 266
$D(u v)$	The relative entropy of u with respect to v , for vectors u and v with nonnegative real number entries.	252
$H(\mathbf{X} \mathbf{Y})$	The conditional Shannon entropy or von Neumann entropy of a register \mathbf{X} with respect to a register \mathbf{Y} .	252, 267
$I(\mathbf{X} : \mathbf{Y})$	The mutual information or quantum mutual information between registers \mathbf{X} and \mathbf{Y} .	253, 267
$H(P)$	The von Neumann entropy of a positive semidefinite operator P .	265
$D(P Q)$	The quantum relative entropy of P with respect to Q , for positive semidefinite operators P and Q .	266

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$T_{n,\varepsilon}(p)$	The set of ε -typical strings of length n with respect to the probability vector p .	286
$\Pi_{n,\varepsilon}$	Projection operator corresponding to the ε -typical subspace of $\mathcal{X}^{\otimes n}$ with respect to a given state.	291
$I_{\text{acc}}(\eta)$	The accessible information of an ensemble η .	295
$\chi(\eta)$	The Holevo information of an ensemble η .	297
$\text{Sep}(\mathcal{X} : \mathcal{Y})$	The set of separable operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between \mathcal{X} and \mathcal{Y} .	311
$\text{SepD}(\mathcal{X} : \mathcal{Y})$	The set of separable density operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between \mathcal{X} and \mathcal{Y} .	311
$\text{Ent}_r(\mathcal{X} : \mathcal{Y})$	The set of operators acting on the tensor product space $\mathcal{X} \otimes \mathcal{Y}$ having entanglement rank bounded by r , with respect to the bipartition between \mathcal{X} and \mathcal{Y} .	322
$\text{SepCP}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of separable maps from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between \mathcal{X} and \mathcal{Y} and between \mathcal{Z} and \mathcal{W} .	325
$\text{SepC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of separable channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between \mathcal{X} and \mathcal{Y} and between \mathcal{Z} and \mathcal{W} .	326
$\text{LOCC}(\mathcal{X}, \mathcal{Z} : \mathcal{Y}, \mathcal{W})$	The set of LOCC channels from $L(\mathcal{X} \otimes \mathcal{Y})$ to $L(\mathcal{Z} \otimes \mathcal{W})$, respecting the bipartition between \mathcal{X} and \mathcal{Y} and between \mathcal{Z} and \mathcal{W} .	330
$E_{\text{D}}(\mathbf{X} : \mathbf{Y})$	The distillable entanglement of the state of a pair of registers (\mathbf{X}, \mathbf{Y}) .	347

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$E_C(\mathsf{X} : \mathsf{Y})$	The entanglement cost of the state of a pair of registers (X, Y) .	347
$\text{PPT}(\mathcal{X} : \mathcal{Y})$	The set of PPT operators acting on $\mathcal{X} \otimes \mathcal{Y}$, respecting the bipartition between \mathcal{X} and \mathcal{Y} .	353
$E_F(\mathsf{X} : \mathsf{Y})$	The entanglement of formation of the state of a pair of registers (X, Y) .	385
W_π	A unitary operator acting on $\mathcal{X}^{\otimes n}$, for a complex Euclidean space \mathcal{X} , that permutes tensor factors according to the permutation π .	391
$\mathcal{X}^{\otimes n}$	The symmetric subspace of $\mathcal{X}^{\otimes n}$, for \mathcal{X} a complex Euclidean space. Also denoted $\mathcal{X}_1 \otimes \cdots \otimes \mathcal{X}_n$ when $\mathcal{X}_1, \dots, \mathcal{X}_n$ are identical copies of \mathcal{X} .	392
$\text{Bag}(n, \Sigma)$	The set of functions describing a bag of n items, each labeled by an element of an alphabet Σ .	393
\mathbb{N}	The set of nonnegative integers $\{0, 1, 2, \dots\}$.	393
Σ_ϕ^n	The subset of Σ^n consistent with a given function $\phi \in \text{Bag}(n, \Sigma)$.	393
$\mathcal{X}^{\odot n}$	The anti-symmetric subspace of $\mathcal{X}^{\otimes n}$, for \mathcal{X} a complex Euclidean space.	398
$L(\mathcal{X})^{\otimes n}$	The algebra of permutation-invariant operators acting on $\mathcal{X}^{\otimes n}$, for \mathcal{X} a complex Euclidean space.	400
μ	Symbol used to denote uniform spherical measure.	408
η	Symbol used to denote Haar measure.	411

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$H_{\min}(\Phi)$	The minimum output entropy of a channel Φ .	451
$C(\Phi)$	The classical capacity of a channel Φ .	466
$C_{\text{E}}(\Phi)$	The entanglement-assisted classical capacity of a channel Φ .	469
$\chi(\Phi)$	The Holevo capacity of a channel Φ .	470
$\chi_{\text{E}}(\Phi)$	The entanglement-assisted Holevo capacity of a channel Φ .	474
$I_{\text{C}}(\rho; \Phi)$	The coherent information of a state ρ through a channel Φ .	474
$I_{\text{C}}(\Phi)$	The maximum coherent information of a channel Φ .	474
$K_{a_1 \dots a_n, \varepsilon}(p)$	The set of ε -typical strings of length n , conditioned on a string $a_1 \dots a_n$, with respect to the probability vector p .	479
$\Lambda_{a_1 \dots a_n, \varepsilon}$	Projection onto the ε -typical subspace of $\mathcal{X}^{\otimes n}$, for \mathcal{X} a complex Euclidean space, conditioned on a string $a_1 \dots a_n$.	481
$S_{n, \varepsilon}(p)$	The set of ε -strongly typical strings of length n with respect to the probability vector p .	498
$Q(\Phi)$	The quantum capacity of a channel Φ .	513
$Q_{\text{EG}}(\Phi)$	The entanglement generation capacity of a channel Φ .	514
$Q_{\text{E}}(\Phi)$	The entanglement-assisted quantum capacity of a channel Φ .	519
$\Phi_0 \oplus \Phi_1$	The direct sum of maps Φ_0 and Φ_1 .	540

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