Analytical Methods in Marine Hydrodynamics

The value of analytical solutions relies on the rigorous formulation, and a strong mathematical background. This comprehensive volume unifies the most important geometries, which allow for the development of analytical solutions for hydrodynamic boundary value problems. It offers detailed explanations of the Laplace domain and numerical results associated with such problems, providing deep insight into the theory of hydrodynamics. Extended numerical calculations are provided and discussed, allowing the reader to use them as benchmarks for their own computations and making this an invaluable resource for specialists in various disciplines, including hydrodynamics, acoustics, optics, electrostatics, and brain imaging.

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To the memory of Rannia, my beloved sister

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Foreword

This book gives an encyclopedic description of the analytical methods based on separable solutions of the Laplace equation in various coordinate systems that are useful in marine hydrodynamics. This approach is classical, and exemplified by the works of Sir Thomas Henry Havelock (1877–1968), Georg Weinblum (1897–1974), and others who preceded the widespread use of computers and numerical solutions. But the present work goes much further, providing a unified description of body shapes and including contemporary contributions by the author and others that go well beyond the earlier work.

Chapter 1 reviews the basic theory to provide the background for the subsequent applications. This is relevant not only for the analytical approaches that follow but also for numerical studies of more general body shapes. The primary application is to analyze the diffraction and radiation of waves by structures on or near the free surface that are stationary or oscillating about a fixed mean position. Nonlinear effects up to third order are included.

Chapter 2 describes the solutions for circular cylinders with vertical axes. In the simplest case, the cylinder extends throughout the fluid column with constant cross section. More complicated configurations are described, including floating or bottom-mounted cylinders with depths less than the fluid, the floating cylinder with an internal "moonpool," compound cylinders with discontinuous radii, and the interactions between multiple cylinders in an array. These problems are among the most common and useful in the field of offshore engineering. They offer a stimulating introduction to the applications of Bessel functions, the vertical eigenfunctions used for water waves in finite depth, and the matching of solutions in different domains.

Chapter 3 considers the same problems for circular cylinders including second- and third-order nonlinear effects. The theoretical background is presented in a thorough manner, and results are described for practical problems including sum- and difference-frequency wave loads in a spectrum, the mean drift force and moment, and second-order effects on the wave elevation close to a structure.

Chapter 4 extends the analysis to cylinders with elliptic sections. This extension has obvious applications to streamlined bodies including ships. The transformation to elliptical coordinates is outlined, with the radial solutions represented by Mathieu functions in place of the simpler Bessel functions used for circular cylinders. The complexity of this topic is alleviated by a section describing the properties of the

Mathieu functions and relations that are useful in their applications. Solutions are presented not only for the simplest cylindrical case but also for truncated cylinders, compound cylinders, and arrays. The second-order solution is also described.

Chapter 5 describes the corresponding analyses for spheres and spheroids. Solutions are presented for the diffraction and radiation by stationary bodies, the steady wave resistance problem in calm water, and the combined problem where a spheroid is moving with forward speed in the presence of incident waves.

Chapter 6 is devoted to the ellipsoid with three unequal axes, perhaps the closest approximation that can be found for most ships and submerged vessels. This was recognized especially by Georg Weinblum, an international leader in twentieth-century ship research and the founding director of the Institut für Schiffbau at the University of Hamburg; he kept a half-model of an ellipsoid on the wall of his office and often stated that "the ellipsoid is God's gift to naval architects!" Here it is necessary to use Lamé functions. Their properties are described, including expansions suitable for computations, and the fundamental solution for the source singularity is derived in terms of ellipsoidal harmonics. Solutions are derived for the motion of an ellipsoid in a uniform stream, added mass and moment of inertia, and for translational motion parallel to a rigid wall or in a channel. Applications to ship maneuvering are described including problems of ship maneuvers in restricted waters and ship-to-ship interactions. Wave diffraction and wave resistance are also considered for a submerged ellipsoid.

I would have treasured this book when I was a student and starting my research career. It covers a broad range of problems in a unified manner, and includes the necessary background information for the special functions that are required. References to other work are cited throughout the text and listed in the extensive bibliography.

With widespread use of numerical solutions that are applicable to arbitrary body shapes, it is logical to question the need for this book. In my view there are several answers. It is essential that engineers and scientists who use numerical solutions understand their underlying theory and assumptions, and confirm their validity by comparisons with simpler known results. It is often said that a good experimentalist should know the answer before conducting an experiment, and the same applies to numerical investigations. In addition to these important practical concerns, there is great beauty in the analytical methods described here, which is rewarding to engineers and scientists with overlapping interests in hydrodynamics and mathematics. In the author's own words, "the free surface, namely the boundary surface where the water meets the air, is what makes hydrodynamics difficult, challenging but at the same time, a fascinating discipline."

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Preface

The first time I was seriously involved with hydrodynamics was during 2004–2005 when my esteemed teacher, Professor Spyros A. Mavrakos, suggested to me the analytical elaboration of the second-order diffraction problem. Indeed that problem had been tackled in the past by others, notable researchers in the field, with obvious success. The first successful experience was the solution of the second-order, double-frequency, hydrodynamic problem by a compound cylinder. My engagement in the area allowed me to realize the attraction provided by the analytical treatment of complicated dynamical systems, such as the hydrodynamical systems are, because, in any event, hydrodynamic boundary value problems still remain dynamical systems. It was evident that in order to treat those types of problems successfully, full devotion and countless working hours are required. The joy I felt when the first result, a single point actually, coincided explicitly with already published data was unimaginable. The second run provided the same coincidence, and then again and again, with new results, a curve was created that reflected the accuracy of the method followed, the assumptions made, the numerical techniques adopted, and the numerical results computed. Then I realized that, regardless of the effort needed, if one achieves an analytical solution, for a specific problem, then he has THE solution.

Accordingly, in a time span of ten years I tried to treat similar analytic problems involving geometries that allow analytical solutions in potential theory using the associated separable solutions of the Laplace equation, namely the field equation for an ideal fluid. Elliptical cylinders, spheroids, and ellipsoids followed next. My engagement revealed my lack in knowledge, which had to be reinforced, but at the same time opened new horizons in terms of understanding what has been done and what could be done more. And as always, the solution of a problem opens numerous challenges defining unsolved problems such as the Hercules Hydra.

This book is the outcome of my involvement in hydrodynamics. It contains topics of my own work and the work done by others. It is about methods for developing analytic solutions of hydrodynamical boundary value problems. The issues treated are of substantial practical importance in several cases, as, for example, the linear and nonlinear hydrodynamics of cylinders, the wave resistance problem of solids below a free surface, the two-ship encounter problem, and the attraction force exerted on solids in rectilinear translatory motion close to a rigid wall.

The development of analytic solution methods discussed in the book is allowed by the geometries of the bodies assumed and considered. These bodies are often characterized as orthotropic, or in direct translation from Greek, *properly shaped*. An orthotropic shape is not necessarily axisymmetric. For example, elliptical cylinders or ellipsoids are not axisymmetric shapes. However, all geometries considered correspond to a special coordinate system that allows separable solutions of the Laplace equation. The Laplace equation is transformed into the associated underlying coordinate system, always being more complicated compared to its simpler formulation, i.e., the Cartesian representation. The products of the separable solutions are called harmonics, which correspond to the particular coordinate system. The underlying wave structure interaction problems are boundary value problems that generally involve a Neumann condition on the structure's surface. Sometimes, Dirichlet conditions may be involved depending on the problem's particulars. In any event, what distinguishes the hydrodynamic boundary value problems combined with free-surface flows (the free surface is not necessarily taken into account for deeply submerged bodies) is the Robin-type boundary condition of the free surface, which, to make things more complicated, is actually inhomogeneous. The free-surface boundary condition is homogeneous only to leading order (the linearized problem). The Neumann boundary condition, typically, is the last to be employed, and given that it is a no-flux condition, should be applied normally to the body surface. The value of the separable solutions of the Laplace equation is exactly that: one of its solutions, which is also involved in the fundamental harmonics, is defined in a coordinate normal to the body surface. Thus the employment of the Neumann condition is assumed by a simple derivative that, because of orthogonality, considerably simplifies the procedure and allows for an efficient solution.

Nevertheless, the underlying solution of the Laplace equation is governed by special functions, Bessel, Mathieu, Legendre, and Lamé, along with their modified counterparts. The theory of special functions is a highly explored vast area of applied mathematics, which opens enormous perspectives for research. Special functions are admittedly a generally unfamiliar topic, with the exception perhaps of the Bessel functions. Many of their properties are still concealed (e.g., representations, integral formulations, connection with other special functions, integral results, orthogonality properties, addition theorems, etc.). It is evident that for analytical studies, the availability of relevant theorems would make the efforts simpler. In fact, dealing with a typical hydrodynamical boundary value problem, shall, in most cases, reveal the necessity of dealing with some fundamental theorems, regardless of whether they have a solution or not. And this is exactly, in my opinion, the magic of applying analytical methods. You never solve a single problem; in fact you solve a series of problems, and eventually you achieve THE solution, in terms of robustness, efficacy, and certainty about its accuracy.

The fundamental difference between employing a numerical methodology and an analytic method lies in the fact that with analytical methods one is able to gain a deep insight into the task undertaken and understand the particulars of each case. Calculated peaks in force, for example using a numerical methodology, could be anticipated by a very simple formula when analytical methods are applied. Why? Because analytical methods allow further processing and elaboration of the sought solution by looking for, say, zero denominators, existence of singularities, etc. Some relevant issues are discussed in this book.

In writing this book, I tried to include all the necessary theory for the tasks described and undertaken. I admit that many more things could have been included. However, the theory of hydrodynamics is so vast that it definitely cannot be embraced in a single monograph. My sincere ambition in writing this book is to try to attract scientists and researchers to the efficacy, aside from the beauty, of analytical methodologies. If I am allowed to give a single piece of advice, it would be: don't be afraid of the long complicated relations. THE solution will come at the end and that will be your reward! I hope that hydrodynamicists will find the material contained in the book valuable for their research and I would be very happy if it could also be used as a tool for referring to the theorems and special relations it contains. The majority of the material presented here has been developed by leading hydrodynamicists, applied scientists, and mathematicians who are cited, together with their studies.

Authoring the book was, by no means, an easy task. However, in this endeavor I was not alone. Hydrodynamics was taught to me by leading scientists in the field, globally renowned for their extraordinary contributions. Indeed I was fortunate in meeting them! It is impossible to thank all the people who helped me over the years. However, I am deeply indebted to teachers, colleagues, friends, and collaborators who, in one way or another, assisted and urged me to complete this book. Special thanks go to Professor Odd M. Faltinsen, who encouraged me to propose this book to Cambridge University Press and to Professor Alexander A. Korobkin, who taught me that hydrodynamics is not only about regular waves. I am deeply obliged to my friends and colleagues Professor Bernard Molin and Dr. Šime Malenica for their constant support, which was not always simply scientific. I sincerely thank them for the trust they showed me. I also thank Professor Jo Pinkster, who shared with me his original seminal study on the near-field method for mean drift forces. I was very privileged to meet and communicate with Professor John Nicholas Newman. Professor Newman kindly accepted my invitation to write the foreword to this book and I am deeply indebted to him. He also allowed me to include some of his results obtained by the respected WAMIT® program to compare with my own calculations. My short encounter with Professor Newman allowed me to understand the qualities that make great scientists, which among others include hard work, devotion, diligence, quest for the perfect, deep understanding, knowledge, inspiration, ingenuity, and curiosity. I sincerely thank Professor George Dassios, who gave me the path to calculate Lamé functions and for the long discussions that helped me understand, to a sufficient extent, the underlying theory. The introduction of Chapter 6 was actually written by Professor Dassios. It is impossible to forget my esteemed teacher and now colleague Professor Spyros A. Mavrakos for his support and guidance over the years. The positive effect of his personality on my progress, and life in general, is unimaginable.

But above all, I would like to thank my teacher and friend Professor Touvia Miloh for his continuous support and constant communication while I was writing this book. Many of the theorems described in this book are actually his own work and the product of his ingenuity. Even new theorems included here, such as the image singularity system of oblate spheroids, have been developed by him and he was so generous to share them with me. Much of the material presented in Chapter 5 is based on his studies and our close collaborations. Nearly all issues studied in Chapter 6 were conceived and elaborated by Professor Miloh. I also thank him for sharing with me the notes of the late Professor Louis Landweber on Lamé functions and related topics. Those notes actually constitute a forgotten art and they still remain the state-of-the-art on ellipsoidal harmonics and their applications in theoretical hydrodynamics. Thanks, Touvia!

Indeed, I was truly fortunate and blessed in my career!

Note on Symbols and Notations

This book uses standard notations for the various physical quantities and mathematical factors. The gravitational acceleration is always denoted by g, the density by ρ, and the water depth by h. The Greek letter ϕ or Φ has been retained to always denote the velocity potential. As regards the mathematical symbols, the Greek letter δ is used to denote either the Kronecker delta function or the Dirac function. The Greek letter ε is used to denote the Neumann symbol (or Jacobi factor) while ϵ is always the scaling factor for wave steepness or the elliptic eccentricity. The equations presented in this book are sometimes lengthy and complicated, requiring a large number of notations for the various parameters. Any effort has been made to use a variety of symbols from the arsenal of editing software to avoid confusion. However, use of the standard notations, such as those appearing in the literature, implies that sometimes the same symbol needs to be used for different quantities. In any event, when necessary the text makes clear if a symbol is used for a different parameter rather than the standard.

As regards the notation of the special functions mentioned in the book, again the standard symbols have been used. Some attention needs to be given to the notation of the Mathieu functions as, generally, the literature does not actually agree to a uniform path. Here the notations suggested in Abramowitz and Stegun (1970) have been adopted. Also, a remark needs to be made with regard to the derivatives of the special functions. To denote a derivative with respect to the argument, the prime and the dot symbols are used alternatively. For Bessel and Mathieu functions, the derivatives are denoted by the prime symbol, while for the Legendre and the Lamé functions the dot symbol is adopted. This has been done because the contents of this book have direct connections with studies appearing in the literature and the author tried to comply with the most generally accepted and followed symbolization.

Finally, some attention needs to be paid as regards the direction and the origin of the vertical coordinate z in a Cartesian representation. Again, the literature does not actually agree on that and the coordinate z is alternatively placed on the undisturbed free surface or in the horizontal bottom. In addition, for immersed bodies, and especially in the presence of a horizontal bottom, the origin of the z-axis is alternatively set on the undisturbed free surface or on the center of the body. Also, the orientation of the z-axis may change, pointing either downward or upward. In any event, any effort has been made to always, and repeatedly, specify both the origin and the orientation of the z-axis.