

Index

- analytical dynamics software, 263–268
 - dedicated multibody dynamics software, 267–268
 - general purpose mathematical software, 263–267
- angular acceleration, 60
- angular momentum, 86–93
- angular velocity, 46, 79
- angular velocity addition, 52–54
- angular velocity tensor, 46
- Appell, Paul, 4
- arc length, 23–24, 32–33
- axis-angle representation, 38–39

- biomechanical systems, 235–262
- Bizzarie di Varie Figure, 1
- Braccelli, Giovanni Battista, 1
- branching chain devices, 5

- calculus of rotations, 44–54, 79
 - angular velocity, 46, 52–54, 79
 - Euler angles, 47–48
 - quaternion, 46–47
 - rotation matrix, 45–46
- center of mass, 85
- centrifugal and Coriolis vector, 111
- configuration space, 9, 70, 76–77
 - generalized coordinates, 3, 9, 70, 76–77, 105, 114, 142, 143, 145, 146
- coning angle, 64, 69
- conservation of angular momentum, 84
- conservation of translational momentum, 84
- conservation principles, 83–100
 - Newton's Laws, 83–84
 - Newton-Euler Principle, 83–99
- constrained biomechanical systems, 245–262
 - muscle-based actuation, 246–253
- constrained Hamiltonian systems, 143–145, 150
- constrained least action, 140–142
- constrained-motion manifold, 116
- constrained muscle-based actuation, 246–253
- constrained task space dynamics, 227–232, 233–234
 - direct mapping of constraints, 227–229
 - task/constraint partitioning, 229–232, 233–234
- constraint matrix, 115
- constraints, 76–77, 81–82, 101, 113–128, 140, 148–149, 150, 151, 160–178, 185–187, 188, 197–200, 211–213, 224, 226, 227–232, 233–234
 - holonomic constraints, 113–128, 148–149, 150, 211–213, 227–232, 233–234
 - nonholonomic constraints, 160–178, 185–187, 197–200, 213
 - rheonomic constraints, 101, 224, 226
 - scleronomic constraints, 101, 128, 140, 151, 188
- covariant derivative, 26
- curvature tensor, 25, 35
- curves, 23–24, 32–33
 - arc length, 23–24, 32–33
 - extrinsic curvature, 24, 32–33
 - tangent vector, 23, 32–33

- d'Alembert's Principle, 2, 101–128, 147–149
 - constraints, 113–128
 - single particle, 101–102
 - single rigid body, 102–104
 - system of particles, 104–106
 - system of rigid bodies, 106–113
 - virtual work, 107
- d'Alembert, Jean Le Rond, 2
- dedicated multibody dynamics software, 267–268
 - commercial software, 267–268
 - open source software, 268
- deformation gradient, 272
- degrees of freedom, 76–77, 81–82
- Delta mechanism, 5
- Denavit-Hartenberg parameters, 69–70, 79–80
- derivatives in rotating reference frames, 59–61
- diffeomorphism, 23, 269
- differential geometry, 22–28, 32–35, 116, 269
 - curves, 23–24, 32–33
 - k-manifolds, 23, 116, 269
 - surfaces, 24–28, 33–35
- double pendulum, 5
- dynamically consistent inverse, 216

- eigenbasis, 18–19
- eigenspace, 18
- eigenvalue problem, 18–19, 32
- elimination of Lagrange multipliers, 145–147
- Euler angles, 39–41, 78
- Euler's disk, 7
- Euler, Leonhard, 1
- Euler-Lagrange equations, 3, 128–140, 149–150
 - kinetic energy ellipsoid, 131–132
 - single particle, 130
 - single rigid body, 130–132
 - system of particles, 132–134
 - system of rigid bodies, 134–140, 149–150
- extrinsic curvature, 24, 32–33

- first-order nonholonomic constraints, 160–178
 - rolling ball, 168–172
 - rolling disk, 163–168
 - two-wheeled assembly, 172–178
- first-order variational principles, 151, 187
 - Jourdain's Principle, 151–178
 - Kane's method, 179–184
 - virtual power, 151, 152, 154, 156, 157, 158, 159
- four-bar linkage, 5, 6, 148, 211–213

- Gauss map, 24, 33–35
- Gauss's Principle, 188, 200
 - constraints, 197–200
 - single particle, 188–189
 - single rigid body, 189–191
 - system of particles, 191–193
 - system of rigid bodies, 193–197
- Gauss's Principle of Least Constraint, 4
- Gauss, Carl Friedrich, 4
- Gaussian curvature, 25, 35
- general purpose mathematical software, 263–267
 - packages for multibody dynamics, 263–267
- generalized accelerations, 4, 111, 192, 197, 201, 202
- generalized constrained equation of motion, 127–128
- generalized constraint forces, 115, 116, 121, 145, 201
- generalized coordinates, 3, 9, 70, 76–77, 105, 114, 142, 143, 145, 146
- generalized forces, 94, 105, 133, 134, 147
- generalized momenta, 142
- generalized velocities, 4, 114, 142, 143, 144, 155, 159, 179, 182, 192, 207
- geodesic curvature, 26–27, 33
- geodesics, 26–28, 35, 213
 - covariant derivative, 26
 - geodesic curvature, 26–27, 33
- Gibbs, Josiah Willard, 4
- Gibbs-Appell method, 4, 206–210, 213
 - system of rigid bodies, 206–210
- Gram-Schmidt orthogonalization, 16, 31

- gravity vector, 111
- gyroscope, 5, 135–140

- Hamilton's canonical equations, 142–143, 150
- Hamilton's Principle, 128–142
 - constrained least action, 140–142
 - Euler-Lagrange equations, 128–130
 - Lagrangian, 128
- Hamilton, William Rowan, 3
- Hamiltonian, 143
- Hamiltonian dynamics, 142–145, 150
 - constrained dynamics, 143–145, 150
 - Hamiltonian, 143
 - unconstrained dynamics, 142–143, 150
- Hertz's Principle of Least Curvature, 4, 202–206
- Hertz, Heinrich Rudolf, 4
- historical background, 1–4
- Hoberman Sphere, 6
- holonomic constraints, 76–77, 81–82, 113–128, 148–149, 150, 211–213, 227–232, 233–234
 - constrained-motion manifold, 116
 - constraint matrix, 115
 - four-bar linkage, 148, 211–213
 - Lagrange multipliers, 116
 - parallel shoulder mechanism, 253–262
 - planar parallel mechanism, 122–126, 230–232
 - slider-crank, 148–149, 150, 233–234
 - Stewart platform, 118–122
- holonomically constrained devices, 5–6
 - Delta mechanism, 5
 - four-bar linkage, 5, 6, 148, 211–213
 - Hoberman Sphere, 6
 - parallel shoulder mechanism, 253–262
 - planar parallel mechanism, 122–126, 230–232
 - slider-crank, 6, 148–149, 150, 233–234
 - Stewart platform, 118–122
- homogeneous transform matrix, 54–55

- inertia tensor, 88
- inertial properties, 85–93, 99–100
 - center of mass, 85–93, 99–100
 - inertia tensor, 85–93, 99–100
- inertial reference frame, 83
- inner product, 10–12

- Jacobian matrix, 70–72, 80–81
 - relationship with screws, 72
- Jourdain's Principle, 4, 151–178, 185–187
 - constraints, 160–178
 - single particle, 151–152
 - single rigid body, 152–154
 - system of particles, 154–156
 - system of rigid bodies, 156–160
 - virtual power, 151, 152, 154, 156, 157, 158, 159
- Jourdain, Philip E. B., 4

- k-manifolds, 23, 116, 269
 - constrained-motion manifold, 116
 - diffeomorphism, 23, 269
- Kane's method, 4, 179–184, 185
- Kane, Thomas, 4
- kinematic chains, 5, 69–76, 79–81, 94–99, 100, 111–113, 135–140, 147–148, 149–150, 217–219, 220–222, 233
 - Denavit-Hartenberg parameters, 69–70, 79–80
 - double pendulum, 5
 - gyroscope, 5, 135–140
 - Jacobian matrix, 70–72, 80–81
 - rate vector propagation, 72–76, 81
 - serial chain robot, 81, 94–99, 100, 111–113, 147–148, 149–150, 217–219, 220–222, 233
 - tree-structured mechanism, 5
- kinematics, 36–82
 - degrees of freedom, 76–77, 81–82
 - kinematic chains, 69–76, 79–81
 - spatial kinematics, 54–69, 79
 - spherical kinematics, 36–54, 77–79
- kinetic energy, 130, 131, 133, 134, 204
- kinetic energy ellipsoid, 131–132

- Lagrange multipliers, 116, 145
- Lagrange, Joseph-Louis, 3
- Lagrangian, 128, 142, 204
- least norm, 17–18, 31–32
- least squares, 16–17, 31
- Leibniz, Gottfried, 2
- linear systems, 10–22, 31–32
 - tensors in \mathbb{R}^3 , 21
 - vector and tensor operations, 21–22
- vector spaces in \mathbb{R}^n , 10–19, 31–32
- vectors in \mathbb{R}^3 , 19–21

- mass matrix, 111
- mean curvature, 25, 35
- motors, 57–58
 - spatial velocity, 57, 72, 79
- muscle activation dynamics, 236
- muscle contraction dynamics, 236–238
 - pennation angle, 236
- muscle Jacobian, 236
- muscle moment arm matrix, 236
- musculoskeletal dynamics, 235–236
 - muscle Jacobian, 236
 - muscle moment arm matrix, 236

- neuro-musculoskeletal dynamics, 235–243
 - musculoskeletal dynamics, 235–236
 - neuromuscular dynamics, 236–238
 - stiff tendon model, 238–243
- neuromuscular dynamics, 236–238
 - activation dynamics, 236
 - contraction dynamics, 236–238

- Newton's Laws, 83–84
- Newton, Isaac, 1
- Newton-Euler Principle, 83–99, 100
 - conservation of angular momentum, 84
 - conservation of translational momentum, 84
 - Newton's Laws, 83–84
 - single particle, 84–85
 - single rigid body, 85–93
 - system of particles, 93
 - system of rigid bodies, 93–99
- nonholonomic constraints, 160–178, 185–187, 197–200, 213
 - first-order nonholonomic constraints, 160–178
 - rolling disk, 185, 213
 - second-order nonholonomic constraints, 197–200
 - two-wheeled assembly, 185–187
- nonholonomically constrained devices, 6–7
 - Euler's disk, 7
 - rolling ball, 7, 168–172
 - rolling disk, 163–168, 185, 213
 - two-wheeled assembly, 7, 172–178, 185–187
- normal curvature, 27, 33
- null space projection matrix, 216

- orthogonal complement, 15–16
- orthogonal projection, 16, 31

- parallel shoulder mechanism, 253–262
- pennation angle, 236
- Piola-Kirchoff stress tensor, 272
- planar parallel mechanism, 122–126, 230–232
- positive definite matrix, 19, 88, 111, 131, 142, 216
- potential energy, 130, 131, 133, 134
- principal curvatures, 25, 35
- Principle of Least Constraint, 200–206
 - Hertz's Principle of Least Curvature, 202–206
 - system of rigid bodies, 201–202
- propagation of kinematic rate vectors, 72–76

- quasi-accelerations, 115, 207, 208, 211
- quasi-forces, 180, 208
- quasi-Jacobians, 179, 207
- quasi-velocities, 115, 179, 180, 182, 207, 210
- quaternion derivative, 46–47
- quaternions, 41–44

- rheonomic constraints, 101
- rolling ball, 7, 168–172
- rolling disk, 163–168, 185, 213
- rotation matrices, 36–38
- rotation matrix derivative, 45–46

- scleronomic constraints, 101, 128, 140, 151, 188
- screws, 55–57, 79
- second-order nonholonomic constraints, 197–200

- second-order variational principles, 188, 213
 - Gauss's Principle, 188, 200
 - Gibbs-Appell method, 206–211
 - Principle of Least Constraint, 200–206
- serial chain robot, 81, 94–99, 100, 111–113, 147–148, 149–150, 217–219, 220–222, 233
- slider-crank, 6, 148–149, 150, 233–234
- spatial kinematics, 54–69, 79
 - derivatives in rotating reference frames, 59–61
 - homogeneous transform, 54–55
 - motors, 57–58, 79
 - screws, 55–57, 79
 - wrenches, 58–59
- spatial velocity, 57, 72, 79
- spherical kinematics, 36–54, 77–79
 - axis-angle, 38–39
 - calculus of rotations, 44–54, 79
 - Euler angles, 39–41, 78
 - quaternions, 41–44
 - rotation matrices, 36–38
- static optimization, 28–30
- Stewart platform, 118–122
- stiff tendon model, 238–243
- surface traction, 272
- surfaces, 24–28, 33–35, 213
 - curvature tensor, 25, 35
 - Gauss map, 24, 33–35
 - Gaussian curvature, 25, 35
 - geodesic curvature, 26–27, 33
 - geodesics, 26–28, 35, 213
 - mean curvature, 25, 35
 - normal curvature, 27, 33
 - principal curvatures, 25, 35
- tangent vector, 23, 32–33
- task space belted ellipsoid, 220
- task space centrifugal and Coriolis vector, 216
- task space dynamics, 214–234
 - belted ellipsoid, 220
 - centrifugal and Coriolis vector, 216
 - constrained task space dynamics, 227–232
 - dynamically consistent inverse, 216
 - energy minimization, 223–224
 - gravity vector, 216
 - inertial properties, 219–222
 - kinetic energy ellipsoid, 220
 - kinetic energy minimization, 223–224
 - least action, 226–227
 - mass matrix, 216
 - null space projection matrix, 216
 - potential energy minimization, 223
 - serial chain robot, 217–219
 - task/constraint symmetry, 224–227
- task space gravity vector, 216
- task space kinetic energy, 219
- task space kinetic energy ellipsoid, 220
- task space mapping of constraints, 227–229
- task space mass matrix, 216
- task/constraint partitioning, 229–232, 233–234
- task/constraint symmetry, 224–227
- tensors in \mathbb{R}^3 , 21
- tree-structured mechanism, 5
- two-wheeled assembly, 7, 172–178, 185–187
- variational principles, 101–213
 - first-order variational principles, 151, 187
 - second-order variational principles, 188–213
 - zeroth-order variational principles, 101–150
- vector and tensor operations, 21–22
- vector basis, 12–15
- vector spaces in \mathbb{R}^n , 10–19, 31–32
 - eigenbasis, 18–19
 - eigenspace, 18
 - eigenvalue problem, 18–19, 32
 - Gram-Schmidt, 16, 31
 - inner product, 10–12
 - least norm, 17–18, 31–32
 - least squares, 16–17, 31
 - orthogonal complement, 15–16
 - orthogonal projection, 16, 31
 - vector basis, 12–15
- vectors in \mathbb{R}^3 , 19–21
- virtual power, 151, 152, 154, 156, 157, 158, 159
- virtual work, 107
- vis viva, 2
- wrenches, 58–59
- zeroth-order variational principles, 101–150
 - d'Alembert's Principle, 101–128
 - Hamilton's Principle, 128–142
 - virtual work, 107