Advanced Analytical Dynamics

This book provides a unique bridge between the foundations of analytical mechanics and application to multibody dynamical systems. It is intended as a textbook, particularly well suited for graduate students seeking an understanding of the theoretical underpinnings of analytical mechanics, as well as modern task space approaches for representing the resulting dynamics that can be exploited for real-world problems in areas such as biomechanics and robotics.

Established principles in mechanics are presented in a thorough and modern way. The chapters build up from general mathematical foundations to an extensive treatment of kinematics and then to a rigorous treatment of conservation and variational principles in mechanics. Parallels are drawn between the different approaches, providing the reader with insights that unify his or her understanding of analytical dynamics. Additionally, a unique treatment is presented on task space dynamical formulations that map traditional configuration space representations into more intuitive geometric spaces.

Vincent De Sapio is a research scientist at HRL Laboratories in Malibu, California. He has more than 40 publications and 16 patents (issued or pending) in the areas of multibody dynamics, robotics, biomechanics, control theory, and human motion synthesis and simulation. Dr. De Sapio is a senior member of IEEE and a member of ASME and Sigma Xi. He is a founding member of the IEEE Technical Committee on Human Movement Understanding. He received his Ph.D. and M.S. from Stanford University and his B.S. from Rensselaer Polytechnic Institute, all in mechanical engineering.

καὶ τὸ φῶς ἐν τῇ σκοτίᾳ φαίνει, καὶ ἡ σκοτία αὐτὸ οὐ κατέλαβεν.

The light shines in the darkness, and the darkness has not overcome it.

John 1:5

S. D. G.

Advanced Analytical Dynamics

Theory and Applications

VINCENT DE SAPIO
HRL Laboratories LLC

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi - 110002, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107179608
DOI: 10.1017/9781316832301

First published 2017

Printed in the United States of America by Sheridan Books, Inc.

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: De Sapio, Vincent, 1968– author.
Title: Advanced analytical dynamics : theory and applications / Vincent De Sapio (HRL Laboratories LLC).
Description: Cambridge, United Kingdom ; New York, NY : Cambridge University Press, 2017. | Includes bibliographical references and index.
Identifiers: LCCN 2016036860| ISBN 9781107179608 (hardback ; alk. paper) | ISBN 1107179602 (hardback ; alk. paper)
Subjects: LCSH: Dynamics. | Mechanics, Analytic.
Classification: LCC QA845 .D42 2017 | DDC 620.1/04–dc23 LC record available at https://lccn.loc.gov/2016036860

ISBN 978-1-107-17960-8 Hardback

Cambridge University Press
978-1-107-17960-8 — Advanced Analytical Dynamics
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Preface

This book addresses the analytical dynamics of multibody systems and is intended for a one- to two-semester advanced graduate-level course in analytical dynamics. The emphasis is on a solid theoretical foundation with examples that concretely illustrate the theory. I have included a chapter on the fundamental mathematics that is helpful in navigating the principles of dynamics. This includes coverage of linear systems and differential geometry. A chapter on kinematics, the study of the geometry of motion, follows. The first chapter, on dynamics, addresses conservation principles, fundamentally the conservation of momentum embodied in the Newton-Euler Principles. Historically, analytical mechanics (dynamics) has referred to the so-called variational principles, rooted in the calculus of variations. Three chapters cover zeroth-, first-, and second-order variational principles, respectively. Lagrangian and Hamiltonian mechanics are among the more well-known formulations arising from variational principles covered in this book. I also cover important, but lesser known, higher-order principles, including Jourdain's Principle of Virtual Power, Gauss's Principle of Least Constraint, and Hertz's Principle of Least Curvature, as well as Kane's formulation and the Gibbs-Appell formulation.

As an aside, it is worth noting that modern theoretical physics emerged out of the classical variational principles. Einstein's general theory of relativity is commonly formulated using Lagrangian mechanics. Dirac was the first to use the Lagrangian in quantum mechanics and provided separate formulations of quantum mechanics and general relativity based on the Hamiltonian formalism. He also provided a generalized formulation of constrained Hamiltonian systems. Additionally, Feynman's path integral formulation of quantum mechanics has its classical ancestry in Hamilton's Principle of Least Action.

After the chapters on the variational principles, I have included a chapter on an alternate formulation of classical dynamics that has found significant utility in the control of robotic systems. The so-called task space formulation of dynamics was pioneered by Khatib under the name of operational space dynamics. It provides a transformation of the configuration space description of system dynamics into a more convenient task-oriented description. An applications-oriented chapter is included on biomechanical systems. This provides a basic overview of musculoskeletal and neuromuscular biomechanics with extensive coverage of application examples using actual anthropometric and muscle property data. The final chapter provides a brief survey of some analytical dynamics software. This is not intended to provide exhaustive coverage but only some examples of general purpose mathematical software with extensions for multibody dynamics, as well as dedicated multibody dynamics software, both commercial and open source. An appendix is included that touches upon the application of continuum mechanics to flexible multibody systems, which include both rigid and deformable bodies. It is my intention to extend this appendix into a fully integrated book chapter in a future edition.

I have tried to provide at least one example to illustrate each important concept covered in the book. The examples presented are thoroughly worked out in the text, so the reader should have no trouble following the methodology. In many cases, numerical simulation results are also provided. Some simple integration schemes are covered, and the reader is encouraged to explore the examples on his or her own and use any preferred mathematical software (e.g., Mathematica, Matlab) to generate simulation results. The reader would also benefit from employing mathematical software in addressing some of the exercises at the end of selected chapters. These exercises are often multipart and can involve fairly extensive mathematical computations.

I would like to thank a number of my past teachers who helped impart to me not only an understanding and insight related to the topics covered in this book but also a sense of the vast expanse of applications for analytical dynamics. These include Oussama Khatib, Scott Delp, Jean Heegaard, Bernie Roth, Ken Waldron, and others. A number of other teachers have contributed to my understanding of areas related to analytical dynamics. These include Stephen Boyd, Steve Rock, Sanjay Lall, Ron Fedkiw, Peter Pinsky, Charles Steele, Edward Goldstein, and Terry Sanger. My academic colleagues have provided productive interaction as well. These include Jaeheung Park, Luis Sentis, François Conti, Mike Zinn, James Warren, Emel Demircan, Jinsung Kwon, Dongjun Shin, Anya Petrovskaya, Peter Thaulad, Oliver Brock, Vincent Padois, Kate Saul, Rob Siston, Jeff Reinbolt, and others.

Although not directly related to the content of this book, many colleagues at HRL (formerly Hughes Research Laboratories) whom I have worked with on various projects have provided professional support and fruitful interaction. Among these individuals are Rajan Bhattacharyya, Jaehoon Choe, Yang Chen, Jose Cruz-Albrecht, Mike Daily, Son Dao, Chong Ding, Darren Earl, Karim El DeFrawy, Stephanie Goldfarb, Heiko Hoffmann, Mike Howard, Qin Jiang, Deepak Khosla, Ken Kim, Tiffany Kim, Dmitiry Korchev, Joonho Lee, Tsai-ching Lu, Charles Martin, Kevin Martin, Connie Ni, Aleksey Nogin, Yuri Owechko, Dave Payton, Matt Phillips, Praveen Pilly, Amir Rahimi, Shankar Rao, Shane Roach, Stephan Salas, Narayan Srinivasan, Nigel Stepp, Ryan Uhlenbrock, and others. Special thanks go to HRL's patent counsel, George Rapacki, my administrative assistant, Jennifer Greene, and Joonho Lee, who performed a technical proofread of part of the manuscript. In addition to working with a talented group of people, HRL's location provides me with inspiring views of the Pacific Ocean from a bluff overlooking the Malibu coastline in Southern California.

It has been a great pleasure working with Cambridge University Press on this book. I would like to thank Steve Elliot, Mark Fox, and Rebecca Rom-Frank at Cambridge. I would also like to thank Holly Monteith for copy editing the manuscript, and Vijay Bhatia for managing the book composition and typesetting. The external technical reviewers provided valuable constructive assessments of the early draft, for which I am

grateful. A special extra note of thanks goes to Steve Elliot who guided me through the manuscript evaluation, approval, and contract-related processes.

A number of people in my life made this book possible, not by technical contribution, but by the consistent impact they have had on my life. First and foremost, my parents, Martin and Lucia, shaped my life in ways that were profound. As immigrants from Italy, both of them embodied the American dream and built their lives upon hard work, commitment to family, and an enduring faith. My siblings, Gaetano, Antonio, Maria, Carmine, and particularly my sisters, Rita and Sally, nurtured me throughout my life and continue to impact my life in positive ways. I can not sufficiently express my gratitude to them for the counsel they provide me in life. I would also like to thank Tom Stephen, who has provided me with his friendship and steadfast spiritual guidance, and my in-laws, Bob and Joyce Prindle, who have always encouraged me.

The greatest joys in my life are my wife, Robin, and my two boys, Robbie and Marty. Robin has stuck by me when I thought no one would. She is a source of enthusiasm, compassion, and humor, and she is my rock. Robbie and Marty inspire me in ways that only a toddler and a six-year-old can. In them I see innocence (and a little mischief), a complete passion for life, and warm hearts. They are my dearest treasures in life.

Finally, I thank God for all the blessings that He has provided in my life. It is through Him that my life has meaning.

Malibu, USA
November 2016

Vincent De Sapio
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Notation

A specific set of notational standards is employed in this book. In the following sections we build up the notational standards that are used in the following chapters, starting with general mathematical objects and proceeding to kinematic objects, dynamical objects, and block diagram elements.

General Mathematical Objects

Sets

The following standard set notation is employed:

$\{\}$	designation of a set
$\mid$	such that
$\forall$	for all
$\in$	element of
$\perp$	orthogonal to
$\square^{\perp}$	orthogonal complement
$\mathbb{R}$	set of real numbers
$\mathbb{R}^n$	set of real n-dimensional vectors
$\mathbb{R}^{m\times n}$	set of real $m \times n$ matrices
$\mathbb{C}$	set of complex numbers
$\mathbb{H}$	set of quaternions
$\mathbb{S}^n$	set of points on an n-dimensional unit sphere

An example is as follows:

$$\begin{gathered} \boldsymbol{A}^T\boldsymbol{\lambda} \perp \boldsymbol{x}, \\ \forall \boldsymbol{\lambda} \in \mathbb{R}^m \quad \text{and} \quad \forall \boldsymbol{x} \in \ker(\boldsymbol{A}), \\ \text{where} \quad \ker(\boldsymbol{A}) = \{\boldsymbol{x} \in \mathbb{R}^n \mid \boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}\}. \end{gathered}$$

This would read as follows: $\boldsymbol{A}^T\boldsymbol{\lambda}$ is orthogonal to $\boldsymbol{x}$ for all $\boldsymbol{\lambda}$ in the set of real m-dimensional vectors and for all $\boldsymbol{x}$ in the kernel of $\boldsymbol{A}$, where the kernel of $\boldsymbol{A}$ is the set of all real n-dimensional vectors, $\boldsymbol{x}$, such that $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}$.

Scalars

Scalars (rank 0 tensors) are represented with nonbold italic characters (e.g., a). These include scalars as well as scalar components of vectors and matrices. Scalar components of vectors and matrices are denoted with a subscripted index to the right of the scalar symbol (e.g., v_i, M_{ij}). The following standard operators are employed:

δ	variation
$\frac{d}{d\square}$	derivative
$\dot{\square}$	time derivative

Complex Numbers and Quaternions

Complex numbers and quaternions are represented with nonbold lowercase (typically) italic characters. The components can be expressed as a sum of the real and imaginary parts, for example,

$$z = a + \boldsymbol{i}b$$
$$h = h_0 + h_1\boldsymbol{i} + h_2\boldsymbol{j} + h_3\boldsymbol{k}.$$

Vectors, Points, and Line Segments

Vectors (rank 1 tensors) are represented with bold lowercase (typically) italic characters. Vectors can be expressed as a 1-dimensional array or as a linear combination of basis vectors, with indexed scalar components (displayed as nonbold italic characters). Basis vectors are denoted as $\hat{\boldsymbol{e}}$. An example follows:

$$\boldsymbol{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \sum_{i=1}^{3} v_i\hat{\boldsymbol{e}}_i = v_1\hat{\boldsymbol{e}}_1 + v_2\hat{\boldsymbol{e}}_2 + v_3\hat{\boldsymbol{e}}_3.$$

Points are represented using nonbold italic characters (e.g., A). Line segments between two points are represented using an arrow (e.g., $\overrightarrow{AB}$).

Matrices and Tensors

Matrices (rank 2 tensors) are represented with bold uppercase (typically) italic characters. Matrices can be expressed as a 2-dimensional array or as a linear combination of dyads, with indexed scalar components (displayed as nonbold italic characters). Dyads consist of a pair of base vectors separated by an outer product symbol, $\otimes$, for example,

$$\boldsymbol{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} = \sum_{i=1}^{2}\sum_{j=1}^{2} M_{ij}\hat{\boldsymbol{e}}_i \otimes \hat{\boldsymbol{e}}_j = M_{11}\hat{\boldsymbol{e}}_1 \otimes \hat{\boldsymbol{e}}_1 + M_{12}\hat{\boldsymbol{e}}_1 \otimes \hat{\boldsymbol{e}}_2$$
$$+ M_{21}\hat{\boldsymbol{e}}_2 \otimes \hat{\boldsymbol{e}}_1 + M_{22}\hat{\boldsymbol{e}}_2 \otimes \hat{\boldsymbol{e}}_2.$$

The identity matrix is denoted as **1** and the *zero* matrix is denoted as **0**.

Vector and Matrix Operators

The following standard vector and matrix operators are employed:

$\cdot$	dot product
$\langle \square, \square \rangle$	inner product
$\Vert \square \Vert$	norm
$\times$	cross-product
$\otimes$	outer product
δ	variation
$\frac{d}{ds}$	derivative with respect to a scalar, s
$\dot{\square}$	time derivative
$\frac{\partial}{\partial \square}$, ∇	partial derivative, gradient
$\mathrm{im}()$	image or range of a matrix
$\ker()$	kernel or null space of a matrix
$\mathrm{proj}_{\square}()$	projection of a vector onto a subspace
$T()$	tangent space operator
$\bar{\square}$	dynamically consistent (mass-weighted) inverse of a matrix

The partial derivative/gradient operators are overloaded for scalars and vectors. For example, given a scalar, $U \in \mathbb{R}$, and a vector, $\boldsymbol{v} \in \mathbb{R}^m$, the respective gradients are

$$\nabla U = \frac{\partial U}{\partial \boldsymbol{q}} = \sum_{i=1}^{n} \frac{\partial U}{\partial q_i} \hat{\boldsymbol{e}}_i = \begin{pmatrix} \frac{\partial U}{\partial q_1} \\ \vdots \\ \frac{\partial U}{\partial q_n} \end{pmatrix} \in \mathbb{R}^n$$

and

$$\boldsymbol{v}\nabla = \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{q}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{\partial v_i}{\partial q_j} \hat{\boldsymbol{e}}_i \otimes \hat{\boldsymbol{e}}_j = \begin{pmatrix} \frac{\partial v_1}{\partial q_1} & \cdots & \frac{\partial v_1}{\partial q_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial v_m}{\partial q_1} & \cdots & \frac{\partial v_m}{\partial q_n} \end{pmatrix} \in \mathbb{R}^{m \times n}.$$

Kinematic Objects

Objects having a kinematic meaning inherit all of the aforementioned rules with respect to their mathematical type. Additionally, they adhere to the following with regard to their physical type.

A position vector, $\boldsymbol{r}$, uses a right subscript to denote the *material point* it refers to and a left superscript to denote the basis it is expressed in. Velocity, $\boldsymbol{v}$, and acceleration, $\boldsymbol{a}$, vectors additionally denote the frame that motion is relative to using a ":" separator in the right subscript. Angular velocity, $\boldsymbol{\omega}$, and angular acceleration vectors, $\boldsymbol{\alpha}$, use a right subscript to denote the *body* they refer to and a left superscript to denote the basis they are expressed in. As with velocity, they additionally denote the frame that motion is relative to, using a ":". Any annotation can be omitted if the information conveyed by it is already clear from context. Generalized coordinates are denoted as $\boldsymbol{q}$ and operational space coordinates are denoted as $\boldsymbol{x}$.

Coordinate transformation matrices, including both orthogonal rotation matrices, $\boldsymbol{Q}$, and homogenous transformation matrices, $\boldsymbol{T}$, denote the frame of interest using a left subscript and the embedding frame using a left superscript. Unit quaternions, h, use similar annotation. Jacobian matrices use a right subscript to denote the object (material point, body, etc.) they refer to and a left superscript to denote the basis they are expressed in. Again, any annotation can be omitted if the information conveyed by it is already clear from context.

Cartesian Space Quantities

⊕, G	center of mass point
${}^{\mathcal{A}}\frac{d}{dt}{}_{\mathcal{O}}$	time derivative relative to $\mathcal{O}$, expressed in $\mathcal{A}$
${}^{\mathcal{A}}\Delta_{\mathcal{O}}$	change relative to $\mathcal{O}$, expressed in $\mathcal{A}$
${}^{\mathcal{B}}\boldsymbol{d}_{\overrightarrow{AB}}$	displacement vector between points A and B, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{r}_{G_{\mathcal{A}}}$	position of center of mass, G, of body $\mathcal{A}$ expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{r}^{\mathcal{A}}_{\mathcal{B}}$	point on body $\mathcal{B}$ to which body $\mathcal{A}$ attaches, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{v}_{G_{\mathcal{A}}:\mathcal{O}}$	velocity of center of mass, G, of body $\mathcal{A}$, relative to $\mathcal{O}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{a}_{G_{\mathcal{A}}:\mathcal{O}}$	acceleration of center of mass, G, of body $\mathcal{A}$, relative to $\mathcal{O}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{A}:\mathcal{O}}$	angular velocity of body $\mathcal{A}$, relative to $\mathcal{O}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{\alpha}_{\mathcal{A}:\mathcal{O}}$	angular acceleration of body $\mathcal{A}$, relative to $\mathcal{O}$, expressed in $\mathcal{B}$
${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{Q}$	rotation matrix of $\mathcal{B}$ with respect to $\mathcal{A}$
$\boldsymbol{Q}_k(\theta)$	rotation matrix representing a spin of θ about axis $\boldsymbol{k}$
${}^{\mathcal{A}}_{\mathcal{B}}h$	quaternion of $\mathcal{B}$ with respect to $\mathcal{A}$
$h_k(\theta)$	quaternion representing a spin of θ about axis $\boldsymbol{k}$
$\$$	screw displacement
${}^{\mathcal{A}}_{\mathcal{B}}\boldsymbol{T}$	homogenous transformation matrix of $\mathcal{B}$ with respect to $\mathcal{A}$
${}^{\mathcal{B}}\boldsymbol{\Gamma}_{G_{\mathcal{A}}}$	Jacobian of position of center of mass, G, of body $\mathcal{A}$ expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{\Pi}_{\mathcal{A}}$	Jacobian of body $\mathcal{A}$ expressed in $\mathcal{B}$

Configuration Space Quantities

$\boldsymbol{q}$	generalized coordinate vector

Constraint Space Quantities

$\boldsymbol{\phi}$	holonomic constraint vector (general zeroth-order constraints)
$\boldsymbol{\Phi}$	holonomic constraint Jacobian matrix
$\boldsymbol{C}$	nonholonomic constraint matrix (linear first-order constraints)
$\boldsymbol{W}$	constraint null space matrix
$\boldsymbol{\psi}$	nonholonomic constraint vector (general first-order constraints)
$\boldsymbol{A}$	nonholonomic constraint matrix (linear second-order constraints)

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978-1-107-17960-8 — Advanced Analytical Dynamics
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Task Space Quantities

$\boldsymbol{x}$	task space coordinate vector
$\boldsymbol{J}$	task Jacobian matrix

Dynamic Objects

Objects having a dynamical meaning inherit all of the aforementioned rules with respect to their mathematical type. Additionally, they adhere to the following with regard to their physical type.

Translational momentum vectors, $\boldsymbol{p}$, have the same scripting as velocity vectors. Angular momentum vectors, $\boldsymbol{H}$, use the same scripting as angular velocity vectors. Additionally, angular momentum vectors denote the point about which they are evaluated using a right superscript. Inertia tensors, $\boldsymbol{I}$, have scripting similar to angular momentum vectors. Again, any annotation can be omitted if the information conveyed by it is already clear from context.

Cartesian Space Quantities

M	mass of point or body
g	acceleration due to gravity (e.g.,$\approx$ 9.8m/s^2 on earth)
${}^{\mathcal{B}}\boldsymbol{f}_{\mathcal{B}}^{\mathcal{A}}$	force that body $\mathcal{A}$ exerts on body $\mathcal{B}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{\varphi}_{\mathcal{B}}^{\mathcal{A}}$	moment that body $\mathcal{A}$ exerts on body $\mathcal{B}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{p}_{G_{\mathcal{A}}:\mathcal{O}}$	translational momentum of center of mass, G, of body $\mathcal{A}$, relative to $\mathcal{O}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{H}_{\mathcal{A}:\mathcal{O}}^{G_{\mathcal{A}}}$	angular momentum of body $\mathcal{A}$, relative to $\mathcal{O}$, about center of mass, G, of body $\mathcal{A}$, expressed in $\mathcal{B}$
${}^{\mathcal{B}}\boldsymbol{I}_{\mathcal{A}:\mathcal{O}}^{G_{\mathcal{A}}}$	inertia tensor of body $\mathcal{A}$, relative to $\mathcal{O}$, about center of mass, G, of body $\mathcal{A}$, expressed in $\mathcal{B}$

Configuration Space Quantities

$\boldsymbol{p}$	generalized momentum vector
$\boldsymbol{\tau}$	generalized force vector
$\boldsymbol{M}$	generalized mass matrix
$\boldsymbol{b}$	generalized Coriolis-centrifugal vector
$\boldsymbol{g}$	generalized gravity vector
$\mathcal{L}$	Lagrangian
$\mathcal{H}$	Hamiltonian
$\mathcal{G}$	Gauss function
$\mathcal{S}$	Gibbs function

Constraint Space Quantities

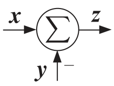
$\boldsymbol{\lambda}$	Lagrange multipliers (constraint forces)
$\mathbf{H}$	constraint space mass matrix
$\boldsymbol{\alpha}$	constraint space Coriolis-centrifugal vector
$\boldsymbol{\rho}$	constraint space gravity vector
$\boldsymbol{\Theta}^T$	constraint null space projection matrix

Task Space Quantities

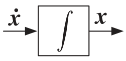
$\boldsymbol{N}^T$	task null space projection matrix
$\boldsymbol{f}$	task space force vector
$\boldsymbol{\Lambda}$	task space mass matrix
$\boldsymbol{\mu}$	task space Coriolis-centrifugal vector
$\boldsymbol{p}$	task space gravity vector

Block Diagrams

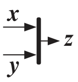
Block diagrams use a number of common schematic elements, as follows. For general (nonlinear) operators, a dashed line and an unfilled arrow are used to denote the input argument into the block:



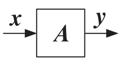
summation, $\boldsymbol{z} = \boldsymbol{x} - \boldsymbol{y}$



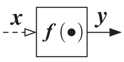
integration, $\boldsymbol{x} = \int \dot{\boldsymbol{x}}\, dt$



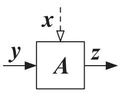
concatenation, $\boldsymbol{z} = (\boldsymbol{x}^T\ \boldsymbol{y}^T)^T$



linear operator, $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$



general (nonlinear) operator, $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$



mixed operator, $\boldsymbol{z} = \boldsymbol{A}(\boldsymbol{x})\boldsymbol{y}$