

Market Design

A Linear Programming Approach to Auctions and Matching

The digital economy has led to many new services where supply is matched with demand for various types of goods and services. More and more people and organizations are now in a position to design market rules that are being implemented in software.

The design of markets is challenging as it is necessary to consider the strategic behavior of market participants, psychological factors, and computational problems in order to implement the objectives of a designer. Market models in economics have not lost their importance, but recent years have led to many new insights and principles for the design of markets which are beyond traditional economic theory. This book introduces the fundamentals of market design, an engineering field concerned with the design of real-world markets.

Martin Bichler is Professor of Informatics at the Technical University of Munich (TUM), and a faculty member at the TUM School of Management. He is known for his academic work on market design, and he has acted as a consultant for private and public organizations including regulators, telecoms, and procurement organizations. Projects in which he is involved include the design of auctions for industrial procurement, logistics, advertising, fishery access rights, and spectrum sales. His research addresses algorithmic, game-theoretical, and behavioral questions and has appeared in leading journals in computer science, economics, operations research, and management science. He is currently Editor of *Business and Information Systems Engineering* and serves on the editorial boards of several academic journals.

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A Linear Programming Approach to Auctions and Matching

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To my wife Claudia and my daughters Mona and Sonja

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