Market Design

A Linear Programming Approach to Auctions and Matching

The digital economy has led to many new services where supply is matched with demand for various types of goods and services. More and more people and organizations are now in a position to design market rules that are being implemented in software.

The design of markets is challenging as it is necessary to consider the strategic behavior of market participants, psychological factors, and computational problems in order to implement the objectives of a designer. Market models in economics have not lost their importance, but recent years have led to many new insights and principles for the design of markets which are beyond traditional economic theory. This book introduces the fundamentals of market design, an engineering field concerned with the design of real-world markets.

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To my wife Claudia and my daughters Mona and Sonja
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