

Cambridge University Press 978-1-107-17287-6 — Random Graphs and Complex Networks Remco van der Hofstad Table of Contents More Information

## Contents

Preface		page xi	
Course Outline			
1	Tutus leaders	1	
1 1.1	Introduction  Motivation	1 1	
1.1	Graphs and Their Degree and Connectivity Structure	2	
1.3	Complex Networks: the Infamous Internet Example	5	
1.4	Scale-Free, Highly Connected and Small-World Graph Sequences	11	
1.5	Further Network Statistics	16	
1.6	Other Real-World Network Examples	20	
1.7	Tales of Tails	40	
1.8	Random Graph Models for Complex Networks	45	
1.9	Notation	49	
1.10	Notes and Discussion	50	
1.11	Exercises for Chapter 1	52	
	· · · · · · · · · · · · · · · · · · ·		
Part l	Preliminaries	55	
2	Probabilistic Methods	57	
2.1	Convergence of Random Variables	57	
2.2	Coupling	62	
2.3	Stochastic Ordering	65	
2.4	Probabilistic Bounds	69	
2.5	Martingales	73	
2.6	Order Statistics and Extreme Value Theory	78	
2.7	Notes and Discussion	82	
2.8	Exercises for Chapter 2	83	
3	Branching Processes	87	
3.1	Survival versus Extinction	87	
3.2	Family Moments	91	
3.3	Random-Walk Perspective to Branching Processes	92	
3.4	Supercritical Branching Processes	96	
3.5	Hitting-Time Theorem and the Total Progeny	100	
3.6	Properties of Poisson Branching Processes	102	
3.7	Binomial and Poisson Branching Processes	107	

vii



viii

Cambridge University Press 978-1-107-17287-6 — Random Graphs and Complex Networks Remco van der Hofstad Table of Contents More Information

> 3.8 Notes and Discussion 109 3.9 Exercises for Chapter 3 111 Part II Basic Models 115 Phase Transition for the Erdős-Rényi Random Graph 117 4.1 117 Introduction 4.2 Comparisons to Branching Processes 122 4.3 The Subcritical Regime 124 4.4 130 The Supercritical Regime 4.5 CLT for the Giant Component 139 4.6 Notes and Discussion 145 4.7 Exercises for Chapter 4 146 Erdős-Rényi Random Graph Revisited 5 150 5.1 The Critical Behavior 150 5.2 Critical Erdős-Rényi Random Graphs with Martingales 156 5.3 Connectivity Threshold 164 5.4 Degree Sequence of the Erdős-Rényi Random Graph 168 5.5 Notes and Discussion 172 5.6 Exercises for Chapter 5 174 Part III Models for Complex Networks 177 Intermezzo: Back to Real-World Networks... 179 6 **Generalized Random Graphs** 183

Contents

6.4	Degree Sequence of Generalized Random Graph	194
6.5	Generalized Random Graph with I.I.D. Weights	197
6.6	Generalized Random Graph Conditioned on Its Degrees	199
6.7	Asymptotic Equivalence of Inhomogeneous Random Graphs	203
6.8	Related Inhomogeneous Random Graph Models	207
6.9	Notes and Discussion	209
6.10	Exercises for Chapter 6	210
7	Configuration Model	216
<b>7</b> 7.1	Configuration Model  Motivation for the Configuration Model	216 216
•	8	
7.1	Motivation for the Configuration Model	216
7.1 7.2	Motivation for the Configuration Model Introduction to the Model	216 218
7.1 7.2 7.3	Motivation for the Configuration Model Introduction to the Model Erased Configuration Model	216 218 227
7.1 7.2 7.3 7.4	Motivation for the Configuration Model Introduction to the Model Erased Configuration Model Repeated Configuration Model: Simplicity Probability	216 218 227 232
7.1 7.2 7.3 7.4 7.5	Motivation for the Configuration Model Introduction to the Model Erased Configuration Model Repeated Configuration Model: Simplicity Probability Uniform Simple Graphs and Generalized Random Graphs	216 218 227 232 236

7.8

6.1

6.2

6.3

Motivation of the Model

Introduction of the Model

Related Random Graph Models

Degrees in the Generalized Random Graph

248

183

184

190



Cambridge University Press 978-1-107-17287-6 — Random Graphs and Complex Networks Remco van der Hofstad Table of Contents More Information

	Contents	ix
7.9	Notes and Discussion	250
7.10	Exercises for Chapter 7	252
8	Preferential Attachment Models	256
8.1	Motivation for the Preferential Attachment Model	256
8.2	Introduction of the Model	259
8.3	Degrees of Fixed Vertices	262
8.4	Degree Sequences of Preferential Attachment Models	264
8.5	Concentration of the Degree Sequence	267
8.6	Expected Degree Sequence	270
8.7	Maximal Degree in Preferential Attachment Models	283
8.8	Related Results for Preferential Attachment Models	288
8.9	Related Preferential Attachment Models	290
8.10	Notes and Discussion	294
8.11	Exercises for Chapter 8	297
Appendix		301
Glossary		304
References		306
Index		317