

ADVANCED STRUCTURAL DYNAMICS

Advanced Structural Dynamics will appeal to a broad readership that includes both undergraduate and graduate engineering students, doctoral candidates, engineering scientists working in various technical disciplines, and practicing professionals in an engineering office. The book has broad applicability and draws examples from aeronautical, civil, earthquake, mechanical, and ocean engineering, and at times it even dabbles in issues of geophysics and seismology. The material presented is based on miscellaneous course and lecture notes offered by the author at the Massachusetts Institute of Technology for many years. The modular approach allows for a selective use of chapters, making it appropriate for use not only as an introductory textbook but later on functioning also as a treatise for an advanced course, covering materials not typically found in competing textbooks on the subject.

Professor Eduardo Kausel is a specialist in structural dynamics in the Department of Civil Engineering at the Massachusetts Institute of Technology. He is especially well known for two papers on the collapse of the Twin Towers on September 11, 2001. The first of this pair, published on the web at MIT only a few days after the terrorist act, attracted more readers around the world than all other works and publications on the subject combined. Professor Kausel is the author of the 2006 book *Fundamental Solutions in Elastodynamics* (Cambridge University Press).

Advanced Structural Dynamics

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CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India
79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.
It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107171510
10.1017/9781316761403

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First published 2017

Printed in the United States of America by Sheridan Books, Inc.

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Kausel, E.

Title: Advanced structural dynamics / by Eduardo Kausel, Massachusetts Institute of Technology.

Other titles: Structural dynamics

Description: Cambridge [England]: Cambridge University Press, 2017. |

Includes bibliographical references and index.

Identifiers: LCCN 2016028355 | ISBN 9781107171510 (hard back)

Subjects: LCSH: Structural dynamics – Textbooks. | Structural analysis (Engineering) – Textbooks.

Classification: LCC TA654.K276 2016 | DDC 624.1/71–dc23

LC record available at <https://lcn.loc.gov/2016028355>

ISBN 978-1-107-17151-0 Hardback

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*To my former graduate student and dear Guardian Angel Hyangly Lee,
in everlasting gratitude for her continued support of my work at MIT.*

Contents

<i>Preface</i>	<i>page</i> xxi
<i>Notation and Symbols</i>	xxv
<i>Unit Conversions</i>	xxix
1 Fundamental Principles	1
1.1 Classification of Problems in Structural Dynamics	1
1.2 Stress–Strain Relationships	2
1.2.1 Three-Dimensional State of Stress–Strain	2
1.2.2 Plane Strain	2
1.2.3 Plane Stress	2
1.2.4 Plane Stress versus Plane Strain: Equivalent Poisson’s Ratio	3
1.3 Stiffnesses of Some Typical Linear Systems	3
1.4 Rigid Body Condition of Stiffness Matrix	11
1.5 Mass Properties of Rigid, Homogeneous Bodies	12
1.6 Estimation of Miscellaneous Masses	17
1.6.1 Estimating the Weight (or Mass) of a Building	17
1.6.2 Added Mass of Fluid for Fully Submerged Tubular Sections	18
1.6.3 Added Fluid Mass and Damping for Bodies Floating in Deep Water	20
1.7 Degrees of Freedom	20
1.7.1 Static Degrees of Freedom	20
1.7.2 Dynamic Degrees of Freedom	21
1.8 Modeling Structural Systems	22
1.8.1 Levels of Abstraction	22
1.8.2 Transforming Continuous Systems into Discrete Ones Heuristic Method	25
1.8.3 Direct Superposition Method	26
1.8.4 Direct Stiffness Approach	26
1.8.5 Flexibility Approach	27
1.8.6 Viscous Damping Matrix	29

1.9	Fundamental Dynamic Principles for a Rigid Body	31
1.9.1	Inertial Reference Frames	31
1.9.2	Kinematics of Motion	31
	Cardanian Rotation	32
	Eulerian Rotation	33
1.9.3	Rotational Inertia Forces	34
1.9.4	Newton's Laws	35
	(a) Rectilinear Motion	35
	(b) Rotational Motion	36
1.9.5	Kinetic Energy	36
1.9.6	Conservation of Linear and Angular Momentum	36
	(a) Rectilinear Motion	37
	(b) Rotational Motion	37
1.9.7	D'Alembert's Principle	37
1.9.8	Extension of Principles to System of Particles and Deformable Bodies	38
1.9.9	Conservation of Momentum versus Conservation of Energy	38
1.9.10	Instability of Rigid Body Spinning Freely in Space	39
1.10	Elements of Analytical Mechanics	39
1.10.1	Generalized Coordinates and Its Derivatives	40
1.10.2	Lagrange's Equations	42
	(a) Elastic Forces	42
	(b) Damping Forces	43
	(c) External Loads	44
	(d) Inertia Forces	45
	(e) Combined Virtual Work	45
2	Single Degree of Freedom Systems	55
2.1	The Damped SDOF Oscillator	55
2.1.1	Free Vibration: Homogeneous Solution	56
	Underdamped Case ($\xi < 1$)	57
	Critically Damped Case ($\xi = 1$)	58
	Overdamped Case ($\xi > 1$)	59
2.1.2	Response Parameters	59
2.1.3	Homogeneous Solution via Complex Frequencies: System Poles	60
2.1.4	Free Vibration of an SDOF System with Time-Varying Mass	61
2.1.5	Free Vibration of SDOF System with Frictional Damping	63
	(a) System Subjected to Initial Displacement	64
	(b) Arbitrary Initial Conditions	65
2.2	Phase Portrait: Another Way to View Systems	67
2.2.1	Preliminaries	67
2.2.2	Fundamental Properties of Phase Lines	69
	Trajectory Arrows	69
	Intersection of Phase Lines with Horizontal Axis	70
	Asymptotic Behavior at Singular Points and Separatrix	70
	Period of Oscillation	71

Contents

ix

2.2.3	Examples of Application	71
	Phase Lines of a Linear SDOF System	71
	Ball Rolling on a Smooth Slope	71
2.3	Measures of Damping	73
2.3.1	Logarithmic Decrement	74
2.3.2	Number of Cycles to 50% Amplitude	75
2.3.3	Other Forms of Damping	76
2.4	Forced Vibrations	76
2.4.1	Forced Vibrations: Particular Solution	76
	(a) Heuristic Method	77
	(b) Variation of Parameters Method	78
2.4.2	Forced Vibrations: General Solution	79
2.4.3	Step Load of Infinite Duration	80
2.4.4	Step Load of Finite Duration (Rectangular Load, or Box Load)	81
2.4.5	Impulse Response Function	81
2.4.6	Arbitrary Forcing Function: Convolution	83
	Convolution Integral	83
	Time Derivatives of the Convolution Integral	84
	Convolution as a Particular Solution	84
2.5	Support Motion in SDOF Systems	85
2.5.1	General Considerations	85
2.5.2	Response Spectrum	88
	Tripartite Spectrum	88
2.5.3	Ship on Rough Seas, or Car on Bumpy Road	89
2.6	Harmonic Excitation: Steady-State Response	92
2.6.1	Transfer Function Due to Harmonic Force	92
2.6.2	Transfer Function Due to Harmonic Support Motion	96
2.6.3	Eccentric Mass Vibrator	100
	Experimental Observation	101
2.6.4	Response to Suddenly Applied Sinusoidal Load	102
2.6.5	Half-Power Bandwidth Method	103
	Application of Half-Power Bandwidth Method	105
2.7	Response to Periodic Loading	106
2.7.1	Periodic Load Cast in Terms of Fourier Series	106
2.7.2	Nonperiodic Load as Limit of Load with Infinite Period	107
2.7.3	System Subjected to Periodic Loading: Solution in the Time Domain	109
2.7.4	Transfer Function versus Impulse Response Function	111
2.7.5	Fourier Inversion of Transfer Function by Contour Integration	111
	Location of Poles, Fourier Transforms, and Causality	113
2.7.6	Response Computation in the Frequency Domain	114
	(1) Trailing Zeros	115
	(2) Exponential Window Method: The Preferred Strategy	115
2.8	Dynamic Stiffness or Impedance	115
2.8.1	Connection of Impedances in Series and/or Parallel Standard Solid	117
		118

2.9	Energy Dissipation through Damping	118
2.9.1	Viscous Damping	119
	Instantaneous Power and Power Dissipation	119
	Human Power	120
	Average Power Dissipated in Harmonic Support Motion	120
	Ratio of Energy Dissipated to Energy Stored	121
	Hysteresis Loop for Spring–Dashpot System	122
2.9.2	Hysteretic Damping	123
	Ratio of Energy Dissipated to Energy Stored	123
	Instantaneous Power and Power Dissipation via the Hilbert Transform	124
2.9.3	Power Dissipation during Broadband Base Excitation	124
2.9.4	Comparing the Transfer Functions for Viscous and Hysteretic Damping	125
	Best Match between Viscous and Hysteretic Oscillator	126
2.9.5	Locus of Viscous and Hysteretic Transfer Function	127
3	Multiple Degree of Freedom Systems	131
3.1	Multidegree of Freedom Systems	131
3.1.1	Free Vibration Modes of Undamped MDOF Systems	131
	Orthogonality Conditions	132
	Normalized Eigenvectors	134
3.1.2	Expansion Theorem	134
3.1.3	Free Vibration of Undamped System Subjected to Initial Conditions	137
3.1.4	Modal Partition of Energy in an Undamped MDOF System	137
3.1.5	What If the Stiffness and Mass Matrices Are Not Symmetric?	138
3.1.6	Physically Homogeneous Variables and Dimensionless Coordinates	139
3.2	Effect of Static Loads on Structural Frequencies: Π - Δ Effects	141
3.2.1	Effective Lateral Stiffness	141
3.2.2	Vibration of Cantilever Column under Gravity Loads	144
3.2.3	Buckling of Column with Rotations Prevented	145
3.2.4	Vibration of Cantilever Shear Beam	146
3.3	Estimation of Frequencies	146
3.3.1	Rayleigh Quotient	147
	Rayleigh–Schwarz Quotients	149
3.3.2	Dunkerley–Mikhlin Method	149
	Dunkerley’s Method for Systems with Rigid-Body Modes	154
3.3.3	Effect on Frequencies of a Perturbation in the Structural Properties	157
	Perturbation of Mass Matrix	158
	Perturbation of Stiffness Matrix	159
	Qualitative Implications of Perturbation Formulas	160

Contents

xi

3.4	Spacing Properties of Natural Frequencies	162
3.4.1	The <i>Minimax</i> Property of Rayleigh's Quotient	162
3.4.2	Interlacing of Eigenvalues for Systems with Single External Constraint	165
	Single Elastic External Support	166
3.4.3	Interlacing of Eigenvalues for Systems with Single Internal Constraint	167
	Single Elastic Internal Constraint	167
3.4.4	Number of Eigenvalues in Some Frequency Interval	167
	Sturm Sequence Property	167
	The Sign Count of the Shifted Stiffness Matrix	168
	Root Count for Dynamically Condensed Systems	170
	Generalization to Continuous Systems	173
3.5	Vibrations of Damped MDOF Systems	176
3.5.1	Vibrations of Proportionally Damped MDOF Systems	176
3.5.2	Proportional versus Nonproportional Damping Matrices	181
3.5.3	Conditions under Which a Damping Matrix Is Proportional	181
3.5.4	Bounds to Coupling Terms in Modal Transformation	183
3.5.5	Rayleigh Damping	184
3.5.6	Caughey Damping	185
3.5.7	Damping Matrix Satisfying Prescribed Modal Damping Ratios	189
3.5.8	Construction of Nonproportional Damping Matrices	191
3.5.9	Weighted Modal Damping: The Biggs–Roësset Equation	194
3.6	Support Motions in MDOF Systems	196
3.6.1	Structure with Single Translational DOF at Each Mass Point	197
	Solution by Modal Superposition (Proportional Damping)	198
3.6.2	MDOF System Subjected to Multicomponent Support Motion	200
3.6.3	Number of Modes in Modal Summation	203
3.6.4	Static Correction	205
3.6.5	Structures Subjected to Spatially Varying Support Motion	207
3.7	Nonclassical, Complex Modes	209
3.7.1	Quadratic Eigenvalue Problem	210
3.7.2	Poles or Complex Frequencies	210
3.7.3	Doubled-Up Form of Differential Equation	213
3.7.4	Orthogonality Conditions	215
3.7.5	Modal Superposition with Complex Modes	216
3.7.6	Computation of Complex Modes	221
3.8	Frequency Domain Analysis of MDOF Systems	223
3.8.1	Steady-State Response of MDOF Systems to Structural Loads	223
3.8.2	Steady-State Response of MDOF System Due to Support Motion	224

3.8.3	In-Phase, Antiphase, and Opposite-Phase Motions	231
3.8.4	Zeros of Transfer Functions at Point of Application of Load	233
3.8.5	Steady-State Response of Structures with Hysteretic Damping	234
3.8.6	Transient Response of MDOF Systems via Fourier Synthesis	235
3.8.7	Decibel Scale	236
3.8.8	Reciprocity Principle	236
3.9	Harmonic Vibrations Due to Vortex Shedding	238
3.10	Vibration Absorbers	239
3.10.1	Tuned Mass Damper	239
3.10.2	Lanchester Mass Damper	243
3.10.3	Examples of Application of Vibration Absorbers	244
3.10.4	Torsional Vibration Absorber	249
4	Continuous Systems	251
4.1	Mathematical Characteristics of Continuous Systems	251
4.1.1	Taut String	251
4.1.2	Rods and Bars	252
4.1.3	Bending Beam, Rotational Inertia Neglected	252
4.1.4	Bending Beam, Rotational Inertia Included	254
4.1.5	Timoshenko Beam	254
4.1.6	Plate Bending	256
4.1.7	Vibrations in Solids	257
4.1.8	General Mathematical Form of Continuous Systems	258
4.1.9	Orthogonality of Modes in Continuous Systems	259
4.2	Exact Solutions for Simple Continuous Systems	260
4.2.1	Homogeneous Rod	260
	Normal Modes of a Finite Rod	262
	Fixed–Fixed Rod	262
	Free–Free Rod	263
	Fixed–Free Rod	264
	Normal Modes of a Rod without Solving a Differential Equation	264
	Orthogonality of Rod Modes	265
4.2.2	Euler–Bernoulli Beam (Bending Beam)	267
	Normal Modes of a Finite-Length Euler–Bernoulli Beam	268
	Simply Supported Beam	269
	Other Boundary Conditions	269
	Normal Modes of a Free–Free Beam	270
	Normal Modes of a Cantilever Beam	273
	Orthogonality Conditions of a Bending Beam	274
	Strain and Kinetic Energies of a Beam	274
4.2.3	Bending Beam Subjected to Moving Harmonic Load	274
	Homogeneous Solution	275
	Particular Solution	275
4.2.4	Nonuniform Bending Beam	277

Contents

xiii

4.2.5	Nonclassical Modes of Uniform Shear Beam	279
	Dynamic Equations of Shear Beam	280
	Modes of Rotationally Unrestrained Shear Beam	281
	Concluding Observations	287
4.2.6	Inhomogeneous Shear Beam	287
	Solution for Shear Modulus Growing Unboundedly with Depth	288
	Finite Layer of Inhomogeneous Soil	289
	Special Case: Shear Modulus Zero at Free Surface	290
	Special Case: Linearly Increasing Shear Wave Velocity	291
4.2.7	Rectangular Prism Subjected to SH Waves	292
	Normal Modes	292
	Forced Vibration	293
4.2.8	Cones, Frustums, and Horns	295
	(a) Exponential Horn	296
	(b) Frustum Growing as a Power of the Axial Distance	299
	(c) Cones of Infinite Depth with Bounded Growth of Cross Section	301
4.2.9	Simply Supported, Homogeneous, Rectangular Plate	302
	Orthogonality Conditions of General Plate	302
	Simply Supported, Homogeneous Rectangular Plate	303
4.3	Continuous, Wave-Based Elements (Spectral Elements)	305
4.3.1	Impedance of a Finite Rod	306
4.3.2	Impedance of a Semi-infinite Rod	311
4.3.3	Viscoelastic Rod on a Viscous Foundation (Damped Rod)	311
	Stress and Velocity	313
	Power Flow	314
4.3.4	Impedance of a Euler Beam	318
4.3.5	Impedance of a Semi-infinite Beam	322
4.3.6	Infinite Euler Beam with Springs at Regular Intervals	323
	Cutoff Frequencies	326
	Static Roots	327
4.3.7	Semi-infinite Euler Beam Subjected to Bending Combined with Tension	328
	Power Transmission	331
	Power Transmission after Evanescent Wave Has Decayed	331
5	Wave Propagation	333
5.1	Fundamentals of Wave Propagation	333
5.1.1	Waves in Elastic Bodies	333
5.1.2	Normal Modes and Dispersive Properties of Simple Systems	334
	An Infinite Rod	334
	Gravity Waves in a Deep Ocean	336
	An Infinite Bending Beam	337

	A Bending Beam on an Elastic Foundation	338
	A Bending Beam on an Elastic Half-Space	340
	Elastic Thick Plate (Mindlin Plate)	341
5.1.3	Standing Waves, Wave Groups, Group Velocity, and Wave Dispersion	342
	Standing Waves	342
	Groups and Group Velocity	343
	Wave Groups and the Beating Phenomenon	344
	Summary of Concepts	344
5.1.4	Impedance of an Infinite Rod	345
5.2	Waves in Layered Media via Spectral Elements	348
5.2.1	SH Waves and Generalized Love Waves	349
	(A) Normal Modes	353
	(B) Source Problem	355
	(C) Wave Amplification Problem	355
5.2.2	SV-P Waves and Generalized Rayleigh Waves	358
	Normal Modes	362
5.2.3	Stiffness Matrix Method in Cylindrical Coordinates	362
5.2.4	Accurate Integration of Wavenumber Integrals	365
	Maximum Wavenumber for Truncation and Layer Coupling	366
	Static Asymptotic Behavior: Tail of Integrals	367
	Wavenumber Step	369
6	Numerical Methods	371
6.1	Normal Modes by Inverse Iteration	371
6.1.1	Fundamental Mode	371
6.1.2	Higher Modes: Gram–Schmidt Sweeping Technique	374
6.1.3	Inverse Iteration with Shift by Rayleigh Quotient	374
6.1.4	Improving Eigenvectors after Inverse Iteration	376
6.1.5	Inverse Iteration for Continuous Systems	377
6.2	Method of Weighted Residuals	378
6.2.1	Point Collocation	381
6.2.2	Sub-domain	381
6.2.3	Least Squares	381
6.2.4	Galerkin	381
6.3	Rayleigh–Ritz Method	384
6.3.1	Boundary Conditions and Continuity Requirements in Rayleigh–Ritz	385
6.3.2	Rayleigh–Ritz versus Galerkin	386
6.3.3	Rayleigh–Ritz versus Finite Elements	387
6.3.4	Rayleigh–Ritz Method for Discrete Systems	388
6.3.5	Trial Functions versus True Modes	390
6.4	Discrete Systems via Lagrange’s Equations	391
6.4.1	Assumed Modes Method	391
6.4.2	Partial Derivatives	391
6.4.3	Examples of Application	392
6.4.4	What If Some of the Discrete Equations Remain Uncoupled?	399

Contents

xv

6.5	Numerical Integration in the Time Domain	400
6.5.1	Physical Approximations to the Forcing Function	401
6.5.2	Physical Approximations to the Response	403
	Constant Acceleration Method	403
	Linear Acceleration Method	404
	Newmark's β Method	404
	Impulse Acceleration Method	405
6.5.3	Methods Based on Mathematical Approximations	406
	Multistep Methods for First-Order Differential Equations	407
	Difference and Integration Formulas	409
	Multistep Methods for Second-Order Differential Equations	409
6.5.4	Runge–Kutta Type Methods	410
	Euler's Method	411
	Improved and Modified Euler Methods	411
	The Normal Runge–Kutta Method	412
6.5.5	Stability and Convergence Conditions for Multistep Methods	413
	Conditional and Unconditional Stability of Linear Systems	413
6.5.6	Stability Considerations for Implicit Integration Schemes	416
6.6	Fundamentals of Fourier Methods	417
6.6.1	Fourier Transform	417
6.6.2	Fourier Series	420
6.6.3	Discrete Fourier Transform	422
6.6.4	Discrete Fourier Series	423
6.6.5	The Fast Fourier Transform	426
6.6.6	Orthogonality Properties of Fourier Expansions	427
	(a) Fourier Transform	427
	(b) Fourier Series	427
	(c) Discrete Fourier Series	427
6.6.7	Fourier Series Representation of a Train of Periodic Impulses	428
6.6.8	Wraparound, Folding, and Aliasing	428
6.6.9	Trigonometric Interpolation and the Fundamental Sampling Theorem	430
6.6.10	Smoothing, Filtering, Truncation, and Data Decimation	432
6.6.11	Mean Value	432
6.6.12	Parseval's Theorem	433
6.6.13	Summary of Important Points	434
6.6.14	Frequency Domain Analysis of Lightly Damped or Undamped Systems	434
	Exponential Window Method: The Preferred Tool	435
6.7	Fundamentals of Finite Elements	440
6.7.1	Gaussian Quadrature	441
	Normalization	442

6.72	Integration in the Plane	444
	(a) Integral over a Rectangular Area	446
	(b) Integral over a Triangular Area	447
	(c) Curvilinear Triangle	448
	(d) Quadrilateral	450
	(e) Curvilinear Quadrilateral	451
	Inadmissible Shapes	451
6.73	Finite Elements via Principle of Virtual Displacements	451
	(a) Consistency	454
	(b) Conformity	454
	(c) Rigid Body Test	454
	(d) Convergence (Patch Test)	454
6.74	Plate Stretching Elements (Plane Strain)	455
	(a) Triangular Element	455
	(b) Rectangular Element	457
6.75	Isoparametric Elements	459
	Plane Strain Curvilinear Quadrilaterals	459
	Cylindrical Coordinates	463
7	Earthquake Engineering and Soil Dynamics	481
7.1	Stochastic Processes in Soil Dynamics	481
7.1.1	Expectations of a Random Process	481
7.1.2	Functions of Random Variable	482
7.1.3	Stationary Processes	482
7.1.4	Ergodic Processes	483
7.1.5	Spectral Density Functions	483
7.1.6	Coherence Function	484
7.1.7	Estimation of Spectral Properties	484
7.1.8	Spatial Coherence of Seismic Motions	488
	Coherency Function Based on Statistical	
	Analyses of Actual Earthquake Motions	488
	Wave Model for Random Field	490
	Simple Cross-Spectrum for SH Waves	490
	Stochastic Deconvolution	493
7.2	Earthquakes, and Measures of Quake Strength	494
7.2.1	Magnitude	495
	Seismic Moment	495
	Moment Magnitude	497
7.2.2	Seismic Intensity	497
7.2.3	Seismic Risk: Gutenberg–Richter Law	499
7.2.4	Direction of Intense Shaking	500
7.3	Ground Response Spectra	502
7.3.1	Preliminary Concepts	502
7.3.2	Tripartite Response Spectrum	504
7.3.3	Design Spectra	505
7.3.4	Design Spectrum in the style of ASCE/SEI-7-05	506
	Design Earthquake	506

Contents

xvii

	Transition Periods	506
	Implied Ground Motion Parameters	507
73.5	MDOF Systems: Estimating Maximum Values from Response Spectra	507
	Common Error in Modal Combination	510
	General Case: Response Spectrum Estimation for Complete Seismic Environment	511
74	Dynamic Soil–Structure Interaction	513
74.1	General Considerations	513
	Seismic Excitation (Free-Field Problem)	514
	Kinematic Interaction	514
	Inertial Interaction	515
74.2	Modeling Considerations	515
	Continuum Solutions versus Finite Elements	515
	Finite Element Discretization	515
	Boundary Conditions	516
74.3	Solution Methods	517
	Direct Approach	517
	Superposition Theorem	518
	Three-Step Approach	519
	Approximate Stiffness Functions	520
74.4	Direct Formulation of SSI Problems	522
	The Substructure Theorem	522
	SSI Equations for Structures with Rigid Foundation	524
74.5	SSI via Modal Synthesis in the Frequency Domain	526
	Partial Modal Summation	529
	What If the Modes Occupy Only a Subspace?	531
	Member Forces	533
74.6	The Free-Field Problem: Elements of 1-D Soil Amplification	534
	Effect of Location of Control Motion in 1-D Soil Amplification	537
74.7	Kinematic Interaction of Rigid Foundations	540
	Iguchi's Approximation, General Case	541
	Iguchi Approximation for Cylindrical Foundations Subjected to SH Waves	544
	Geometric Properties	545
	Free-Field Motion Components at Arbitrary Point, Zero Azimuth	546
	Surface Integrals	546
	Volume Integrals	548
	Effective Motions	549
75	Simple Models for Time-Varying, Inelastic Soil Behavior	551
75.1	Inelastic Material Subjected to Cyclic Loads	551
75.2	Masing's Rule	553
75.3	Ivan's Model: Set of Elastoplastic Springs in Parallel	555
75.4	Hyperbolic Model	556
75.5	Ramberg–Osgood Model	558

7.6	Response of Soil Deposits to Blast Loads	561
7.6.1	Effects of Ground-Borne Blast Vibrations on Structures	561
	Frequency Effects	561
	Distance Effects	562
	Structural Damage	563
8	Advanced Topics	565
8.1	The Hilbert Transform	565
8.1.1	Definition	565
8.1.2	Fourier Transform of the Sign Function	566
8.1.3	Properties of the Hilbert Transform	567
8.1.4	Causal Functions	569
8.1.5	Kramers–Kronig Dispersion Relations	570
	Minimum Phase Systems	572
	Time-Shifted Causality	573
8.2	Transfer Functions, Normal Modes, and Residues	573
8.2.1	Poles and Zeros	573
8.2.2	Special Case: No Damping	574
8.2.3	Amplitude and Phase of the Transfer Function	575
8.2.4	Normal Modes versus Residues	577
8.3	Correspondence Principle	580
8.4	Numerical Correspondence of Damped and Undamped Solutions	582
8.4.1	Numerical Quadrature Method	582
8.4.2	Perturbation Method	584
8.5	Gyroscopic Forces Due to Rotor Support Motions	585
8.6	Rotationally Periodic Structures	590
8.6.1	Structures Composed of Identical Units and with Polar Symmetry	590
8.6.2	Basic Properties of Block-Circulant Matrices	593
8.6.3	Dynamics of Rotationally Periodic Structures	594
8.7	Spatially Periodic Structures	596
8.7.1	Method 1: Solution in Terms of Transfer Matrices	596
8.7.2	Method 2: Solution via Static Condensation and Cloning	602
	Example: Waves in a Thick Solid Rod Subjected to Dynamic Source	603
8.7.3	Method 3: Solution via Wave Propagation Modes	604
	Example 1: Set of Identical Masses Hanging from a Taut String	605
	Example 2: Infinite Chain of Viscoelastically Supported Masses and Spring-Dashpots	608
8.8	The Discrete Shear Beam	610
8.8.1	Continuous Shear Beam	611
8.8.2	Discrete Shear Beam	611

Contents

xix

9	Mathematical Tools	619
9.1	Dirac Delta and Related Singularity Functions	619
9.1.1	Related Singularity Functions	620
	Doublet Function	620
	Dirac Delta Function	620
	Unit Step Function (Heaviside Function)	620
	Unit Ramp Function	621
9.2	Functions of Complex Variables: A Brief Summary	621
9.3	Wavelets	626
9.3.1	Box Function	626
9.3.2	Hanning Bell (or Window)	626
9.3.3	Gaussian Bell	627
9.3.4	Modulated Sine Pulse (Antisymmetric Bell)	628
9.3.5	Ricker Wavelet	628
9.4	Useful Integrals Involving Exponentials	630
9.4.1	Special Cases	630
9.5	Integration Theorems in Two and Three Dimensions	630
9.5.1	Integration by Parts	631
9.5.2	Integration Theorems	631
9.5.3	Particular Cases: Gauss, Stokes, and Green	633
9.6	Positive Definiteness of Arbitrary Square Matrix	633
9.7	Derivative of Matrix Determinant: The Trace Theorem	640
9.8	Circulant and Block-Circulant Matrices	642
9.8.1	Circulant Matrices	642
9.8.2	Block-Circulant Matrices	644
10	Problem Sets	647
	<i>Author Index</i>	713
	<i>Subject Index</i>	714

Preface

The material in this book slowly accumulated, accreted, and grew out of the many lectures on structural dynamics, soil dynamics, earthquake engineering, and structural mechanics that I gave at MIT in the course of several decades of teaching. At first these constituted mere handouts to the students, meant to clarify further the material covered in the lectures, but soon the notes transcended the class environment and began steadily growing in size and content as well as complication. Eventually, the size was such that I decided that it might be worthwhile for these voluminous class notes to see the light as a regular textbook, but the sheer effort required to clean out and polish the text so as to bring it up to publication standards demanded too much of my time and entailed sacrifices elsewhere in my busy schedule that I simply couldn't afford. Or expressing it in MIT-speak, I applied the *Principle of Selective Neglect*. But after years (and even decades) of procrastination, eventually I finally managed to break the vicious cycle of writer's block and brought this necessary task to completion.

Make no mistake: the material covered in this book far exceeds what can be taught in any one-semester graduate course in structural dynamics or mechanical vibration, and indeed, even in a sequence of two such courses. Still, it exhaustively covers the fundamentals in vibration theory, and then goes on well beyond the standard fare in – and conventional treatment of – a graduate course in structural dynamics, as a result of which most can (and should) be excluded from an introductory course outline, even if it can still be used for that purpose. Given the sheer volume of material, the text is admittedly terse and at times rather sparse in explanations, but that is deliberate, for otherwise the book would have been unduly long, not to mention tedious to read and follow. Thus, the reader is expected to have some background in the mechanical sciences such that he or she need not be taken by the hand. Still, when used in the classroom for a first graduate course, it would suffice to jump over advanced sections, and do so without sacrifices in the clarity and self-sufficiency of the retained material.

In a typical semester, I would start by reviewing the basic principles of dynamics, namely Newton's laws, impulse and conservation of linear and angular momenta, D'Alembert's principle, the concept of point masses obtained by means of mass lumping and tributary areas, and most importantly, explicating the difference between static and dynamic degrees of freedom (or master–slave DOF), all while assuming small displacements and skipping initially over the section that deals with Lagrange's equations. From

there on I would move on to cover the theory of single-DOF systems and devote just about half of the semester to that topic, inasmuch as multi-DOF systems and continuous systems can largely be regarded as generalizations of those more simple systems. In the lectures, I often interspersed demonstration experiments to illustrate basic concepts and made use of brief Matlab® models to demonstrate the application of the concepts being learned. I also devoted a good number of lectures to explain harmonic analysis and the use of complex Fourier series, which in my view is one of the most important yet difficult concepts for students to comprehend and assimilate properly. For that purpose, I usually started by explaining the concepts of amplitude and phase by considering a simple complex number of the form $z = x + iy$, and then moving on to see what those quantities would be for products and ratios of complex numbers of the form $z = z_1/z_2$, $z = z_1/z_2 = |z_1|/|z_2| e^{i(\phi_1 - \phi_2)}$, and in particular $z = 1/z_2 = e^{-i\phi_2}/|z_2|$. I completely omitted the use of sine and cosine Fourier series, and considered solely the complex exponential form of Fourier series and the Fourier transform, which I used in the context of periodic loads, and then in the limit of an infinite period, namely a transient load. From there the relationship between impulse response function and transfer functions arose naturally. In the context of harmonic analysis, I would also demonstrate the great effectiveness of the (virtually unknown) *Exponential Window Method* (in essence, a numerical implementation of the Laplace Transform) for the solution of lightly damped system via complex frequencies, which simultaneously disposes of the problems of added trailing zeroes and undesired periodicity of the response function, and thus ultimately of the “wraparound” problem, that is, causality.

Discrete systems would then take me some two thirds of the second half of the semester, focusing on classical modal analysis and harmonic analysis, and concluding with some lectures on the vibration absorber. This left me just about one third of the half semester (i.e., some two to three weeks) for the treatment of continuous systems, at which time I would introduce the use of Lagrange’s equations as a tool to solve continuous media by discretizing those systems via the *Assumed Modes Method*.

In the early version of the class lecture notes I included support motions and ground response spectra as part of the single-DOF lectures. However, as the material dealing with earthquake engineering grew in size and extent, in due time I moved that material out to a separate section, even if I continued to make seamless use of parts of those in my classes.

Beyond lecture materials for the classroom, this book contains extensive materials not included in competing books on structural dynamics, of which there already exist a plethora of excellent choices, and this was the main reason why I decided it was worthwhile to publish it. For this reason, I also expect this book to serve as a valuable reference for practicing engineers, and perhaps just as importantly, to aspiring young PhD graduates with academic aspirations in the fields of structural dynamics, soil dynamics, earthquake engineering, or mechanical vibration.

Last but not least, I wish to acknowledge my significant indebtedness and gratitude to Prof. José Manuel Roësset, now retired from the Texas A&M University, for his most invaluable advice and wisdom over all of the years that have spanned my academic career at MIT. It was while I was a student and José a tenured professor here that I learned with him mechanics and dynamics beyond my wildest expectations and

Preface

xxiii

dreams, and it could well be said that everything I know and acquired expertise in is ultimately due to him, and that in a very real sense he has been the ghost writer and coauthor of this book.

In problems relating to vibrations, nature has provided us with a range of mysteries which for their elucidation require the exercise of a certain amount of mathematical dexterity. In many directions of engineering practice, that vague commodity known as common sense will carry one a long way, but no ordinary mortal is endowed with an inborn instinct for vibrations; mechanical vibrations in general are too rapid for the utilization of our sense of sight, and common sense applied to these phenomena is too common to be other than a source of danger.

C. E. Inglis, FRS, James Forrest Lecture, 1944

Notation and Symbols

Although we may from time to time change the meaning of certain symbols and deviate temporarily from the definitions given in this list, by and large we shall adopt in this book the notation given herein, and we shall do so always in the context of an upright, right-handed coordinate system.

Vectors and matrices: we use **boldface** symbols, while non-boldface symbols (in italics) are scalars. Capital letters denote matrices, and lowercase letters are vectors. (Equivalence with blackboard symbols: \underline{q} is the same as \mathbf{q} , and \underline{M} is the same as \mathbf{M}).

Special Constants (non-italic)

- e Natural base of logarithms = 2.71828182845905...
 i Imaginary unit = $\sqrt{-1}$
 π 3.14159265358979...

Roman Symbols

- a Acceleration
 \mathbf{a} Acceleration vector
 A Amplitude of a transfer function or a wave; also area or cross section
 A_s Shear area
 b Body load, $b = b(\mathbf{x}, t)$
 \mathbf{b} Vector of body loads, $\mathbf{b} = \mathbf{b}(\mathbf{x}, t)$
 c Viscous damping (dashpot) constant
 C_1, C_2 Constants of integration
 C_s Shear wave velocity ($\sqrt{G/\rho}$)
 C_r Rod wave velocity ($\sqrt{E/\rho}$)
 C_f Flexural wave velocity ($\sqrt{C_r R \omega}$)
 \mathbf{C} Viscous damping matrix
 \mathbb{C} Modally transformed, diagonal damping matrix ($\Phi^T \mathbf{C} \Phi$)
 D Diameter
 f Frequency in Hz; it may also denote a flexibility

f_d	Damped natural frequency, in Hz
f_n	Natural frequency, in Hz
$\hat{\mathbf{e}}$	Cartesian, unit base vector ($\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3 \equiv \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$)
E	Young's modulus, $E = 2G(1 + \nu)$
E_d	Energy dissipated
E_s	Elastic energy stored
$\hat{\mathbf{g}}$	Curvilinear base vector ($\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3$)
g	Acceleration of gravity
$g(t)$	Unit step-load response function
G	Shear modulus
h	Depth or thickness of beam, element, or plate
$h(t)$	Impulse response function
H	Height
$H(\omega)$	Transfer function (frequency response function for a unit input)
I	Area moment of inertia
j	Most often an index for a generic mode
J	Mass moment of inertia
k	Usually stiffness, but sometimes a wavenumber
k_c	Complex stiffness or impedance
K	Kinetic energy
\mathbf{K}	Stiffness matrix
L	Length of string, rod, beam, member, or element
m	Mass
\mathbf{M}	Mass matrix
n	Abbreviation for <i>natural</i> ; also, generic degree of freedom
N	Total number of degrees of freedom
$p(t)$	Applied external force
$\tilde{p}(\omega)$	Fourier transform of $p(t)$, i.e., load in the frequency domain
p_0	Force magnitude
\mathbf{p}	External force vector, $\mathbf{p} = \mathbf{p}(t)$
$q(t)$	Generalized coordinate, or modal coordinate
$\mathbf{q}(t)$	Vector of generalized coordinates
r	Tuning ratio $r = \omega / \omega_n$; radial coordinate
\mathbf{r}	Radial position vector
R	Radius of gyration or geometric radius
S_a	Ground response spectrum for absolute acceleration (pseudo-acceleration)
S_d	Ground response spectrum for relative displacements
S_v	Ground response spectrum for relative pseudo-velocity
t	Time
t_d	Time duration of load
t_p	Period of repetition of load
T	Period ($= 1/f$), or duration
T_d	Damped natural period

Notation and Symbols

xxvii

T_n	Natural period
$u(t)$	Absolute displacement. In general, $u = u(\mathbf{x}, t) = u(x, y, z, t)$
$\tilde{u}(\omega)$	Fourier transform of $u(t)$; frequency response function
u_0	Initial displacement, or maximum displacement
\dot{u}_0	Initial velocity
u_g	Ground displacement
u_h	Homogeneous solution (free vibration)
u_p	Particular solution
u_{p0}	Initial displacement <i>value</i> (not condition!) of particular solution
\dot{u}_{p0}	Initial velocity <i>value</i> (not condition!) of particular solution
\mathbf{u}	Absolute displacement vector
$\dot{\mathbf{u}}$	Absolute velocity vector
$\ddot{\mathbf{u}}$	Absolute acceleration vector
v	Relative displacement (scalar)
\mathbf{v}	Relative displacement vector
V	Potential energy; also, magnitude of velocity
V_{ph}	Phase velocity
x, y, z	Cartesian spatial coordinates
\mathbf{x}	Position vector
Z	Dynamic stiffness or impedance (ratio of complex force to complex displacement)
\mathbf{Z}	Impedance matrix

Greek Symbols

α	Angular acceleration
$\boldsymbol{\alpha}$	Angular acceleration vector
γ	Specific weight; direction cosines; participation factors
$\delta(t)$	Dirac-delta function (singularity function)
Δ	Determinant, or when used as a prefix, finite increment such as Δt
ε	Accidental eccentricity
λ	Lamé constant $\lambda = 2G\nu/(1-2\nu)$; also wavelength $\lambda = V_{ph}/f$
ϕ_{ij}	i th component of j th mode of vibration
$\boldsymbol{\phi}_j$	Generic, j th mode of vibration, with components $\boldsymbol{\phi}_j = \{\phi_{ij}\}$
φ	Rotational displacement or degree of freedom
$\boldsymbol{\Phi}$	Modal matrix, $\boldsymbol{\Phi} = \{\boldsymbol{\phi}_j\} = \{\phi_{ij}\}$
θ	Azimuth; rotational displacement, or rotation angle
ρ	Mass density
ρ_w	Mass density of water
ξ	Fraction of critical damping; occasionally dimensionless coordinate
μ	Mass ratio
τ	Time, usually as dummy variable of integration
ν	Poisson's ratio

- ω Driving (operational) frequency, in radians/second
- ω_d Damped natural frequency
- ω_n Natural frequency, in rad/s
- ω_j Generic j th modal frequency, in rad/s, or generic Fourier frequency
- $\boldsymbol{\omega}$ Rotational velocity vector
- $\boldsymbol{\Omega}$ Spectral matrix (i.e., matrix of natural frequencies), $\boldsymbol{\Omega} = \{\omega_j\}$

Derivatives, Integrals, Operators, and Functions

- Temporal derivatives $\frac{\partial u}{\partial t} = \dot{u}, \quad \frac{\partial^2 u}{\partial t^2} = \ddot{u}$
- Spatial derivatives $\frac{\partial u}{\partial x} = u', \quad \frac{\partial^2 u}{\partial x^2} = u''$
- Convolution $f * g \equiv f(t) * g(t) = \int_0^T f(\tau)g(t-\tau)d\tau = \int_0^T f(t-\tau)g(\tau)d\tau$
- Real and imaginary parts: If $z = x + iy$ then $x = \text{Re}(z), y = \text{Im}(z)$.
 (Observe that the imaginary part does *not* include the imaginary unit!).
- Signum function $\text{sgn}(x-a) = \begin{cases} 1 & x > a \\ 0 & x = a \\ -1 & x < a \end{cases}$
- Step load function $\mathcal{H}(t-t_0) = \begin{cases} 1 & t > t_0 \\ \frac{1}{2} & t = t_0 \\ 0 & t < t_0 \end{cases}$
- Dirac-delta function $\delta(t-t_0) = \begin{cases} 0 & t > t_0 \\ \infty & t = t_0, \\ 0 & t < t_0 \end{cases} \quad \int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t-t_0)dt = 1, \quad \varepsilon > 0$
- Kronecker delta $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$
- Split summation $\sum_{j=m}^n a_j = \frac{1}{2}a_m + a_{m+1} + \dots + a_{n-1} + \frac{1}{2}a_n$ (first and last element halved)

Unit Conversions

Fundamental Units

	Metric	
Length	Mass	Time
(m)	(kg)	(s)

	English	
Length	Force	Time
(ft)	(lb)	(s)

Length

Distance

1 m = 100 cm = 1000 mm
 1 dm = 10 cm = 0.1 m

1 ft = 12 in.
 1 yd = 3' = 0.9144 m
 1 mile = 5280 ft = 1609.344 m

1 in.	=	2.54 cm
1 ft	=	30.48 cm

Volume

1 dm³ = 1 [l]

Until 1964, the liter (or *litre*) was defined as the volume occupied by 1 kg of water at 4°C = 1.000028 dm³. Currently, it is defined as being *exactly* 1 dm³.

1 gallon = 231 in³ (exact!)
 1 pint = 1/8 gallon
 = 1/2 quart
 1 cu-ft = 1728 cu-in.
 = 7.48052 gallon
 1 quart = 2 pints = 0.03342 ft³
 = 0.946353 dm³ (liters)

1 gallon	=	3.785412 dm ³
1 cu-ft	=	28.31685 dm ³
1 pint	=	0.473176 dm ³

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Unit Conversions

Mass

$$\begin{array}{lcl}
 1 \text{ (kg)} & = & 1000 \text{ g} \\
 1 \text{ (t)} & = & 1000 \text{ kg (metric ton)} \\
 & = & 1 \text{ Mg}
 \end{array}
 \qquad
 \begin{array}{lcl}
 1 \text{ slug} & = & 32.174 \text{ lb-mass} \\
 & = & 14.594 \text{ kg}
 \end{array}$$

1 lb-mass	=	0.45359237 kg (exact!)
	=	453.59237 g
1 kg	=	2.2046226 lb-mass

Time

Second (s), also (sec)

Derived Units

Acceleration of Gravity

$$\begin{array}{lcl}
 G & = & 9.80665 \text{ m/s}^2 \quad (\text{exact normal value!}) \\
 & = & 980.665 \text{ cm/s}^2 \quad (\text{gals})
 \end{array}
 \qquad
 \begin{array}{lcl}
 G & = & 32.174 \text{ ft/s}^2 \\
 & = & 386.09 \text{ in./s}^2
 \end{array}$$

Useful approximation: $g \text{ (in m/s}^2) \approx \pi^2 = 9.8696 \approx 10$

Density and Specific Weight

$$\begin{array}{lcl}
 1 \text{ kg/dm}^3 & = & 1000 \text{ kg/m}^3 \\
 & = & 62.428 \text{ lb/ft}^3 \\
 & = & 8.345 \text{ lb/gal} \\
 & = & 1.043 \text{ lb/pint}
 \end{array}$$

1 ounce/ft³ = 1.0012 kg/m³ (an interesting near coincidence!)

Some specific weights and densities (approximate values):

		Spec. weight		Density
Steel	=	490 lb/ft ³	=	7850 kg/m ³
Concrete	=	150 lb/ft ³	=	2400 kg/m ³
Water	=	62.4 lb/ft ³	=	1000 kg/m ³
Air	=	0.0765 lb/ft ³	=	1.226 kg/m ³

