ADVANCED STRUCTURAL DYNAMICS

Advanced Structural Dynamics will appeal to a broad readership that includes both undergraduate and graduate engineering students, doctoral candidates, engineering scientists working in various technical disciplines, and practicing professionals in an engineering office. The book has broad applicability and draws examples from aeronautical, civil, earthquake, mechanical, and ocean engineering, and at times it even dabbles in issues of geophysics and seismology. The material presented is based on miscellaneous course and lecture notes offered by the author at the Massachusetts Institute of Technology for many years. The modular approach allows for a selective use of chapters, making it appropriate for use not only as an introductory textbook but later on functioning also as a treatise for an advanced course, covering materials not typically found in competing textbooks on the subject.

Professor Eduardo Kausel is a specialist in structural dynamics in the Department of Civil Engineering at the Massachusetts Institute of Technology. He is especially well known for two papers on the collapse of the Twin Towers on September 11, 2001. The first of this pair, published on the web at MIT only a few days after the terrorist act, attracted more readers around the world than all other works and publications on the subject combined. Professor Kausel is the author of the 2006 book *Fundamental Solutions in Elastodynamics* (Cambridge University Press).

Advanced Structural Dynamics

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To my former graduate student and dear Guardian Angel Hyangly Lee,
in everlasting gratitude for her continued support of my work at MIT.

Contents

Preface

The material in this book slowly accumulated, accreted, and grew out of the many lectures on structural dynamics, soil dynamics, earthquake engineering, and structural mechanics that I gave at MIT in the course of several decades of teaching. At first these constituted mere handouts to the students, meant to clarify further the material covered in the lectures, but soon the notes transcended the class environment and began steadily growing in size and content as well as complication. Eventually, the size was such that I decided that it might be worthwhile for these voluminous class notes to see the light as a regular textbook, but the sheer effort required to clean out and polish the text so as to bring it up to publication standards demanded too much of my time and entailed sacrifices elsewhere in my busy schedule that I simply couldn't afford. Or expressing it in MIT-speak, I applied the *Principle of Selective Neglect*. But after years (and even decades) of procrastination, eventually I finally managed to break the vicious cycle of writer's block and brought this necessary task to completion.

Make no mistake: the material covered in this book far exceeds what can be taught in any one-semester graduate course in structural dynamics or mechanical vibration, and indeed, even in a sequence of two such courses. Still, it exhaustively covers the fundamentals in vibration theory, and then goes on well beyond the standard fare in – and conventional treatment of – a graduate course in structural dynamics, as a result of which most can (and should) be excluded from an introductory course outline, even if it can still be used for that purpose. Given the sheer volume of material, the text is admittedly terse and at times rather sparse in explanations, but that is deliberate, for otherwise the book would have been unduly long, not to mention tedious to read and follow. Thus, the reader is expected to have some background in the mechanical sciences such that he or she need not be taken by the hand. Still, when used in the classroom for a first graduate course, it would suffice to jump over advanced sections, and do so without sacrifices in the clarity and self-sufficiency of the retained material.

In a typical semester, I would start by reviewing the basic principles of dynamics, namely Newton's laws, impulse and conservation of linear and angular momenta, D'Alembert's principle, the concept of point masses obtained by means of mass lumping and tributary areas, and most importantly, explicating the difference between static and dynamic degrees of freedom (or master–slave DOF), all while assuming small displacements and skipping initially over the section that deals with Lagrange's equations. From there on I would move on to cover the theory of single-DOF systems and devote just about half of the semester to that topic, inasmuch as multi-DOF systems and continuous systems can largely be regarded as generalizations of those more simple systems. In the lectures, I often interspersed demonstration experiments to illustrate basic concepts and made use of brief Matlab® models to demonstrate the application of the concepts being learned. I also devoted a good number of lectures to explain harmonic analysis and the use of complex Fourier series, which in my view is one of the most important yet difficult concepts for students to comprehend and assimilate properly. For that purpose, I usually started by explaining the concepts of amplitude and phase by considering a simple complex number of the form $z = x + \mathrm{i}\, y$, and then moving on to see what those quantities would be for products and ratios of complex numbers of the form $z = z_1 z_2$, $z = z_1/z_2 = |z_1|/|z_2|\,\mathrm{e}^{\mathrm{i}(\phi_1 - \phi_2)}$, and in particular $z = 1/z_2 = \mathrm{e}^{-\mathrm{i}\phi_2}/|z_2|$. I completely omitted the use of sine and cosine Fourier series, and considered solely the complex exponential form of Fourier series and the Fourier transform, which I used in the context of periodic loads, and then in the limit of an infinite period, namely a transient load. From there the relationship between impulse response function and transfer functions arose naturally. In the context of harmonic analysis, I would also demonstrate the great effectiveness of the (virtually unknown) *Exponential Window Method* (in essence, a numerical implementation of the Laplace Transform) for the solution of lightly damped system via complex frequencies, which simultaneously disposes of the problems of added trailing zeroes and undesired periodicity of the response function, and thus ultimately of the "wraparound" problem, that is, causality.

Discrete systems would then take me some two thirds of the second half of the semester, focusing on classical modal analysis and harmonic analysis, and concluding with some lectures on the vibration absorber. This left me just about one third of the half semester (i.e., some two to three weeks) for the treatment of continuous systems, at which time I would introduce the use of Lagrange's equations as a tool to solve continuous media by discretizing those systems via the *Assumed Modes Method*.

In the early version of the class lecture notes I included support motions and ground response spectra as part of the single-DOF lectures. However, as the material dealing with earthquake engineering grew in size and extent, in due time I moved that material out to a separate section, even if I continued to make seamless use of parts of those in my classes.

Beyond lecture materials for the classroom, this book contains extensive materials not included in competing books on structural dynamics, of which there already exist a plethora of excellent choices, and this was the main reason why I decided it was worthwhile to publish it. For this reason, I also expect this book to serve as a valuable reference for practicing engineers, and perhaps just as importantly, to aspiring young PhD graduates with academic aspirations in the fields of structural dynamics, soil dynamics, earthquake engineering, or mechanical vibration.

Last but not least, I wish to acknowledge my significant indebtedness and gratitude to Prof. José Manuel Roësset, now retired from the Texas A&M University, for his most invaluable advice and wisdom over all of the years that have spanned my academic career at MIT. It was while I was a student and José a tenured professor here that I learned with him mechanics and dynamics beyond my wildest expectations and

dreams, and it could well be said that everything I know and acquired expertise in is ultimately due to him, and that in a very real sense he has been the ghost writer and coauthor of this book.

> In problems relating to vibrations, nature has provided us with a range of mysteries which for their elucidation require the exercise of a certain amount of mathematical dexterity. In many directions of engineering practice, that vague commodity known as common sense will carry one a long way, but no ordinary mortal is endowed with an inborn instinct for vibrations; mechanical vibrations in general are too rapid for the utilization of our sense of sight, and common sense applied to these phenomena is too common to be other than a source of danger.
>
> C. E. Inglis, FRS, James Forrest Lecture, 1944

Notation and Symbols

Although we may from time to time change the meaning of certain symbols and deviate temporarily from the definitions given in this list, by and large we shall adopt in this book the notation given herein, and we shall do so always in the context of an upright, right-handed coordinate system.

Vectors and matrices: we use **boldface** symbols, while non-boldface symbols (in italics) are scalars. Capital letters denote matrices, and lowercase letters are vectors. (Equivalence with blackboard symbols: $\underset{\sim}{q}$ is the same as $\mathbf{q}$, and $\underline{M}$ is the same as $\mathbf{M}$).

Special Constants (non-italic)

e	Natural base of logarithms = 2.71828182845905…
i	Imaginary unit = $\sqrt{-1}$
π	3.14159265358979…

Roman Symbols

a	Acceleration
$\mathbf{a}$	Acceleration vector
A	Amplitude of a transfer function or a wave; also area or cross section
A_s	Shear area
b	Body load, $b = b(\mathbf{x}, t)$
$\mathbf{b}$	Vector of body loads, $\mathbf{b} = \mathbf{b}(\mathbf{x}, t)$
c	Viscous damping (dashpot) constant
C_1, C_2	Constants of integration
C_S	Shear wave velocity $\left(\sqrt{G/\rho}\right)$
C_r	Rod wave velocity $\left(\sqrt{E/\rho}\right)$
C_f	Flexural wave velocity $\left(\sqrt{C_r R\,\omega}\right)$
$\mathbf{C}$	Viscous damping matrix
$\mathbb{C}$	Modally transformed, diagonal damping matrix ($\mathbf{\Phi}^T\mathbf{C}\,\mathbf{\Phi}$)
D	Diameter
f	Frequency in Hz; it may also denote a flexibility

f_d	Damped natural frequency, in Hz
f_n	Natural frequency, in Hz
$\hat{\mathbf{e}}$	Cartesian, unit base vector $\left(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2, \hat{\mathbf{e}}_3 \equiv \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}\right)$
E	Young's modulus, $E = 2G(1+\nu)$
E_d	Energy dissipated
E_s	Elastic energy stored
$\hat{\mathbf{g}}$	Curvilinear base vector $(\hat{\mathbf{g}}_1, \hat{\mathbf{g}}_2, \hat{\mathbf{g}}_3)$
g	Acceleration of gravity
$g(t)$	Unit step-load response function
G	Shear modulus
h	Depth or thickness of beam, element, or plate
$h(t)$	Impulse response function
H	Height
$H(\omega)$	Transfer function (frequency response function for a unit input)
I	Area moment of inertia
j	Most often an index for a generic mode
J	Mass moment of inertia
k	Usually stiffness, but sometimes a wavenumber
k_c	Complex stiffness or impedance
K	Kinetic energy
$\mathbf{K}$	Stiffness matrix
L	Length of string, rod, beam, member, or element
m	Mass
$\mathbf{M}$	Mass matrix
n	Abbreviation for *natural*; also, generic degree of freedom
N	Total number of degrees of freedom
$p(t)$	Applied external force
$\tilde{p}(\omega)$	Fourier transform of $p(t)$, i.e., load in the frequency domain
p_0	Force magnitude
$\mathbf{p}$	External force vector, $\mathbf{p} = \mathbf{p}(t)$
$q(t)$	Generalized coordinate, or modal coordinate
$\mathbf{q}(t)$	Vector of generalized coordinates
r	Tuning ratio $r = \omega / \omega_n$; radial coordinate
$\mathbf{r}$	Radial position vector
R	Radius of gyration or geometric radius
S_a	Ground response spectrum for absolute acceleration (pseudo-acceleration)
S_d	Ground response spectrum for relative displacements
S_v	Ground response spectrum for relative pseudo- velocity
t	Time
t_d	Time duration of load
t_p	Period of repetition of load
T	Period $(= 1/f)$, or duration
T_d	Damped natural period

T_n	Natural period
$u(t)$	Absolute displacement. In general, $u = u(\mathbf{x},t) = u(x,y,z,t)$
$\tilde{u}(\omega)$	Fourier transform of $u(t)$; frequency response function
u_0	Initial displacement, or maximum displacement
$\dot{u}_0$	Initial velocity
u_g	Ground displacement
u_h	Homogeneous solution (free vibration)
u_p	Particular solution
u_{p0}	Initial displacement *value* (not condition!) of particular solution
$\dot{u}_{p0}$	Initial velocity *value* (not condition!) of particular solution
$\mathbf{u}$	Absolute displacement vector
$\dot{\mathbf{u}}$	Absolute velocity vector
$\ddot{\mathbf{u}}$	Absolute acceleration vector
v	Relative displacement (scalar)
$\mathbf{v}$	Relative displacement vector
V	Potential energy; also, magnitude of velocity
V_{ph}	Phase velocity
x, y, z	Cartesian spatial coordinates
$\mathbf{x}$	Position vector
Z	Dynamic stiffness or impedance (ratio of complex force to complex displacement)
$\mathbf{Z}$	Impedance matrix

Greek Symbols

α	Angular acceleration
$\boldsymbol{\alpha}$	Angular acceleration vector
γ	Specific weight; direction cosines; participation factors
$\delta(t)$	Dirac-delta function (singularity function)
Δ	Determinant, or when used as a prefix, finite increment such as Δt
ε	Accidental eccentricity
λ	Lamé constant $\lambda = 2G\nu/(1-2\nu)$; also wavelength $\lambda = V_{ph}/f$
ϕ_{ij}	ith component of jth mode of vibration
$\boldsymbol{\phi}_j$	Generic, jth mode of vibration, with components $\boldsymbol{\phi}_j = \{\phi_{ij}\}$
φ	Rotational displacement or degree of freedom
$\boldsymbol{\Phi}$	Modal matrix, $\boldsymbol{\Phi} = \{\boldsymbol{\phi}_j\} = \{\phi_{ij}\}$
θ	Azimuth; rotational displacement, or rotation angle
ρ	Mass density
ρ_w	Mass density of water
ξ	Fraction of critical damping; occasionally dimensionless coordinate
μ	Mass ratio
τ	Time, usually as dummy variable of integration
ν	Poisson's ratio

ω Driving (operational) frequency, in radians/second
ω_d Damped natural frequency
ω_n Natural frequency, in rad/s
ω_j Generic jth modal frequency, in rad/s, or generic Fourier frequency
$\boldsymbol{\omega}$ Rotational velocity vector
$\boldsymbol{\Omega}$ Spectral matrix (i.e., matrix of natural frequencies), $\boldsymbol{\Omega} = \{\omega_j\}$

Derivatives, Integrals, Operators, and Functions

Temporal derivatives $\frac{\partial u}{\partial t} = \dot{u}, \qquad \frac{\partial^2 u}{\partial t^2} = \ddot{u}$

Spatial derivatives $\frac{\partial u}{\partial x} = u', \qquad \frac{\partial^2 u}{\partial x^2} = u''$

Convolution $f * g \equiv f(t) * g(t) = \int_0^T f(\tau)\, g(t-\tau)\, d\tau = \int_0^T f(t-\tau)\, g(\tau)\, d\tau$

Real and imaginary parts: If $z = x + \mathrm{i}\, y$ then $x = \mathrm{Re}(z), \quad y = \mathrm{Im}(z)$.

(Observe that the imaginary part does *not* include the imaginary unit!).

Signum function $\operatorname{sgn}(x-a) = \begin{cases} 1 & x > a \\ 0 & x = a \\ -1 & x < a \end{cases}$

Step load function $\mathcal{H}(t-t_0) = \begin{cases} 1 & t > t_0 \\ \frac{1}{2} & t = t_0 \\ 0 & t < t_0 \end{cases}$

Dirac-delta function $\delta(t-t_0) = \begin{cases} 0 & t > t_0 \\ \infty & t = t_0 \\ 0 & t < t_0 \end{cases}, \qquad \int_{t_0-\varepsilon}^{t_0+\varepsilon} \delta(t-t_0)\, dt = 1, \qquad \varepsilon > 0$

Kronecker delta $\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$

Split summation $\sum_{j=m}^{n}{}' a_j = \frac{1}{2} a_m + a_{m+1} + \cdots + a_{n-1} + \frac{1}{2} a_n$ (first and last element halved)

Unit Conversions

Fundamental Units

	Metric	
Length	Mass	Time
(m)	(kg)	(s)

	English	
Length	Force	Time
(ft)	(lb)	(s)

Length

Distance

1 m = 100 cm = 1000 mm
1 dm = 10 cm = 0.1 m

1 ft = 12 in.
1 yd = 3'= 0.9144 m
1 mile = 5280 ft = 1609.344 m

1 in.	=	2.54 cm
1 ft	=	30.48 cm

Volume

1 dm^3 = 1 [l]

Until 1964, the liter (or *litre*) was defined as the volume occupied by 1 kg of water at 4°C = 1.000028 dm^3. Currently, it is defined as being *exactly* 1 dm^3.

1 gallon = 231 in^3 (exact!)
1 pint = 1/8 gallon
= ½ quart
1 cu-ft = 1728 cu-in.
= 7.48052 gallon
1 quart = 2 pints = 0.03342 ft^3
= 0.946353 dm^3 (liters)

1 gallon	=	3.785412 dm^3
1 cu-ft	=	28.31685 dm^3
1 pint	=	0.473176 dm^3

Mass

1 (kg)	=	1000 g	1 slug	=	32.174 lb-mass
1 (t)	=	1000 kg (metric ton)		=	14.594 kg
	=	1 Mg			

1 lb-mass	=	0.45359237 kg (exact!)
	=	453.59237 g
1 kg	=	2.2046226 lb-mass

Time

Second (s), also (sec)

Derived Units

Acceleration of Gravity

G	=	9.80665 m/s^2	(exact normal value!)	G	=	32.174 ft/s^2
	=	980.665 cm/s^2	(gals)		=	386.09 in./s^2

Useful approximation: g (in m/s^2) $\approx \pi^2 = 9.8696 \approx 10$

Density and Specific Weight

1 kg/dm^3	=	1000 kg/m^3	=	62.428 lb/ft^3
			=	8.345 lb/gal
			=	1.043 lb/pint

1 ounce/ft^3 = 1.0012 kg/m^3 (an interesting near coincidence!)

Some specific weights and densities (approximate values):

		Spec. weight		Density
Steel	=	490 lb/ft^3	=	7850 kg/m^3
Concrete	=	150 lb/ft^3	=	2400 kg/m^3
Water	=	62.4 lb/ft^3	=	1000 kg/m^3
Air	=	0.0765 lb/ft^3	=	1.226 kg/m^3

Force

1 N [Newton] = force required to accelerate 1 kg by 1 m/s^2

9.81 N	=	1 kg-force	= 1 kp	(“kilopond”; widely used in Europe in the past, it is a metric, non-SSI unit!)
1 lb	=	4.44822 N		(related unit: 1 Mp =1 “megapond” = 1 metric ton)
		0.45359 kg-force		

Pressure

1 Pa	=	1 N/m^2	Normal atmospheric pressure	= 1.01325 bar (15°C, sea level)
1 kPa	=	10^3 Pa		= 101.325 kPa (exact!)
1 bar	=	10^5 Pa		= 14.696 lb/in^2 = 0.014696 ksi
1 ksi	=	6.89476 MPa		= 2116.22 lb/ft^2

Power

1 kW = 1000 W = 1 kN-m/s = 0.948 BTU/s

1 HP = 550 lb-ft/s = 0.707 BTU/s = 0.7457 kW
1 BTU/s = 778.3 lb-ft/s = 1.055 KW
1 CV = 75 kp × m/s = 0.7355 kW “Cheval Vapeur”

Temperature

T(°F) = 32 + 9/5 T(°C) (some exact values: –40°F = –40°C, 32°F = 0°C and 50°F = 10°C)