Bond Pricing and Yield-Curve Modelling

A Structural Approach

This book provides the theoretical foundations (no-arbitrage, convexity, expectations, affine modelling) for a treatment of government bond markets; presents and critically discusses the wealth of empirical findings that have appeared in the literature in the last decade; and introduces the 'structural' models that are used by central banks, institutional investors, sovereign wealth funds, academics and advanced practitioners to model the yield curve, to answer policy questions, to estimate the magnitude of the risk premium, to gauge market expectations and to assess investment opportunities. The book weaves precise theory with up-to-date empirical evidence to build, with the minimum mathematical sophistication required for the task, a critical understanding of what drives the government bond market.

Riccardo Rebonato is Professor of Finance at the EDHEC Business School and the EDHEC Risk Institute and holds the EDHEC PIMCO Research Chair. He has been Global Head of Fixed Income and FX Analytics at PIMCO and Head of Research, Risk Management and Derivatives Trading at several major international banks. He has previously held academic positions at Imperial College and Oxford University, and has been a board director for the International Swaps and Derivatives Association (ISDA) and the Global Association of Risk Professionals (GARP). He currently sits on the board of The Nine Dots Prize. He is the author of several books and articles in finance and risk management, including *Portfolio Management under Stress* (2004).

Bond Pricing and Yield-Curve Modelling

A Structural Approach

Riccardo Rebonato
EDHEC Business School
EDHEC Risk Institute

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi - 110025, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107165854
DOI: 10.1017/9781316694169

First published 2018

Printed in the United Kingdom by Clays, Ltd.

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-16585-4 Hardback

To the memory of my father, to my wife and to my son, with thanks.

Contents

Acknowledgements

I owe a great debt of gratitude to a number of friends and colleagues. First and foremost, an anonymous referee has produced the most thorough and constructively critical review of an earlier draft of this manuscript that I have ever received in my publishing career. If he or she had written this book, I am sure it would have been far better than what I managed to produce. Luckily (for me) he or she didn' t. Thank you.

My ex-colleagues at PIMCO have been a constant source of inspiration, illumination, challenge and insight. I have benefitted greatly from illuminating discussions with (in alphabetical order) Mr Mukundan Deverajan, Dr Soraya Kazziha, Mr Ravi Mattu, Dr Vasant Naik, Dr Niels Pedersen, Dr David Pottinton, Dr Vlad Putiatyn, Dr Wendong Qu, Mr Jeremy Rosten, Mr Ivan Saroka and Dr Chelsea Wang. These ex-colleagues have been generous with their time and suggestions beyond what is reasonable to expect.

Among friends, Dr Andrei Liasenko has shown great interest in several early drafts of the manuscript, corrected small and not-so-small mistakes and suggested a far more elegant (and, the reader will be happy to hear, simpler) proof for an important result in Chapter 33. Ms Jean Whitmore has made use of her unfailing good judgment to suggest ways in which an earlier version of the work could be improved.

Professors Francis Diebold, Michael Dempster, Rene Garcia, Lionel Martinelli and Raman Uppal have provided very useful comments. Prof Paolo Veronesi has pointed me in the direction of important material which has found its way into the book.

Several of my students, both from EDEHC and from my previous academic affiliation (The Mathematical Finance Department at Oxford University), have provided a springboard for discussing the ideas presented in this book. In particular, I would like to thank Dr Daniela Villegas, Dr Tramback, Dr Taku Hatano and Dr Alexey Eherekinski.

I have benefited greatly from discussion with Dr Taylor Spears, and with the delegates at the Global Derivatives conferences delegates in Amsterdam (2014–2015) and Budapest (2016).

The editors at Cambridge University Press have displayed an enthusiasm for the project that has delighted – and surprised – me, all the more because this is my second book with them. Special thanks go to Mr Chris Harrison for his help and guidance, Ms Karen Maloney for her enthusiasm and the production team for their endless patience. I am truly touched.

My father and my wife have given me all the support I could have wished for the project. My eight-year-old son has forgiven me, I hope, for not playing with him as much as I would have otherwise; perhaps he has been so understanding because he has had a chance to build a few thousand paper planes with the earlier drafts of this book.

Despite all this help and encouragement, and despite the care I have devoted to this work, I am sure that many errors still lurk in its 750-plus pages. It goes without saying that I am fully responsible for these errors; but I will say so nonetheless.

Symbols and Abbreviations

LATIN SYMBOLS

A_t^T = A scalar that enters the expression for P_t^T, ie, for the price at time t of a zero-coupon bond of maturity T, as in $P_t^T = e^{A_t^T + (B_t^T)^{\mathrm{T}} x_t}$
B_t^T = A vector that enters the expression for P_t^T, ie, for the price at time t of a zero-coupon bond of maturity T, as in $P_t^T = e^{A_t^T + (B_t^T)^{\mathrm{T}} x_t}$
$BEI_{t,T}$ = Break-even inflation at time t for maturity T
c_t = Consumption at time t
$Conv_t^T$ = Time-t convexity of a T-maturity (discount) bond
$D = \sum_{i=1,n} \omega_i \left(B_t^{T_i}\right) \left(B_t^{T_i}\right)^{\mathrm{T}}$
Dur_t^T = Time-t duration of a T-maturity (discount) bond
EI_t^T = Time-t expected inflation per unit time over the period $[t, T]$
f_t^T = Value at time t of an instantaneous forward rate expiring at time T
$F_t^{T,T+\tau}$ = Value at time t of a discrete forward rate expiring at time T and covering a notional borrowing/lending period of length τ
$\{l_i\}$ = Eiegenvalues of the reversion-speed matrix
m_{t+1} = Stochastic discount factor for time $t+1$
$M_t = \beta u'(c_t)$ stochastic discount factor in continuous time
p_t^T = Log price at time t of a zero-coupon bond of maturity T
p_s = Probability of reaching state s
P_t^T = Price at time t of a zero-coupon bond of maturity T
$P_{t,T}^R$ = Price at time t of a real zero-coupon bond of maturity T
$q_t \equiv \log Q_t$
Q_t = Value of the price process at time t
r_t = Value of the short rate at time t
r_t^R = Value of the real short rate at time t
r_f = Riskless rate
ret_t^T = Annualized return from investing in the T-maturity discount bond
R^f = Gross one-period riskless return
$\underline{S}$= Volatility matrix for an affine model
S_t = Time-t price of a generic security

sp_s = State price at time s
SR = Sharpe Ratio
$u(c_t)$ = Utility for consumption at time t
W = Initial wealth
x_t = A scalar or a vector that denotes the state variables in an affine model
$xret_t^T$ = Annualized *excess* return from investing in the T-maturity discount bond
y_t^T = Yield at time t of a zero-coupon bond of maturity T
$y_t^{nom,T}$ = Nominal yield at time t of a zero-coupon bond of maturity T
$y_t^{\text{real},T}$ = Real yield at time t of a zero-coupon bond of maturity T

GREEK SYMBOLS

$\alpha_t^T = -\frac{1}{T-t}A_t^T$: One of the two quantities that links yields to state variables for affine models, as in $y_t^T = \alpha_t^T + \left(\beta_t^T\right)^{\mathrm{T}} x_t$
$\beta_t^T = -\frac{1}{T-t}A_t^T$: One of the two quantities that links yields to state variables for affine models, as in $y_t^T = \alpha_t^T + \left(\beta_t^T\right)^{\mathrm{T}} x_t$
β = Time impatience term in the stochastic discount factor: $m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$
κ = Reversion speed in a one-dimensional mean-reverting process
θ = Reversion level (vector or scalar) for an affine model
$\Theta_t^T \equiv \left(\frac{Q_t^T}{Q_t^t}\right)^{-1}$
λ = Market price of risk
Λ = Matrix of eigenvalues of the reversion-speed matrix
$\mu_r^{\mathbb{P}(\mathbb{Q})}$ = Drift in the real-world (risk-neutral) measure of the short rate
ν_0 = The price of a security today
π_s = State-price deflator
$\pi_t^{\text{real},T}$ = Real state-price deflator
σ_P = Bond price volatility
σ_r = Volatility of the short rate
$\sigma_{y_t^T}$ = Volatility of a yield of expiry T
$\tau \equiv T - t$
ω_i = The ith weight in a portfolio

FRAKTUR AND CALLIGRAPHIC SYMBOLS

$\mathfrak{CP}$ = Convexity of a portfolio
$\mathfrak{Cp}_t^T(Vasicek)$ = Bond price convexity in the Vasicek model
$\mathfrak{Cy}_t^T(Vasicek)$ = Yield convexity in the Vasicek model
$\mathrm{Ci}_{t,T}$ = Convexity term in the expression for break-even inflation in the real-world measure
$\mathrm{Ci}_{t,T}^{\mathbb{Q}}$ = Convexity term in the expression for break-even inflation in the risk-neutral measure

$\mathrm{Ci}^{\mathbb{T}}_{t,T}$ = Convexity term in the expression for break-even inflation in the risk-neutral measure
$\mathcal{K}$ = Reversion speed matrix for an affine model
$\mathcal{L}^T_t$ = Liquidity component in the decomposition of break-even inflation: $BEI_{t,T} = \mathbb{E}\left(I^T_t\right)^{\mathbb{P}} + \mathcal{P}^T_t + \mathcal{L}^T_t$
$\mathcal{M}^N_t$ = The Ratio of the nominal stochastic discount factors: $\mathcal{M}^N_t = \frac{\pi^N(T)}{\pi^N(t)}$
$\mathcal{N}\left(\mu, \sigma^2\right)$ = Normal distribution with mean, μ, and variance, σ^2
$\mathcal{P}^T_t$ = Risk premium required by an investor in order to bear inflation risk
$\mathbb{P}$ = Real-world measure
$\mathbb{Q}$ = Risk-neutral measure
$\mathbb{S}$ = Subjective measure
$\mathcal{S}_\tau$ = The Set of all the possible future states at time τ
$\mathbb{T}$ = Terminal measure