Theory of Social Choice on Networks

Classical social choice theory relies heavily on the assumption that all individuals have fixed preference orderings. This highly original book presents a new theory of social preferences that explicitly accounts for important social phenomena such as coordination, compromise, negotiation, and altruism. Drawing on network theory, it extends classical social choice theory by constructing a framework that allows for dynamic preferences that are modulated by the situation-dependent social influence that they exert on each other. In this way the book shows how members of a social network may modulate their preferences to account for social context. This important expansion of social choice theory will be of interest to readers in a wide variety of disciplines, including economists and political scientists concerned with choice theory as well computer scientists and engineers working on network theory.

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Theory of Social Choice on Networks

Preference, Aggregation, and Coordination

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To my eternal companion Patti
who, for me, is the epitome of socialis,
Latin for "companionship."

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The only mental tool by means of which a very finite piece of reasoning can cover a myriad cases is called "abstraction" . . . The purpose of abstraction is *not* to be vague, but to create a new semantic level in which one can be absolutely precise.

— Edsger Dijkstra
The Humble Programmer, ACM Turing Lecture, 1972

At root, mathematics is the name we give to the collection of all possible patterns and interrelationships . . . The essence of mathematics lies in the relationships between quantities and qualities.

— John D. Barrow
Impossibility (Oxford University Press, 1998)

Preface and Acknowledgments

Network theory provides a powerful and expressive framework for the analysis and synthesis of collectives whose members exert social influence on each other. When such a collective is engaged in a social choice, all social relationships that could influence the decision must be taken into consideration. This book advances social choice theory by introducing extended concepts of preference, aggregation, deliberation, and coordination that enable the group to incorporate social influence relationships into a comprehensive social model from which a coordinated social choice can be deduced.

Historically, social choice theory has focused mainly on the study of human behavior and has principally fallen under the purview of the social sciences. Increasingly, however, computer science has applied social choice theory to the design and synthesis of artificial societies such as multiagent systems and networks. A principle motivation for this book is to present a view of the theory that is applicable to both cultures.

Although both the social science and computer science disciplines rely on abstract mathematical models, they use them differently. Social science uses social models primarily as *analysis* tools to understand, predict, explain, or recommend behavior for human society. Such models may provide useful insights regarding social behavior, but they are not causal – they do not dictate behavior. They are idealized approximations whose validity hinges on assumptions regarding human social behavior. Computer science and engineering, however, use social models as *synthesis* tools to design and construct artificial social systems populated by autonomous agents who are designed to function in ways that are compatible with human behavior. In this sense, the models are causal, since they generate the behavior of the members of the society as they interact.

The difference between analysis and synthesis is that with analysis, models are used to reduce reality *to* an abstraction, while synthesis

uses models to create a reality *from* an abstraction. The difference between these two applications is important. With analysis, psychological or sociological attributes such as cooperation and altruism, or even such overtly antisocial attributes as conflict and avarice, can be ascribed to individuals as a function of the solution concept, even if such attributes are not formally part of the mathematical model. But when synthesizing an artificial society, such attributes must be explicitly incorporated into the mathematical model or they will not exist.

This book fits at the intersection of social science and computer science. It provides an interface between game theory, social choice theory, and welfare economics on the one hand, and artificial intelligence, multiagent systems, and distributed control theory on the other. It advances social choice theory for both analysis and synthesis applications. It is intended to reach an interdisciplinary audience comprising academics, practitioners, and students of general decision theory from the social science, management science, control engineering, computer science, and biological science disciplines. They will find the material covered in this book to be quite different from the standard treatments of social choice theory, but with the hope of fostering renewed interest in the fundamental assumptions that underlie the theory and its applicability to complex social networks. In addition, since this book touches on such philosophical issues as epistemology and rationality, it should be of interest to philosophers and other students of rational choice theory.

This work is truly an interdisciplinary study; it is a merging of concepts from social science, engineering/computer science, and philosophy. As is the case with many interdisciplinary efforts, it asks readers from all disciplines to tolerate the sometimes awkward attempts to bridge the gap between them. In particular, I ask for the forbearance of social scientists who may consider the treatment of traditional social choice theory as limited. My intent is *not* to criticize a theory that has proven to be of great value. Rather, it is to add to the overall discussion – to supplement, not supplant. Thus, I apologize in advance for any inaccurate representations of social choice theory.

One may indeed wonder how a person trained in electrical engineering with a specialization in optimal control theory would develop an interest in social choice theory. In fact, this book is in many ways the unintended consequence of a rather unconventional intellectual

journey. It began with an engineering question: What are reasonable principles that would govern the design of a collective of autonomous distributed agents who are designed to behave in accord with human notions of rational behavior? The conventional notions of optimization theory do not easily extend to distributed systems, and constrained optimization solutions, such as Nash equilibria, place the accent on defensive behavior and do not foster cooperation (e.g., the Prisoner's Dilemma and other mixed-motive scenarios). Of course, there are many ways to generalize the notion of optimization, such as seeking Pareto efficient solutions to multi-attribute decision problems, but at the end of the day, such approaches rely on some notion of maximization, which is an intrinsically individualized concept. The hypothesis governing the development in this book, however, is that when dealing with multiagent decision making in contexts where individuals exert social influence on each other, it is often more relevant to evaluate behavior according to concepts that are intrinsically social. In that regard, a fundamental attribute of such a collective is its ability to coordinate. Etymologically, coordination comes from the Latin *co* (together) + *ordinare* (to regulate). The Oxford English Dictionary defines *to coordinate* as

> to place or arrange (things) in proper position relative to each other and to the system of which they form parts; to bring into proper combined order as parts of a whole

(Murray et al., 1991). Thus, from a group perspective, the ability of a collective to arrange itself according to some systematic notion of group-level behavior is a more relevant concept than for each individual to pursue a solution that is best for the self according to a measure of individual material benefit, but may not be good for the group or even for the self when viewed from a social perspective.

Placing emphasis on coordination, rather than focusing exclusively on optimization, requires more than a change in perspective. It requires an expanded concept of individually rational behavior and an enhanced mechanism by which individuals may express their preferences in terms of both material and social interests. The pursuit of these goals led to my book *Theory of Conditional Games* (Stirling, 2012). This present work extends that approach to social choice problems where explicit social influence exists among the members of the collective, and where social behavior is characterized in terms

of systematic group-level behavior that emerges as a result of shared interests.

Many colleagues have contributed to the ideas contained in this work. Of particular note are three of my former graduate students – Darryl Morrell, Michael Goodrich, and Matthew Nokleby – whose creativity and enthusiasm have sustained me many times, and whose ideas are integral to the development of this theory. I have also greatly benefited from my collaborations with Richard Frost, Harold Miller, Todd Moon, Dennis Packard, Teppo Felin, and Luca Tummolini. The research environment and support offered by Brigham Young University and, in particular, by my colleagues in the Electrical and Computer Engineering Department who have graciously tolerated my unorthodox research agenda, are greatly appreciated. My greatest debt, however is to my wife Patti, whose love has sustained and nourished me for nearly five decades. Her special insights and wisdom contribute an intangible, but nevertheless essential, element to this enterprise.

Introduction

If we want to start new things rather than trying to elaborate and improve old ones, then we cannot escape reflecting on our basic conceptions.

— Hans Primas
Chemistry, Quantum Mechanics, and Reductionism
(Springer-Verlag, 1981)

A social choice arises when a collective of autonomous individuals must choose one and only one element from a set of distinct and mutually exclusive alternatives. The most widely established way to frame this issue is to assume that each individual comes to the social engagement with a fixed linear preference ordering over the set of alternatives that accounts for all issues deemed to be relevant to the individual's welfare. Given this profile of *ex ante* preference orderings, the social choice problem reduces, essentially, to defining an appropriate aggregation mechanism that produces a rational social choice.

A social influence network comprises a collective whose members are able to influence each other through some direct means of communication or control. When faced with a social choice, the members of such a network may each have preferences over the set of alternatives in terms of individual welfare, but they are also subject to the social influence that others exert on them. As the individuals interact, this influence propagates through the collective, creating a complex social structure as the end result of a deliberative process by which individuals incorporate the influence of others into their own preferences. In such an environment, a set of *ex ante* preference orderings expressed in terms of individual welfare is not a complete manifestation of the social structure and may even obscure the true nature of the society. This is the nub of the issue. A true characterization of the society requires a comprehensive social model defined in terms of social influence relationships as well as individual

welfare, from which a coordinated social choice that incorporates all of the social influence can be deduced.

This book undertakes the task of developing such a model. Unlike many treatments of social choice theory, however, the point of departure is not Arrow's impossibility theorem. Arrow's fundamental hypothesis is that all members of a society come to a social engagement with fully formed preference orderings from which all issues regarding aggregation and the existence of social welfare functions and social choice functions stem. The approach taken by this work, however, is to move upstream to get closer to the headwaters of the way preferences are formed when individuals are subjected to social influence. The focus is on the structure of the social linkages between individuals, that is, on the mechanisms by which influence is transmitted and incorporated, rather than on psychological or sociological motivations for behavior. The distinctive features of this approach are as follows.

- Networks are expressed as directed graphs whose vertices are individuals and whose edges constitute the medium by which social influence is propagated between individuals.
- The individual preference framework is extended to enable members of the network to modulate their preferences in response to the social influence that others exert on them.
- The concept of aggregation is expanded to generate a comprehensive social model that provides a compete characterization of the complex social structure that emerges as influence propagates through the network and social relationships are formed. A coordinated social choice that takes full account of all social relationships can then be deduced from the social model.
- A social deliberation process is developed that enables dialogue among the members of the network in order to pursue a compromise social choice.
- A formal mathematical theory is established to quantify the intrinsic ability of a network to produce coordinated behavior.
- The model is extended to incorporate stochastic agents into a network and to accommodate social choice scenarios where individuals possess randomized preferences.
- The concept of neo-satisficing is applied to social choice theory in order to provide a flexible framework within which groups can develop meaningful negotiation protocols.

All of these features are entirely consistent with conventional social choice theory, which becomes a special case of this more general approach.

Chapter Synopses

Preference Chapter 1 extends the notion of individually rational behavior whereby the members of an influence network expand their interests beyond their own narrowly construed individual welfare in order to incorporate the influence of others into their own rationality. Such a network is expressed as a graph whose vertices are the individuals and whose edges transmit the social influence (using common graph-theoretic terminology) from parent to child. Influence is conveyed as the antecedent of a hypothetical proposition regarding the preferences of the parents, and the consequent is the resulting *conditional preference ordering* for the child given the hypothesized preferences, termed *conjectures*, for the parents. These hypothetical propositions thus serve as the mechanism by which individuals modulate their preferences in response to the social influence that is exerted on them.

Aggregation The propagation of social influence throughout a network raises the possibility that some form of systematic behavior may emerge as a result of nascent social relationships that are thus formed. To the extent that the interests of the individuals are shared through the social linkages, the collective may possess an ability to coordinate in the sense that individual behaviors generate some form of rational behavior for the group. Chapter 2 develops a generalized concept of aggregation that combines the conditional preferences to create a social model that characterizes such emergent behavior. Key features of this model are that a) it is endogenous, in the sense that it is a function of, and only of, the conditional preferences; b) it is comprehensive, in that it captures all of the social relationships that emerge as the individuals interact; and c) it is socially coherent in the sense that no individual can either unilaterally subvert coordinated behavior or is the victim of categorical subjugation. By appealing to the Dutch book theorem and Bayesian network theory, it is established that these conditions can be met if, and only if, the conditional preferences are combined according to the syntax of probability theory.

Deliberation The presence of social influence, coupled with the ability of individuals to modulate their preferences in response, renders it possible for individuals to engage in dialogue. This ability addresses one of the criticisms leveled against social choice theory by advocates of deliberative democracy, who argue that constraining individuals to fixed preferences prohibits any form of deliberation. Chapter 3 extends the basic model structure developed in Chapter 2 to account for influence cycles, which enables individuals to engage in back-and-forth discussions and thereby modify their preferences. The presence of influence cycles, however, introduces the possibility of unstable behavior, where individual preferences continually oscillate. This chapter establishes necessary and sufficient conditions to ensure that cyclic conditional preferences converge to stable preferences. The key mathematical basis for this extension is the Markov convergence theorem.

Coordination The presence of social influence generates an ability for the network to coordinate its behavior *as a function of the social structure*. Chapter 4 introduces a mathematically precise concept of coordination that arises due to the social influence relationships that exist among the members of the network. The key theoretical result, based on Shannon information theory, is the development of a coordinatability index that provides a measure of the intrinsic ability of the network to coordinate as a result of social influence.

Randomization Chapter 5 deals with social choice in the presence of uncertainty. Two manifestations of stochastic social choice are developed. One involves the incorporation of stochastic agents into the network as fully integrated members of the society, and the other involves situations where the agents employ mixed strategies and generate the expected utility of the social choice.

Satisficing Chapter 6 presents a reformulation of social choice in terms of neo-satisficing theory as originally developed in Stirling (2003). Classical concepts of decision making, including social choice theory, are developed according to a concept of rational behavior based on inter-alternative preference comparisons, which are designed to identify optimal solutions. Optimization, however, is an intrinsically individual-based concept. For members of a group to comply with this form of rational behavior, they must seek a constrained

optimal solution (e.g., a Nash equilibrium). The accent falls on defensive behavior, where others are viewed more as constraints than as partners. Neo-satisficing, however, employs a concept of rational behavior based on intra-alternative comparisons, which is designed to identify solutions that are "good enough." This more flexible concept of rational behavior easily accommodates the design of negotiation protocols that lead to compromise solutions (what is best for me may not be best for you, but there may be a compromise that is good enough for both of us).

Appendices Since this book employs several mathematical concepts that are not normally encountered in conventional treatments of social choice theory, several appendices has been included that provide additional detail and proofs in order to make this book as self-contained as possible. Appendix A provides a proof of the Dutch book theorem; Appendix B establishes important features of Bayesian networks; Appendix C summarizes key concepts of measure theory and probability theory; Appendix D provides a proof of the Markov convergence theorem; and Appendix E summarizes key features of Shannon information theory.