Differential Games in Industrial Economics

Game theory has revolutionised our understanding of industrial organisation and the traditional theory of the firm. Despite these advances, industrial economists have tended to rely on a restricted set of tools from game theory, focussing on static and repeated games to analyse firm structure and behaviour. Luca Lambertini, a leading expert on the application of differential game theory to economics, argues that many dynamic phenomena in industrial organisation (such as monopoly, oligopoly, advertising and R&D races) can be better understood and analysed through the use of differential games. After illustrating the basic elements of the theory, Lambertini guides the reader through the main models, spanning from optimal control problems describing the behaviour of a monopolist through to oligopoly games in which firms' strategies include prices, quantities and investments. This approach will be of great value to students and researchers in economics and those interested in advanced applications of game theory.

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To Monica, just because

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Preface

The main aim of this volume is to do justice to a literature which has developed itself at the intersection of several disciplines but, unlike others using similar formal instruments (as, for example, growth theory), has not yet been given a systematic reconstruction. To avoid misunderstandings, I want to point out that the material contained in this volume covers the literature using continuous time models in industrial economics, i.e., either *optimal control problems* or *differential games*. Covering also dynamic models in discrete time would require at least the same space, or transforming the book into a large survey. Either way, the volume would become hardly useful.

After the adoption of the game theory approach, the analysis of oligopolistic competition has taken a completely new angle as compared to the previous view of industrial economics prevailing until the early 1970s. The revolution generating what we now call the theory of industrial organization (IO) has shed light on topics which had remained at the margin of the discipline for decades, creating from scratch a number of research strands. Some of the topics addressed in these fields of IO – if not all – have an explicit and intuitive dynamic nature. Capacity accumulation, research & development (R&D) and advertising are obvious examples. Yet, the game theory toolkit has included static (often multistage) and repeated games, and Markovian games in discrete time. In static multistage games, time is blackboxed, while repeated games take a time invariant constituent stage game and typically insert time and time discounting to look at critical thresholds of the latter (as in folk theorems investigating the stability of implicit collusion in prices or outputs).

Proper dynamic games in either discrete or continuous time have been seldom used. This is apparent from dominant textbook in IO at different levels (Tirole, 1988; Martin, 1993, 2002; Shy, 1995; Belleflamme and Peitz, 2010), where dynamics in continuous time is usually confined to the exposition of models dealing with R&D races. Even in Fudenberg and Tirole (1986), a relevant portion of the text treats repeated games while most of the remainder looks at R&D competition. A relevant exception is Fudenberg and Tirole (1991), in which differential game theory is presented and complemented with an illustration of oligopoly games with capacity accumulation games. However, for a wider perspective on differential games in IO and related fields (environmental and resource economics and trade theory), the choice is confined to Dockner *et al.* (2000), Erickson (2003), Jørgensen and Zaccour (2004), Long (2010), Lambertini (2013), and discrete time models (Sorger 2015).

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In a nutshell, the difference between a static game (whether one-shot, repeated, or multistage) and a truly dynamic game is the presence of at least one state variable whose motion is governed by its own dynamic equation hosting the players' controls and – in turn – affecting the players' performance at any time during the game. To grasp intuitively the implications of the presence of state variables, it suffices to note that this is exactly what is lost for good when one builds up a multistage game to investigate, say, the interplay between a long-run variable (say, R&D or product quality/location and, *a fortiori*, the stock of natural resources or environmental damages) and a short-run one (prices or quantities). As mentioned earlier, this approach is usually thought of as blackboxing a dynamic process which remains behind the curtains of the static model. The latter, including any policy implications it may produce, is indeed reliable if and only if the results are consistent with those one would find solving the dynamic version of the same problem. For instance, to be more explicit, if the limit of the dynamic model coincides with the equilibrium of the static one. In other words, is the static equilibrium always an accurate snapshot of the steady state equilibrium of its truly dynamic version? Even if the answer is affirmative, it is still true that the static model is silent about the transition to the equilibrium.

Moreover, the equilibria of static (and often multistage) games are built to be subgame perfect and therefore credible. This raises another crucial question as to whether and when the limit of a dynamic game confirms the predictions of its static version. One of the pieces of common wisdom about differential games – not entirely justified in itself – holds that the open-loop solution is (quasi-)static. In fact, what usually happens is that the limit of the open-loop equilibrium coincides with the equilibrium of the static game, for instance when discounting is absent. However, this is almost never the case when feedback effects are operating, unless the game features specific properties. Whenever the subgame perfect equilibria of the static and dynamic version of a game do not coincide (even in the limit), the descriptive and normative powers of the static model are compromised and potentially counterproductive.

The foregoing considerations motivate revisiting anew a consolidated theory using the alternative lens of differential game theory, so as to

- deliver a coherent view of the existing research on the dynamics of competition along several dimensions associated with market and non-market variables;
- offer a forward-looking perspective on future developments concerning topics which have recently entered the picture (e.g., portfolio race games) or largely disregarded because of technical difficulties but representing obvious candidate to be investigated with these instruments (e.g., collusive games featuring trigger strategy equilibria); and
- establish a connection with fields in which differential game theory has been extensively applied, but which are traditionally considered as external to IO (e.g., environmental and resource economics and trade theory and even macroeconomic growth models).

Additionally, brief digressions towards discrete time games are also present at some points, as this approach has been intensively used to construct a general framework for

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the numerical and empirical analysis of the evolution of an industry (Doraszelski and Pakes, 2007).

Aims and Style

Given the nature of the subject matter and the level of the mathematics involved, this book is aimed at PhD students carrying out their research in the field of optimal control and differential game theory with applications to industrial economics and related fields, and as a reference for professional researchers working in the same area. Although, at least in line of principle, the ideal reader could be expected to be familiar with the theory of optimal control and dynamic programming, technical aspects will be duly illustrated in Chapters 1 and 9 as well as throughout the volume, where necessary.

The book may be of interest to PhD students and researchers in other areas, such as business and management, engineering, applied mathematics and operations research. Indeed, as mentioned earlier, many contributions – including crucial ones – have been produced by colleagues in all of these areas.

Writing a really exhaustive and self-contained book is hardly possible, if at all. At least this is my sincere opinion. What I had in mind when my ideas about this venture took shape was to produce an instrument using which a reader endowed with some essential skills in optimal control and dynamic programming could understand some important differential games top to bottom. As will become apparent reading the book, I have used (hopefully not abused) my personal judgement in devoting some additional space to core models that, in view of their relevance and/or didactical properties, are illustrated in deeper details than others, which are instead more briefly sketched.

Structure of the Book

Chapter 1 provides the coordinates of the required mathematics, obviously without replicating in full what the reader can find in Dockner *et al.* (2000) and without offering a detailed exposition of the theory of differential equations and stability analysis of dynamic systems. In particular, the chapter illustrates Bellman's dynamic programming principle against Pontryagin's maximum principle, in connection with open-loop, closed-loop and feedback information. Then it also explains the conditions for strong time consistency (i.e., subgame perfection) to arise in differential games, and singles out the classes of differential games where subgame perfection is attained under open-loop information. The latter is an important feature, as many relevant games contained in the chapters to follow indeed share this property. Finally, it paves the way for the analysis of time consistency in Stackelberg differential games, which is the focus of Chapter 9.

The material appearing in Chapter 2 serves the threefold objective of (1) motivating the whole book through the simplest setting – monopoly – in which strategic interaction is absent but time does affect a firm's performance and therefore its strategies;

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(2) illustrating the fact that, under monopoly, the open-loop solution via the Hamiltonian technique (indeed, a single-agent optimal control problem) and the feedback solution via the Bellman equation may not coincide even if no strategic interaction takes place; and (3) offering an overview of those themes (from capacity accumulation and environmental issues to optimal pricing for durables and the impact of network externalities) which will appear again in proper differential games.

Chapter 3 is the largest of the book, as it reviews oligopoly games where firms use market variables only. The exposition sets out with the paradigmatic Cournot game under price stickiness, which is given a careful look as it offers the possibility of comparing open-loop, closed-loop and feedback solutions in detail, using linear and nonlinear demands and also nonlinear feedback strategies. This is followed by the opposite problem posed by sluggish demand and then by the illustration of Bertrand and Cournot games with capacity accumulation dynamics, either à la Solow–Swan or *à la* Ramsey. The survey of oligopoly games is complemented by those featuring price competition with costly price adjustment and durable or addictive goods. The remainder of the chapter is devoted to a reconstruction of the debate about conjectural variations, best reply functions and potential functions in differential games, as well as the little we avail of about the use of trigger strategies and, as mentioned earlier, to the growing literature on the empirical analysis of the intertemporal evolution of industries.

Chapter 4 deals with advertising games. A large portion of the extant material in this subfield has been produced either in industrial economics or in marketing and management, but also in operations research and engineering. Bluntly speaking, this literature stipulates that firms' advertising efforts modify either market size or market shares or goodwill (also labelled as brand equity). The resulting literature is teeming with games which lend themselves to a fully analytical solution under any information structure, and has attracted a considerable amount of empirical research.

Product differentiation based on discrete choice models (Anderson *et al.*, 1992) is the subject matter of Chapter 5. The incentive for firms to invest in product differentiation in order to soften price competition has been investigated IO in full details and for several decades by now. Conversely, the space devoted to this issue in differential game theory is still limited. This might sound surprising, given the inherently dynamic nature of this theme, and the discussion carried out in this chapter is also meant to illustrate the reasons for this apparent lack of activity, in connection with the lack of tractability of discrete choice models when these are translated into continuous time frameworks.

Chapter 6 summarises stochastic and deterministic games of innovation. The first part offers a reconstruction of the transformation of R&D races with exogenous efforts into proper differential games with firms using R&D investments as controls, influencing the expected innovation date in a really strategic way. The second part accounts for games in which uncertainty plays no role and firms invest in either process or product innovation, or both. Here we will come back to the issue of whether the limit properties of differential games coincide with those of their static counterparts. Learning by doing, the use of technological knowledge as a barrier to entry and catching-up in portfolio races are complementary issues also treated in this chapter.

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Differential oligopoly games in environmental and resource economics are illustrated in Chapter 7, where the benchmark Cournot games with either polluting emissions or renewable resource extraction are described in detail, including nonlinear feedback strategies and the voracity effect possibly leading to resource extinction. A large amount of space is devoted to the behaviour of cartels extracting nonrenewables, as well as to R&D for green or backstop technologies.

Chapter 8 deals with intraindustry trade, reviewing dynamic reformulations of intraindustry trade with tariffs, quotas and voluntary export or import restraints in frameworks appearing in Chapter 3, based on either price stickiness or capacity accumulation. This exposition is then complemented by the material pertaining to the intersection between trade theory and environmental economics, in which trade impacts pollution and global warming.

Chapter 9 is a mix of theory and applications meant to illustrate the delicate nature of the Stackelberg solution in differential games. Actually, I might simply say ‘games’ without any further qualification, since the chapter sets out with a compact illustration of the original Cournot-Stackelberg model (Stackelberg, 1934) we are accustomed to from introductory textbooks in microeconomics, in order to outline the concept of time (in)consistency. This is then extensively discussed in the remainder of the chapter by illustrating the property of uncontrollability and the requirements for subgame perfect Stackelberg equilibria to arise under open-loop and feedback information. This discussion is complemented by examples based on games with sticky prices, advertising or taxation of polluting emissions in monopoly. A relatively wider space is for a theme which is widespread in the business and management literature, supply chain coordination, as this is a natural candidate for the analysis of the Stackelberg solution.

At the end of each chapter, there appears a short paragraph offering suggestions for further readings.

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A book is never the outcome of a one-man-band effort. I would like to warmly thank Roberto Cellini, Luca Colombo, Flavio Delbono, Davide Dragone, Paola Labrecciosa, Andrea Mantovani and Arsen Palestini for many fruitful suggestions and comments and for carefully reading each chapter. Needless to say, the responsibility for the contents and any remaining errors or omissions remains with me only. Special thanks to five reviewers for their constructive comments and suggestions on the initial book proposal, and to Phil Good, my editor at Cambridge University Press, who accompanied me most friendly and efficiently throughout the gestation of the book. Last but by no means least, I would like to express my most sincere gratitude and affection to George Leitmann, with a hug to him and Nancy.

During the editorial process of the book, Engelbert Dockner passed away. He largely contributed to the development of differential game theory and its applications in several directions, including the areas treated here. This volume is also a modest tribute to his memory.

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