# **Principles of Photonics**

With this self-contained and comprehensive text, students will gain a detailed understanding of the fundamental concepts and major principles of photonics. Assuming only a basic background in optics, readers are guided through key topics such as the nature of optical fields, the properties of optical materials, and the principles of major photonic functions regarding the generation, propagation, coupling, interference, amplification, modulation, and detection of optical waves or signals. Numerous examples and problems are provided throughout to enhance understanding, and a solutions manual containing detailed solutions and explanations is available online for instructors.

This is the ideal resource for electrical engineering and physics undergraduates taking introductory, single-semester or single-quarter courses in photonics, providing them with the knowledge and skills needed to progress to more advanced courses on photonic devices, systems, and applications.

**Jia-Ming Liu** is Distinguished Professor of Electrical Engineering and Associate Dean for Academic Personnel of the Henry Samueli School of Engineering and Applied Science at the University of California, Los Angeles. Professor Liu has published over 250 scientific papers and holds 12 US patents, and is the author of *Photonic Devices* (Cambridge, 2005). He is a fellow of the Optical Society of America, the American Physical Society, the IEEE, and the Guggenheim Foundation.

## **ENDORSEMENTS FOR LIU, PRINCIPLES OF PHOTONICS**

"With much thoughtfulness and a rigorous approach, Prof. Jia-Ming Liu has put together an excellent textbook to introduce students to the principles of photonics. This book covers a comprehensive list of subjects that allow students to learn the fundamental properties of light as well as key phenomena and functions in photonics. Compared to other textbooks in classical optics, this book places the necessary emphasis on photonics for readers who want to learn about this field. Compared to other textbooks introducing photonics, this book is carefully and well written, with ample examples, illustrations, and well-designed homework problems. Instructors will find this book very helpful in teaching the subjects, and students will find themselves gaining solid understanding of the materials by reading and working through the book." Lih Lin, University of Washington

"For a long while the photonics community has been waiting for a new textbook which is informative, comprehensive, and also contains practical examples for students; in other words, one which describes fundamental concepts and provides working principles in optics. Professor Jia-Ming Liu's book, *Principles of Photonics*, serves very well for these purposes – it covers optical phenomena and optical properties of materials, as well as the basic principles behind light emitting, modulation, amplification and detection devices that are commonly used nowadays in communications, displays, and sensing. A distinguishing feature of this book is its seamless use of "additional space" to ensure that each concept is sufficiently explained in words, coupled with mathematics, simple yet illustrative figures, and/or examples. Each chapter ends with questions/problems followed by key references, making it very self-contained and very easy to follow."

Paul Yu, University of California, San Diego

"A pedagogical tour-de-force. Professor Liu covers the principles of photonics with extreme attention to notation, completeness of derivations, and clear examples matched to the concepts being taught. This is a book one can really *learn* from."

Jeffrey Tsao, Sandia National Lab

# Principles of Photonics

JIA-MING LIU University of California



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To Vida and Janelle

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## CONTENTS

	Preface Partial List of Symbols	<i>page</i> xi xiii
1	Basic Concepts of Optical Fields	1
-	1.1 Nature of Light	1
	1.2 Optical Fields and Maxwell's Equations	4
	1.3 Optical Power and Energy	8
	1.4 Wave Equation	10
	1.5 Harmonic Fields	11
	1.6 Polarization of Optical Fields	13
	1.7 Optical Field Parameters	18
	Problems	20
	Bibliography	21
2	Optical Properties of Materials	22
	2.1 Optical Susceptibility and Permittivity	22
	2.2 Optical Anisotropy	24
	2.3 Resonant Optical Susceptibility	32
	2.4 Optical Conductivity and Conduction Susceptibility	38
	2.5 Kramers–Kronig Relations	44
	2.6 External Factors	44
	2.7 Nonlinear Optical Susceptibilities	55
	Problems	60
	Bibliography	65
3	Optical Wave Propagation	66
	3.1 Normal Modes of Propagation	66
	3.2 Plane-Wave Modes	73
	3.3 Gaussian Modes	86
	3.4 Interface Modes	92
	3.5 Waveguide Modes	108
	3.6 Phase Velocity, Group Velocity, and Dispersion	122
	3.7 Attenuation and Amplification	129
	Problems	132
	Bibliography	139

Cambridge University Press 978-1-107-16428-4 — Principles of Photonics Jia-Ming Liu Frontmatter <u>More Information</u>

viii	Contents	
4	Optical Coupling	141
	4.1 Coupled-Mode Theory	141
	4.2 Two-Mode Coupling	147
	4.3 Codirectional Coupling	154
	4.4 Contradirectional Coupling	156
	4.5 Conservation of Power	159
	4.6 Phase Matching	160
	Problems	165
	Bibliography	168
5	Optical Interference	169
	5.1 Optical Interference	169
	5.2 Optical Gratings	183
	5.3 Fabry–Pérot Interferometer	191
	Problems	200
	Bibliography	203
6	Optical Resonance	204
	6.1 Optical Resonator	204
	6.2 Longitudinal Modes	207
	6.3 Transverse Modes	211
	6.4 Cavity Lifetime and Quality Factor	214
	6.5 Fabry–Pérot Cavity	216
	Problems	221
	Bibliography	223
7	Optical Absorption and Emission	224
	7.1 Optical Transitions	224
	7.2 Transition Rates	234
	7.3 Attenuation and Amplification of Optical Fields	241
	Problems	245
	Bibliography	248
8	Optical Amplification	249
	8.1 Population Rate Equations	249
	8.2 Population Inversion	251
	8.3 Optical Gain	259
	8.4 Optical Amplification	265
	8.5 Spontaneous Emission	267
	Problems	270
	Bibliography	273

Cambridge University Press 978-1-107-16428-4 — Principles of Photonics Jia-Ming Liu Frontmatter <u>More Information</u>

		Contents	ix
9	Laser Oscillation		274
	9.1 Conditions for Laser Oscillation		274
	9.2 Mode-Pulling Effect		277
	9.3 Oscillating Laser Modes		279
	9.4 Laser Power		285
	Problems		293
	Bibliography		296
10	Optical Modulation		297
	10.1 Types of Optical Modulation		297
	10.2 Modulation Schemes		298
	10.3 Direct Modulation		308
	10.4 Refractive External Modulation		319
	10.5 Absorptive External Modulation		344
	Problems		353
	Bibliography		361
11	Photodetection		362
	11.1 Physical Principles of Photodetection		362
	11.2 Photodetection Noise		375
	11.3 Photodetection Measures		382
	Problems		391
	Bibliography		395
	Appendix A Symbols and Notations		396
	Appendix B SI Metric System		403
	Appendix C Fundamental Physical Constants		405
	Appendix D Fourier-Transform Relations		406
	Index		409

## PREFACE

The field of photonics has matured into an important discipline of modern engineering and technology. Its core principles have become essential knowledge for all undergraduate students in many engineering and scientific fields. This fact is fully recognized in the new curriculum of the Electrical Engineering Department at UCLA, which makes the principles of photonics a required course for all electrical engineering undergraduate students. Graduate students study-ing in areas related to photonics also need this foundation.

The most fundamental concepts in photonics are the nature of optical fields and the properties of optical materials because the entire field of photonics is based on the interplay between optical fields and optical materials. Any photonic device or system, no matter how simple or sophisticated it might be, consists of some or all of these functions: the generation, propagation, coupling, interference, amplification, modulation, and detection of optical waves or signals. The properties of optical fields and optical materials are addressed in the first two chapters of this book. The remaining nine chapters cover the principles of the major photonic functions.

This book is written for a one-quarter or one-semester undergraduate course for electrical engineering or physics students. Only some of these students might continue to study advanced courses in photonics, but at UCLA we believe that all electrical engineering students need to have a basic understanding of the core knowledge in photonics because it has become an established key area of modern technology. Many universities already have departments that are entirely devoted to the field of photonics. For the students in such photonics-specific departments or institutions, the subject matter in this book is simply the essential foundation that they must master before advancing to other photonics courses. Based on this consideration, this book emphasizes the principles, not the devices or the systems, nor the applications. Nevertheless, it serves as a foundation for follow-up courses on photonic devices, optical communication systems, biophotonics, and various subjects related to photonics technology. Because this book is meant for a one-quarter or one-semester course, it is kept to a length that can be completed in a quarter or a semester. Because it likely serves the only required undergraduate photonics course in the typical electrical engineering curriculum, it has to cover most of the essential principles. The chapters of this book are organized based on the major principles of photonics rather than based on device or system considerations. These attributes are the key differences between this book and other books in this field.

Through my teaching experience on this subject over many years, I find a need for a textbook that has the following features.

- 1. It is self-contained, and its prerequisites are among the required core courses in the typical electrical engineering curriculum.
- 2. It covers the major principles in a single book that can be completely taught in a one-quarter or one-semester course. And it treats these subjects not superficially but to a sufficient depth

Cambridge University Press 978-1-107-16428-4 — Principles of Photonics Jia-Ming Liu Frontmatter <u>More Information</u>

#### Preface

xii

for a student to gain a solid foundation to move up to advanced photonics courses, if the student stays in the photonics field, or for a student to gain a useful understanding of photonics, if the student moves on to a different field.

3. It has ample examples that illustrate the concepts discussed in the text, and it has plenty of problems that are closely tied to these concepts and examples.

This book is written with the above features to serve the need for a book covering a core photonics course in a modern electrical engineering curriculum.

There are two prerequisites for a course that uses this book: (1) basic electromagnetics up to electromagnetic waves and (2) basic solid-state physics or solid-state electronics. No advanced background in optics beyond what a student normally learns in general physics is required. At UCLA, this course is taught as a required course in the Electrical Engineering Department to undergraduate juniors and seniors. The materials of this book have been test taught for a few years in this one-quarter course, which has 38 hours of lectures, excluding the time for the midterm and final exams. This course is followed by elective courses on photonic devices and circuits, photonic sensors and solar cells, and biophotonics.

Carefully designed examples are given at proper locations to illustrate the concepts discussed in the text and to help students apply what they learn to solving problems. Each example is tied closely to one or more concepts discussed in the text and is placed right after that text; its solution does not simply give the answer but presents a detailed explanation as part of the teaching process. An ample number of problems are given at the end of each chapter. The problems are labeled with the corresponding section numbers and are arranged in the sequence of the material presented in the text. The entire book has 100 examples and 247 problems.

The materials in this book are selected and structured to suit the purpose of a course on the principles of photonics. Besides the newly written materials, text and figures are adopted from my book *Photonic Devices* wherever suitable. All examples and problems, except for the very few that illustrate key concepts, are newly designed specifically to meet the pedagogical purpose of this book.

This book was developed through test teaching a course in the new curriculum at UCLA. In this process, I received much feedback from my colleagues and my students. I would like to thank my editor, Julie Lancashire, for her help at every stage during the development of this book, and my content manager, Jonathan Ratcliffe, for taking care of the production matters of this book. I would like to express my loving appreciation to my daughter, Janelle, who took a special interest in this project and shared my excitement in it. Special thanks are due to my wife, Vida, who gave me constant support and created an original oil painting for the cover art of this book.

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## **PARTIAL LIST OF SYMBOLS**

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
a	none	round-trip intracavity field amplification factor	(6.4)
$a_{\rm E}, a_{\rm M}$	none	asymmetry factors for TE and TM modes	(3.130)
$A, \tilde{A}$	$W^{1/2}$	mode amplitude	(4.23), (4.26)
$A_{v}$	$W^{1/2}$	amplitude of mode v	(4.3)
<i>A</i> <sub>21</sub>	$s^{-1}$	Einstein A coefficient	(7.21)
$\mathcal{A}$	m <sup>2</sup>	area	(11.59)
b	m	confocal parameter of Gaussian beam	(3.69)f
b	none	normalized guide index	(3.129)
b	none	linewidth enhancement factor	(9.39)
$B, \tilde{B}$	$W^{1/2}$	mode amplitude	(4.24), (4.27)
В	Hz	bandwidth	(11.1)
$B_{12}, B_{21}$	$m^3 J^{-1} s^{-1}$	Einstein <i>B</i> coefficients	(7.19), (7.20)
В	Т	real magnetic induction in the time domain	(1.3)
<b>B</b> , <i>B</i>	Т	complex magnetic induction	(1.41)
с	${\rm ms^{-1}}$	speed of light in free space	(1.1)b, (1.39)
$c_{\nu\mu}$	none	overlap coefficient between modes $v$ and $\mu$	(4.19)
C <sub>ijkl</sub>	$\mathrm{m}^2\mathrm{A}^{-2}$	quadratic magneto-optic coefficient	(2.77)
d	m	thickness or distance; $d_{g}$ , $d_{QW}$	(3.127)
$d, d_0$	m	beam spot size diameter, $d = 2w$ , $d_0 = 2w_0$	(3.69)b
$d_{\rm E}, d_{\rm M}$	m	effective waveguide thicknesses for TE and TM modes	(3.138), (3.143)
D	none	group-velocity dispersion; $D_1$ , $D_2$ , $D_\beta$	(3.167)
D	$W^{-1}$	detectivity	(11.58)

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#### xiv

Partial List of Symbols

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$D_{\lambda}$	$\mathrm{s}\mathrm{m}^{-2}$	group-velocity dispersion; $D_{\lambda} = -D/c\lambda$	(3.168)
$D^*$	$mHz^{1/2}W^{-1}$	specific detectivity	(11.59)
D	$\mathrm{C}\mathrm{m}^{-2}$	real electric displacement in the time domain	(1.2)
<b>D</b> , <i>D</i>	$\mathrm{C}\mathrm{m}^{-2}$	complex electric displacement; $\mathbf{D}_{e}, \mathbf{D}_{o}, D_{+}, D_{-}$	(1.42), (1.51)
$\mathcal{D}, \mathcal{D}$	$\mathrm{C}\mathrm{m}^{-2}$	slowly varying amplitudes of $\mathbf{D}$ and $D$ ; $\mathcal{D}_{e}$ , $\mathcal{D}_{0}$	(3.57)
DR	dB	dynamic range	(11.62)
е	С	electronic charge	(2.30)f
ê	none	unit vector of electric field polarization; $\hat{e}_{e}$ , $\hat{e}_{o}$ , $\hat{e}_{+}$ , $\hat{e}_{-}$	(1.61)
$E_1, E_2$	eV	energies of levels $ 1\rangle$ and $ 2\rangle$	(7.1)
$E_{\rm c}, E_{\rm v}$	eV	conduction-band and valence-band edges	(10.106)
$E_{\mathrm{F}}$	eV	Fermi energy	(11.5)b
$E_{\mathrm{g}}$	eV	bandgap	(10.105), (11.7)
$E_{ m th}$	eV	threshold photon energy	(11.5)
E	$V m^{-1}$	real electric field in the time domain	(1.2)
$\boldsymbol{E}_0,  \boldsymbol{E}_0$	$V m^{-1}$	static or low-frequency electric field	(2.54)
$E_{\rm e}, E_{\rm h}$	$V m^{-1}$	electric fields seen by electrons and holes	(10.106)
<b>E</b> , <i>E</i>	$V m^{-1}$	complex electric field	(1.40)
$\mathbf{E}_{v}, E_{v}$	$V m^{-1}$	complex electric field of mode $v$	(3.1)
<b>E</b> , E	$V m^{-1}$	slowly varying amplitudes of <b>E</b> and <i>E</i> ; $\mathcal{E}_{e}$ , $\mathcal{E}_{o}$ , $\mathcal{E}_{+}$ , $\mathcal{E}_{-}$	(1.52)
$\boldsymbol{\mathcal{E}}_{v}, \boldsymbol{\mathcal{E}}_{v}$	$V m^{-1}$	complex electric field profile of mode $v$	(3.1)
$\hat{oldsymbol{\mathcal{E}}}_{v}$	$V m^{-1} W^{-1/2}$	normalized electric mode field distribution, $\boldsymbol{\mathcal{E}}_{\nu} = A_{\nu} \hat{\boldsymbol{\mathcal{E}}}_{\nu}$	(3.18)
ER	dB	extinction ratio	(10.18)
f	Hz	acoustic or modulation frequency, $f=\Omega/2\pi$	(2.79)b, (10.27)b
$f_{\rm 3dB}$	Hz	3-dB modulation bandwidth or cutoff frequency	(10.31), (11.64)
$f_{\rm K}$	m	Kerr focal length	(10.115)
$f_{ijk}$	$m A^{-1}$	linear magneto-optic coefficient, Faraday coefficient	(2.76), (10.77)

#### **Partial List of Symbols**

xv

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$f_{\rm r}$	Hz	relaxation resonance frequency, $f_{\rm r} = \Omega_{\rm r}/2\pi$	(10.41)
F	none	excess noise factor	(11.38)
F	none	finesse of interferometer or optical cavity	(5.49), (6.12)
$\mathbf{F}(z;z_0)$	none	forward-coupling matrix for codirectional coupling	(4.48)
g, g(v)	$\mathrm{m}^{-1}$	gain coefficient, amplification coefficient	(3.183)f, (7.46)
$g_0$	$m^{-1}$	unsaturated gain coefficient	(8.22)
$g_{\mathrm{th}}$	$m^{-1}$	threshold gain coefficient	(9.9), (9.19)
$\hat{g}(v)$	S	lineshape function	(7.2)
g	none	degeneracy factor; $g_1, g_2$	(7.1)f, (7.28)
g	$s^{-1}$	gain parameter	(9.18)
$g_0$	$s^{-1}$	unsaturated gain parameter	(9.22)
g <sub>n</sub>	${ m m}^3~{ m s}^{-1}$	differential gain parameter	(10.36)
g <sub>p</sub>	${\rm m}^3  {\rm s}^{-1}$	nonlinear gain parameter	(10.36)
$g_{\rm th}$	$s^{-1}$	threshold gain parameter	(9.20), (10.34)
G	none	cavity round-trip field gain; $G_c$ , $G_{mn}$ , $G_{mn}^c$	(6.4)
G	none	photodetector current gain	(11.4)f, (11.36)
$G, G_0$	none	optical amplifier power gain, $G_0$ for unsaturated gain	(8.39)
h, ħ	J s	Planck's constant, $\hbar = h/2\pi$	(1.1)
$h_1, h_2, h_3$	$m^{-1}$	transverse oscillation parameters of mode field	(3.104), (3.133)
Н	m	height of acousto-optic transducer	(10.89)
$H(\cdot)$	none	Heaviside step function	(2.24)
$H_m(\cdot)$	none	Hermite function	(3.73)f
H	$A m^{-1}$	real magnetic field in the time domain	(1.3)
$H_0, H_0$	$A m^{-1}$	static or low-frequency magnetic field	(2.68)
<b>H</b> , <i>H</i>	$A m^{-1}$	complex magnetic field	(1.42)
$\mathbf{H}_{v}, H_{v}$	$A m^{-1}$	complex magnetic field of mode v	(3.2)

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#### xvi

#### Partial List of Symbols

(cont.)	
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Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
${\cal H}, {\cal H}$	$A m^{-1}$	slowly varying amplitudes of $\mathbf{H}$ and $H$	(3.5)
$\mathcal{H}_{v}, \mathcal{H}_{v}$	$A m^{-1}$	complex magnetic field profile of mode $v$	(3.2)
$\hat{\mathcal{H}}_{v}$	$A m^{-1} W^{-1/2}$	normalized electric mode field distribution, $\mathcal{H}_{\nu} = A_{\nu} \hat{\mathcal{H}}_{\nu}$	(3.18)
i	none	$\sqrt{-1}$	
i	А	current; $i_{\rm b}$ , $i_{\rm d}$ , $i_{\rm n}$ , $i_{\rm ph}$ , $i_{\rm s}$	(11.4)f
Ι	А	injection current; $I_0$ , $I_m$ , $I_{th}$	(10.22)
Ι	$W m^{-2}$	optical intensity; $I_0$ , $I_i$ , $I_{in}$ , $I_{out}$ , $I_r$ , $I_t$	(1.56)
$I_0$	А	reverse current	(11.15)
I(v)	$\mathrm{W}~\mathrm{m}^{-2}~\mathrm{Hz}^{-1}$	optical spectral intensity distribution	(7.17)
$I_{\rm p}, I_{\rm s}$	$W m^{-2}$	optical pump and signal intensities	(8.36)
I <sub>sat</sub>	$W m^{-2}$	saturation intensity	(8.22)
<b>J</b> , <i>J</i>	$A m^{-2}$	real current density	(1.5)
J	$A m^{-2}$	complex current density	(2.35)
k	$\mathrm{m}^{-1}$	propagation constant, wavenumber; $k_0$ , $k_i$ , $k_r$ , $k_t$	(1.84)
k <sub>B</sub>	$J K^{-1}$	Boltzmann constant	(7.14), (7.25)
k <sub>e</sub> , k <sub>o</sub>	$\mathrm{m}^{-1}$	propagation constants of extraordinary and ordinary waves	(3.57)
k', k"	$\mathrm{m}^{-1}$	real and imaginary parts of $k, k = k' + ik''$	(3.180)
$k^x, k^y, k^z$	$\mathrm{m}^{-1}$	propagation constants of $x$ , $y$ , and $z$ polarized fields	(2.15)
$k^X, k^Y, k^Z$	$\mathrm{m}^{-1}$	propagation constants of $X$ , $Y$ , and $Z$ polarized fields	(2.67)
$k^+, k^-$	$m^{-1}$	propagation constants of circularly polarized fields	(2.21)
ĥ	none	unit vector in the $\mathbf{k}$ direction	(1.84)
k	$m^{-1}$	wavevector; $\mathbf{k}_i$ , $\mathbf{k}_r$ , $\mathbf{k}_t$ , $\mathbf{k}_q$	(1.1)b, (1.52)
$\mathbf{k}_{e},  \mathbf{k}_{o}$	$m^{-1}$	wavevectors of extraordinary and ordinary waves	(3.56)f, (3.57)
$\mathbf{k}^{x}, \mathbf{k}^{y}, \mathbf{k}^{z}$	$m^{-1}$	wavevectors of $x$ , $y$ , and $z$ polarized fields	(3.48)f
$\mathbf{k}^+,\mathbf{k}^-$	$m^{-1}$	wavevectors of circularly polarized fields	(10.74)

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**Partial List of Symbols** 

xvii

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
K	$\mathrm{m}^{-1}$	wavenumber of acoustic wave or grating, $K = 2\pi/\Lambda$	(2.79)f, (4.35)
Κ	S	K factor of a semiconductor laser	(10.43)
К	$m^{-1}$	wavevector of acoustic wave	(2.79)
l	m	length or distance	(3.185)
lc	m	coupling length; $l_{\rm c}^{\rm PM}$	(4.56)
$l_{\rm RT}$	m	round-trip optical path length	(6.1)
$l_{\lambda/4},l_{\lambda/2}$	m	quarter-wave and half-wave lengths	(3.49), (3.50)
L	m	length of acousto-optic transducer	(10.89)
т	none	transverse mode index associated with $x$	(3.1)f
m	none	modulation index	(10.27)
$m_0$	kg	free electron rest mass	Fig. 11.1
$m^*$	kg	effective mass of carriers	(2.31)
$m_{\rm e}^*, m_{\rm h}^*$	kg	effective masses of electrons and holes	(10.107)
Μ	kg	atomic or molecular mass	(7.14)
$M_{\rm TE}, M_{\rm TM}$	none	numbers of guided TE and TM modes	(3.152), (3.153)
M <sub>s</sub>	$A m^{-1}$	saturation magnetization	(10.78)
М	$A m^{-1}$	real magnetic polarization in the time domain	(1.3)
$M_0, M_0$	$A m^{-1}$	static or low-frequency magnetization	(2.70)
n	none	transverse mode index associated with y	(3.1)f
n	none	index of refraction; $n_{\beta}, \overline{n}$	(1.84)
n	$m^{-3}$	electron concentration	(11.9)
<i>n</i> <sub>0</sub>	$m^{-3}$	equilibrium concentration of electrons	(11.9)f
$n_1, n_2, n_3$	none	refractive indices of waveguide layers, $n_1 > n_2 > n_3$	(3.125)
<i>n</i> <sub>2</sub>	$m^2 \ W^{-1}$	coefficient of intensity-dependent index change	(10.101)
$n_{\rm e}, n_{\rm o}$	none	extraordinary and ordinary indices of refraction	(2.15)f, (3.56)
$n_x, n_y, n_z$	none	principal indices of refraction	(2.14)

#### xviii

Partial List of Symbols

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$n_X, n_Y, n_Z$	none	new principal indices of refraction	(2.66)
$n_{+}, n_{-}$	none	principal indices of refraction for circular polarized modes	(2.20)
$n_{\perp}, n_{  }$	none	indices of second-order magneto-optic effect	(2.16)
<i>n'</i> , <i>n''</i>	none	real and imaginary parts of refractive index, n = n' + in''	(3.181)
ĥ	none	unit normal vector	(1.23)
Ν	none	some number	(5.21)
Ν	none	group index; $\cdot N_1$ , $N_2$ , $N_\beta$	(3.171)
Ν	$m^{-3}$	carrier density	(2.31)
Ν	$m^{-3}$	effective population inversion	(8.4)
$N_1, N_2, N_t$	$m^{-3}$	population densities in levels $ 1\rangle, 2\rangle,$ and all levels	(7.26), (8.12)
$N_{ m sp}$	none	spontaneous emission factor	(9.14)
$\mathcal{N}$	none	number of charge carriers	(11.3)
NEP	W	noise equivalent power	(11.55)
р	none	probability	(11.18)
р	none	cross-section ratio for pumping	(8.13)
p	$m^{-3}$	hole concentration	(11.9)
$p_0$	$m^{-3}$	equilibrium concentration of holes	(11.9)f
$p_{ijkl} p'_{ijkl}$	none	elasto-optic and rotation-optic coefficients	(2.83)
$p(v_k)$	$Hz^{-1}$	probability density function	(7.10)
Р	W	power; $P_{\rm a}$ , $P_{\rm in}$ , $P_{\rm out}$ , $P_{\rm pk}$ , $P_{\rm th}$ , $P_{\nu}$	(3.17)
$P_{\rm p}, P_{\rm s}$	W	pump and signal powers; $P_p^{\text{th}}$ , $P_p^{\text{tr}}$ , $P_s^{\text{in}}$ , $P_s^{\text{out}}$	(9.27), (8.37)
P <sub>sat</sub>	W	saturation power	(8.37)
P <sub>sp</sub>	W	spontaneous emission power	(8.44)
$P_{\rm sp}^{ m tr}$	W	critical fluorescence power	(8.46)

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#### **Partial List of Symbols**

xix

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Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$\hat{P}_{\rm sp}$	$W m^{-3}$	spontaneous emission power density	(8.43)
$\hat{p}_{ m sp}^{ m tr}$	$W m^{-3}$	critical fluorescence power density	(8.45)
Р	$C m^{-2}$	real electric polarization in the time domain	(1.2)
<b>P</b> , <i>P</i>	$C m^{-2}$	complex electric polarization	(1.50)
$P^{(n)}$	$C m^{-2}$	<i>n</i> th-order nonlinear real electric polarization	(2.91)
$\mathbf{P}^{(n)}, \mathbf{P}^{(n)}$	$C m^{-2}$	<i>n</i> th-order nonlinear complex electric polarization	(2.91)
P <sub>res</sub>	$C m^{-2}$	complex electric polarization from resonant transition	(7.47)b
q	none	longitudinal mode index	(5.47), (6.9)
q	none	order of coupling or diffraction	(4.36), (5.24)
q	С	charge	(2.30)
q(z)	m	complex radius of curvature of a Gaussian beam	(3.75)
Q	none	quality factor of resonator; $Q_{mnq}$	(6.26), (6.30)
Q	none	acousto-optic diffraction parameter	(10.83)
r	m	radial coordinate, radial distance	
r	none	reflection coefficient; $r_1$ , $r_2$ , $r_p$ , $r_s$	(3.91), (4.67)
r	none	pumping ratio of a laser	(9.26)
$r_{ijk}, r_{\alpha k}$	$m V^{-1}$	linear electro-optic coefficients, Pockels coefficients	(2.58), (2.60)
$r(f), r(\Omega)$	none	complex modulation response function	(10.29), (10.40)
r	m	spatial vector	(1.2)
R	none	reflectance, reflectivity; $R_1$ , $R_2$ , $R_p$ , $R_s$	(3.93)
R	Ω	resistance; $R_i$ , $R_L$	(11.16)
R	$m^{-3} s^{-1}$	effective pumping rate for population inversion	(8.6)
$R_1, R_2$	$m^{-3} s^{-1}$	pumping rates for levels $ 1\rangle$ and $ 2\rangle$	(8.1), (8.2)
R(f)	none	electrical power spectrum of modulation response	(10.30), (10.44)
$\mathbf{R}(z;0,l)$	none	reverse-coupling matrix for contradirectional coupling	(4.59)
$\mathbf{R}, R_{ij}$	none	rotation tensor and elements, $\mathbf{R} = [R_{ij}]$	(2.82)

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## Partial List of Symbols

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$\mathcal{R}$	m	radius of curvature; $\mathcal{R}_1$ , $\mathcal{R}_2$	(3.71), (6.31)
$\mathcal{R}$	none	amplitude of rotation; $\mathcal{R}_{ij}$	(2.87)
$\mathcal{R}$	$A W^{-1}$	responsivity of photodetector with current output; $\mathcal{R}_0$	(11.50)
$\mathcal{R}$	$V W^{-1}$	responsivity of photodetector with voltage output	(11.51)
S	m	separation, spacing; $s_e$	Fig. 4.2, (10.69)
S	none	signal; s <sub>n</sub>	(11.18)
$S_{ijkl}, S_{\alpha kl}$	$m^2 \ V^{-2}$	quadratic electro-optic coefficients, Kerr coefficients	(2.58), (2.60)
S	$m^{-3}$	photon density	(9.21)
S <sub>sat</sub>	$m^{-3}$	saturation photon density	(9.24)
S	$W m^{-2}$	real Poynting vector	(1.32)
S	$W m^{-2}$	complex Poynting vector; $\mathbf{S}_{e}, \mathbf{S}_{o}$	(1.54)
<b>S</b> , $S_{ij}$	none	strain tensor and elements, $\mathbf{S} = \begin{bmatrix} S_{ij} \end{bmatrix}$	(2.81)
S	none	amplitude of strain; $S_{ij}$	(2.87)
S	none	number of photons	(11.2)
SNR	none, dB	signal-to-noise ratio	(11.26)
t	S	time	
t	none	transmission coefficient; $t_{\rm p}$ , $t_{\rm s}$	(3.92)
$t_{\rm r}, t_{\rm f}$	S	risetime and falltime	(11.63)b
Т	Κ	temperature	(7.14)
Т	S	time interval	(1.53)
Т	S	round-trip time of optical cavity	(6.1)
Т	none	transmittance, transmissivity; $T_{\rm p}$ , $T_{\rm s}$	(3.94), (10.108)
<i>u</i> , <i>u</i> <sub>0</sub>	$J m^{-3}$	electromagnetic energy density	(7.16), (1.33)
u(v)	$J\ m^{-3}\ Hz^{-1}$	spectral energy density	(7.16)
<b>u</b> , <i>u</i> <sub><i>i</i></sub>	m	elastic deformation wave and its components	(2.79), (2.81)
U	J	optical energy; $U_{mode}$	(9.28)

**Partial List of Symbols** 

xxi

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
U	m	amplitude of elastic wave	(2.79)
v	V	voltage; $v_n$ , $v_{out}$ , $v_s$	(11.16)
v	${\rm m}~{\rm s}^{-1}$	velocity	Fig. 11.1
$v_{\rm a}$	${\rm m}~{\rm s}^{-1}$	acoustic wave velocity	(2.80)b
$v_{\rm g}$	${\rm m}~{\rm s}^{-1}$	group velocity; $v_{\beta}^{g}$	(3.165)
$v_{\rm p}$	${\rm m}~{\rm s}^{-1}$	phase velocity; $v^{\rm p}_{\beta}$	(3.162)
V	none	normalized frequency and waveguide thickness, V number	(3.128)
V	rad $A^{-1}$	Verdet constant	(10.77)
V	V	voltage; $V_{\rm m}$ , $V_{\pi}$ , $V_{\pi/2}$	(10.51), (11.15)
V <sub>c</sub>	none	cutoff V number; $V_m^c$	(3.147)
$\mathcal{V}$	m <sup>3</sup>	volume; $\mathcal{V}_{gain}$ , $\mathcal{V}_{mode}$	(1.31)b, (6.2)
<i>w</i> , <i>w</i> <sub>0</sub>	m	Gaussian beam radius, spot size	(3.69), (3.70)
W	m	width of acousto-optic cell	(10.91)
W	$s^{-1}$	transition probability rate; $W_{12}$ , $W_{21}$ , $W_p$ , $W_{sp}$	(7.22)–(7.24)
$W_{\rm p}, W_{\rm m}$	$W m^{-3}$	power densities expended by EM field on $P$ and $M$	(1.34), (1.35)
W(v)	none	transition rate per unit frequency; $W_{12}(v), W_{21}(v), W_{sp}(v)$	(7.19)–(7.21)
x	m	spatial coordinate	
<i>x</i>	none	unit coordinate vector or principal dielectric axis	(1.62), (2.13)b
X	m	spatial coordinate along $\hat{X}$	
$\hat{X}$	none	new principal dielectric axis	(2.65)b
у	m	spatial coordinate	
ŷ	none	unit coordinate vector or principal dielectric axis	(1.62), (2.13)b
Y	m	spatial coordinate along $\hat{Y}$	
Ŷ	none	new principal dielectric axis	(2.65)b
z	m	spatial coordinate	

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#### xxii

#### Partial List of Symbols

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Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
<i>î</i> .	none	unit coordinate vector or principal dielectric axis	(3.16), (2.13)b
ZR	m	Rayleigh range of a Gaussian beam	(3.69)
Ζ	m	spatial coordinate along $\hat{Z}$	
Ź	none	new principal dielectric axis	(2.65)b
α	rad	field polarization angle	(1.64)
α	rad	walk-off angle of extraordinary wave	(3.60)
$\alpha, \alpha(v)$	$\mathrm{m}^{-1}$	attenuation coefficient, absorption coefficient	(3.180), (7.45)
α <sub>0</sub>	$m^{-1}$	unsaturated absorption coefficient	(10.110)
$\alpha_{\rm c}$	$\mathrm{m}^{-1}$	propagation parameter for contradirectional coupling	(4.61)
β	none	bottleneck factor	(8.7)
β	$\mathrm{m}^{-1}$	propagation constant of a mode; $\beta_{mn}$ , $\beta_{TE}$ , $\beta_{TM}$	(3.1)
$\beta',\beta''$	$\mathrm{m}^{-1}$	real and imaginary parts of $\beta$ , $\beta = \beta' + i\beta''$	(3.184)
$\beta_{\rm c}$	$\mathrm{m}^{-1}$	propagation parameter for codirectional coupling	(4.50)
γ	$s^{-1}$	relaxation rate, decay rate; $\gamma_{21}$ , $\gamma_i$ , $\gamma_{out}$	(2.23)
$\gamma_1, \gamma_2, \gamma_3$	$m^{-1}$	transverse decay parameters of mode field	(3.118), (3.131)
$\gamma_{\rm a}$	$s^{-1}$	acoustic decay rate	(10.93)
γ <sub>c</sub>	$s^{-1}$	cavity decay rate, photon decay rate; $\gamma^{c}_{mnq}$	(6.25)
γ <sub>n</sub>	$s^{-1}$	differential carrier relaxation rate	(10.37)
γ <sub>p</sub>	$s^{-1}$	nonlinear carrier relaxation rate	(10.37)
γ <sub>r</sub>	$s^{-1}$	total carrier relaxation rate	(10.42)
$\gamma_{\rm s}$	$s^{-1}$	spontaneous carrier relaxation rate	(10.42)
Г	none	overlap factor	(6.2)
δ	$m^{-1}$	phase mismatch parameter for phase mismatch of $2\delta$	(4.31)
$\delta \omega_{mnq}$	$rad s^{-1}$	frequency shift of mode pulling	(9.12)
$\Delta n, \Delta p$	$m^{-3}$	excess electron and hole concentrations	(11.10)
ΔP	$\mathrm{C}~\mathrm{m}^{-2}$	change in electric polarization	(4.8)

Partial List of Symbols

xxiii

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$\Delta t$	8	pulsewidth or time duration; $\Delta t_{\rm ps}$	(10.117), (11.1)
$\Delta\epsilon, \Delta\epsilon$	$\mathrm{F}~\mathrm{m}^{-1}$	change or variation in electric permittivity	(2.55), (4.12)
$\Delta \widetilde{\epsilon}, \Delta \widetilde{\epsilon}$	$\mathrm{F}~\mathrm{m}^{-1}$	amplitudes of $\Delta \epsilon$ and $\Delta \epsilon$	(2.88)
$\Delta \eta$ , $\Delta \eta_{ij}$	none	change or variation in relative impermeability	(2.58)
$\Delta \theta$	rad	divergence angle of a Gaussian beam	(3.72)
Δλ	m	spectral width; $\Delta \lambda_{g}$	Table 7.1
$\Delta v$	Hz	optical linewidth, bandwidth; $\Delta v_D$ , $\Delta v_g$ , $\Delta v_{inh}$ , $\Delta v_h$	(7.4)
$\Delta v_{\rm c}$	Hz	longitudinal mode linewidth	(6.18)
$\Delta v_{\rm L}$	Hz	longitudinal mode frequency spacing	(6.17)
$\Delta v_{mnq}$	Hz	oscillating laser mode linewidth	(9.13)
$\Delta v_{\rm ST}$	Hz	Schawlow–Townes linewidth of laser mode; $\Delta v_{mnq}^{ST}$	(9.14)
$\Delta \varphi$	rad	phase shift or phase retardation	(10.13)
$\Delta \varphi_{\rm c}$	rad	phase width of a cavity resonance peak	(6.11)
$\Delta \varphi_{\rm L}$	rad	phase spacing between cavity resonance peaks	(6.10)
Δχ, Δχ	none	change or variation in electric susceptibility	(2.54)
$\Delta \omega$	rad $s^{-1}$	optical linewidth, bandwidth, $\Delta \omega = 2\pi \Delta v$ ; $\Delta \omega_{inh}$ , $\Delta \omega_h$	(7.3)f, (7.13)
$\epsilon$	$\mathrm{F}~\mathrm{m}^{-1}$	electric permittivity	(2.11), (3.4)
$\epsilon_0$	$\mathrm{F}~\mathrm{m}^{-1}$	electric permittivity of free space	(1.2)
$\epsilon', \epsilon''$	$\mathrm{F}~\mathrm{m}^{-1}$	real and imaginary parts of $\epsilon$ , $\epsilon = \epsilon' + i\epsilon''$	(3.179)
$\epsilon_x, \epsilon_y, \epsilon_z$	$\mathrm{F}~\mathrm{m}^{-1}$	principal dielectric permittivities	(2.13)
$\epsilon_X, \epsilon_Y, \epsilon_Z$	$\mathrm{F}~\mathrm{m}^{-1}$	new principal dielectric permittivities	(2.65)
$\epsilon_+,\epsilon$	$\mathrm{F}~\mathrm{m}^{-1}$	principal dielectric permittivities of circular polarizations	(2.17)
$\boldsymbol{\epsilon}(\mathbf{r},t)$	$F m^{-4} s^{-1}$	real permittivity tensor in the real space and time domain	(1.21)
$\epsilon(\omega), \epsilon_{ij}$	$\mathrm{F}~\mathrm{m}^{-1}$	complex permittivity tensor in the frequency domain	(1.60)
$\epsilon_{ m res}(\omega)$	$\mathrm{F}~\mathrm{m}^{-1}$	permittivity of resonant transition	(6.36)

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#### xxiv

Partial List of Symbols

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Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
З	rad	ellipticity of polarization ellipse	(1.68)
$\zeta_{mn}(z)$	rad	phase variation of Gaussian mode field; $\zeta_{mn}^{\text{RT}}$	(3.76)
η	none	coupling efficiency; $\eta_{\rm PM}$	(4.55)
$\eta_{\rm c}$	none	power conversion efficiency	(9.37)
$\eta_{ m coll}$	none	collection efficiency	(11.48)
$\eta_{\rm e}$	none	external quantum efficiency	(10.24), (11.48)
$\eta_{ m i}$	none	internal quantum efficiency	(11.48)
$\eta_{ m inj}$	none	injection efficiency	(10.22)
$\eta_{\rm s}$	none	slope efficiency, differential power conversion efficiency	(9.38)
$\eta_{\mathrm{t}}$	none	transmission efficiency	(11.48)
$\mathbf{\eta}, \eta_{ij}, \eta_{\alpha}$	none	relative impermeability tensor and its elements, $\mathbf{\eta} = [\eta_{ij}]$	(2.57)
θ	rad	angle, spherical angular coordinate	(3.51)
θ	rad	orientation of the polarization ellipse	(1.69)
$\theta_{\rm B}$	rad	Brewster angle or Bragg angle	(3.100), (10.88)
$ heta_{ m c}$	rad	critical angle	(3.102)
$ heta_{ m d}$	rad	angle of diffraction	(10.87)
$\theta_{\rm def}$	rad	deflection angle	Example 10.9
$\theta_{\mathrm{F}}$	rad	Faraday rotation angle	(10.75)
$\theta_{\rm i},\theta_{\rm r},\theta_{\rm t}$	rad	angles of incidence, reflection, and refraction (transmitted)	(3.88)
κ	$\mathrm{m}^{-1}$	coupling coefficient; $\kappa_{\nu\mu}$	(4.13)
ĸ	$\mathrm{m}^{-1}$	coupling coefficient; $\tilde{\kappa}_{\nu\mu}$	(4.20)
λ	m	optical wavelength in free space	(1.1)
$\lambda_{\rm c}$	m	cutoff wavelength; $\lambda_m^c$	(3.151)
$\lambda_{ m th}$	m	threshold wavelength	(11.5)
Λ	m	acoustic wavelength or grating period	(2.79)b, (4.35)

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**Partial List of Symbols** 

xxv

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$\mu_0$	${\rm H}~{\rm m}^{-1}$	magnetic permeability of free space	(1.3)
$\mu_{\rm e}, \mu_{\rm h}$	$m^2 \; V^{-1} \; s^{-1}$	electron and hole mobilities	(11.9)
v	Hz	optical frequency	(1.1)
$v_0$	Hz	center optical frequency	(2.27)f, (7.12)
<i>v</i> <sub>21</sub>	Hz	resonance frequency between levels $ 1\rangle$ and $ 2\rangle$	(7.1)
ξ	none	duty factor	Fig. 4.3
$\xi,\xi(M_{0z})$	none	permittivity tensor elements for circular birefringence	(2.16), (2.78)
ρ	$C m^{-3}$	charge density	(1.6)
$ ho_{ m F}$	$rad m^{-1}$	specific Faraday rotation	(10.79)
σ	$\mathrm{S}\mathrm{m}^{-1}$	conductivity; $\sigma_0$	(2.33), (11.9)
$\sigma_{12}, \sigma_{21}$	m <sup>2</sup>	transition cross sections	(7.36), (7.37)
$\sigma_{\rm a}, \sigma_{\rm e}$	m <sup>2</sup>	absorption and emission cross sections	(7.38), (7.39)
$\sigma_s^2$	none	variance of s	(11.19)
τ	8	lifetime, decay time, or time constant	(2.30), (7.6)
$\tau_1, \tau_2$	8	fluorescence lifetimes of levels $ 1\rangle$ and $ 2\rangle$	(7.6), (7.8)
$ au_{\rm c}$	8	photon lifetime; $\tau^{c}_{mnq}$	(6.23)
$ au_{ m s}$	8	saturation lifetime or spontaneous carrier lifetime	(8.23), (10.23)
$ au_{ m sp}$	8	spontaneous radiative lifetime	(7.32)
$\phi$	rad	azimuthal angle, azimuthal angular coordinate	(3.52)
$\phi$	V	work function potential; $e\phi = \text{work function}$	(11.6)
φ	rad	phase or phase shift	(1.63), (1.83)
χ	none	electric susceptibility	(2.11)
χ	V	electron affinity potential; $e\chi$ = electron affinity	(11.7)
$\chi_{\rm res}$	none	resonant electric susceptibility	(2.25), (2.26)
$\chi_x, \chi_y, \chi_z$	none	principal dielectric susceptibilities	(2.15)f
χ', χ"	none	real and imaginary parts of $\chi$ , $\chi = \chi' + 1\chi''$	(2.7)b

xxvi

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#### Partial List of Symbols

(cont.)

Symbol	Unit	Meaning; derivatives	References <sup>1</sup>
$\boldsymbol{\chi}(\mathbf{r},t)$	$m^{-3} s^{-1}$	real susceptibility tensor in the real space and time domain	(1.20)
$\chi(\omega), \chi_{ij}$	none	complex susceptibility tensor in the frequency domain	(1.59)
$\chi^{(2)},\chi^{(2)}_{ijk}$	$m V^{-1}$	second-order nonlinear susceptibility in the frequency domain	(2.98), (2.100)
$\chi^{(3)},\chi^{(3)}_{ijkl}$	$m^2 V^{-2}$	third-order nonlinear susceptibility in the frequency domain	(2.99), (2.101)
Ψ	rad	spatial phase of mode field distribution	(3.107)
$\psi_{\rm e}$	rad	angle between $\mathbf{S}_{e}$ and optical axis of crystal	(3.60)
ω	rad $s^{-1}$	optical angular frequency; $\omega = 2\pi v$	(1.1)b
$\omega_0$	rad $s^{-1}$	center optical angular frequency; $\omega_0 = 2\pi v_0$	(2.22), (7.13)
$\omega_{21}$	rad s <sup>-1</sup>	resonance angular frequency between levels $ 1\rangle$ and $ 2\rangle$	(2.22)
$\omega_{\rm c}$	rad $s^{-1}$	cutoff frequency; $\omega_m^c$	(3.151)
Ω	rad $s^{-1}$	acoustic or modulation angular frequency; $\Omega = 2\pi f$	(2.79), (10.27)
$\Omega_{ m r}$	rad $s^{-1}$	relaxation resonance frequency; $\Omega_{\rm r} = 2\pi f_{\rm r}$	(10.41)

<sup>1</sup> Suffixes, f "forward" and b "backward," on the equation number indicate symbols explained for the first time in the text immediately after or before the equation cited.