Limit Order Books

A limit order book is essentially a file in a computer that contains all orders sent to the market, with their characteristics such as the sign of the order, price, quantity and a timestamp. The majority of organized electronic markets rely on limit order books to store lists of the interests of market participants in their central computer. A limit order book contains all information available on a specific market and it reflects the way the market moves under the influence of its participants.

This book discusses several models of limit order books. It begins by assessing the empirical properties of data, and then moves on to mathematical models in order to reproduce the observed properties. It finally presents a framework for numerical simulations. It also covers important modelling techniques including agent-based modelling, and advanced modelling of limit order books based on Hawkes processes. The book also provides in-depth coverage of simulation techniques and introduces general, flexible, open source library concepts useful to readers in studying trading strategies in order-driven markets.

The book will be useful to graduate students in the field of econophysics, financial mathematics and quantitative finance. The contents of this book are taught by the authors at CentraleSupélec (France) for a course on “Physics of Markets”. A short course based on the content of this book has been taught at the Graduate School of Mathematical Sciences, University of Tokyo (Japan), and it will be used at the Université Paris Saclay (France) for a course in quantitative finance.

Frédéric Abergel is a Professor and Director of the Chair of Quantitative Finance, CentraleSupélec, France. Beginning as a CNRS scientist at Université Paris Sud Orsay, he acquired several years of industrial experience in investment banking at BNP Paribas, CAI Cheuvreux, Barclays Capital and Natixis CIB. His research interests include financial markets, pricing and hedging of derivatives, quantitative finance and empirical properties of financial data.

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Foreword

When physicists became convinced that matter was not continuous but made from atoms, new ideas on old subjects started flourishing. Not only well-known macroscopic laws (thermodynamics, hydrodynamics) became better understood and bolstered by a more fundamental underlying reality, but a host of spectacular and often unexpected effects were rationalized, in particular collective emergent phenomena phase transitions, superconductivity, avalanches, etc. Similarly, after decades of mathematical finance devoted to the study of effective low frequency models of markets (chiefly based on variations on the Brownian motion), the increasing availability of high frequency data now allows a comprehensive study of price formation and of the microstructure of supply and demand. A new era of financial modelling is opening up, with the hope of addressing a hitherto neglected yet crucial aspect of price dynamics: feedback effects that can lead to market anomalies, instabilities and crashes. Instead of considering the market as an inert, reliable measurement apparatus that merely reveals the fundamental value of assets without influencing it, the empirical study of the order book reveals that markets do generate their own dynamics. New intuitions about market dynamics are necessary. New fascinating statistical regularities are collected and modelled, in particular using numerical simulations of agent based models. New analytical tools are being built to account for these observations. The final goal is, much as in physics, to understand the emergent phenomena and replace ad-hoc models of prices by micro-founded ones where jumps, fat-tails and clustered volatility would have a clear origin. This is important on many counts: while the intellectual endeavour is of course exciting in itself, its offshoots will deeply influence the way we think about market regulation in the wake of high-frequency trading, and the models we use for financial engineering (from derivative pricing to algorithmic trading and optimal execution).

Limit order books offers a much needed, broad review of a field that has literally exploded in the last 20 years, where researchers from economics, financial mathematics, physics, computer science, etc. compete and confront. This diversity is well illustrated by the content of the present book that covers a very wide ground, from empirical facts to advanced mathematical techniques and numerical simulation tools. It will be a very useful

and inspiring entry point for all scientists, engineers, regulators and traders interested in understanding how financial markets really work at the basic level.

Jean-Philippe Bouchaud
Capital Fund Management & École Polytechnique

Preface

The Chair of Quantitative Finance was created at École Centrale Paris, now CentraleSupélec, in 2007. Since its inception, most of its research activities were devoted to the study of high frequency financial data. The interdisciplinary nature of the team, composed of mathematicians, financial engineers, computer scientists and physicists, gave it a special dimension. A sizeable portion of its research efforts has been focused on the characterization and mathematical modelling of *limit order books*.

Literally at the core of every modern, electronic financial market, the limit order book has triggered a huge amount of research in the past twenty years, marked by the seminal work of Biais et al. (1995) on the empirical analysis of the Paris exchange and revitalized a few years later, in a fascinating manner, by the work of Smith et al. (2003). However, much as this topic is interesting, important and challenging, we realized that there was still no reference book on the subject! We therefore decided to assemble in a single document a survey of the existing literature and our own contributions on limit order books, whether they were pertaining to their statistical properties, mathematical modelling or numerical simulation.

We have tried to follow the intellectual approach of an experimental physicist: empirical data should come first, and only empirical analyses may be considered as a reliable ground for building up any kind of theory. The mathematical modelling follows. Models address the different phenomena that are observed and highlighted, and provide a framework to explain and reproduce these phenomena, and they are studied from theoretical, analytical and numerical perspectives.

The book is thus organized as follows: The first part is devoted to the empirical properties of limit order books; the second part, to their mathematical modelling and the third, to their numerical analysis. The fourth part deals with some advanced topics such as imperfection and predictability. Each part presents a survey of the existing scientific literature, as well as our own contributions.

Significant parts of the material covered in this book have already been presented in bits and pieces in different research and survey articles, in particular Chakraborti et al. (2011a,b); Abergel and Jedidi (2013, 2015); Anane and Abergel (2015); Muni Toke (2015, 2011). However, what was lacking was a consistent and systematic compilation of these,

found in a single place where the emphasis was set on a single object of interest. We hope that this book will fulfil this need and complement the already existing abundant literature on market microstructure. The interdisciplinary approaches, with the stress on both empirical data analyses and theoretical studies, will hopefully render it useful to the reader – researcher, graduate student or practitioner, while facilitating him/her in finding most of the contemporary knowledge on this essential component of financial markets.

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