

Biomechanics

Thoroughly revised and updated for the second edition, this comprehensive textbook integrates basic and advanced concepts of mechanics with numerical methods and biomedical applications. Coverage is expanded to include a complete introduction to vector and tensor calculus, and new or fully updated chapters on biological materials and continuum mechanics, motion, deformation and rotation, and the constitutive modelling of solids and fluids. Topics such as kinematics, equilibrium, and stresses and strains are also included, as well as the mechanical behaviour of fibres and the analysis of one-dimensional continuous elastic media. Numerical solution procedures based on the finite element method are presented, with accompanying MATLAB-based software and dozens of new biomedical engineering examples and exercises allowing readers to practise and improve their skills. Solutions for instructors are also available online. This is the definitive guide for both undergraduate and graduate students taking courses in biomechanics.

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Cambridge Texts in Biomedical Engineering provide a forum for high-quality textbooks targeted at undergraduate and graduate courses in biomedical engineering. They cover a broad range of biomedical engineering topics from introductory texts to advanced topics, including biomechanics, physiology, biomedical instrumentation, imaging, signals and systems, cell engineering, and bioinformatics, as well as other relevant subjects, with a blending of theory and practice. While aiming primarily at biomedical engineering students, this series is also suitable for courses in broader disciplines in engineering, the life sciences and medicine.

“The increased number of exercises and examples used to bring the lectures alive and to illustrate the theory in biomedical applications make this second edition of the book *‘Biomechanics: Concepts and Computation’* definitely the reference to teach classical concepts of mechanics and computational modelling techniques for biomedical engineers at Bachelor level. The authors from Eindhoven University of Technology belong to one of the most prestigious Departments of Biomedical Engineering around the world, with a well-recognized expertise in Soft Tissue Biomechanics and Tissue Engineering. I have no hesitation in recommending that book that should be a prerequisite for any student studying biomechanics.”

Yohan Payan, *Director of Research at Centre National de la Recherche Scientifique (CNRS), Université Grenoble Alpes*

“A comprehensive textbook for learning all important concepts of biomechanics and their possible applications in sports and medicine. Students will enjoy the opportunity of learning computational modeling in biomechanics from scratch, needing only basic mathematical background. Instructors will appreciate the endless source of problems all resulting from successful experiences of teaching in the authors’ career. Definitely recommended in every library.”

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“*‘Biomechanics: Concepts and Computation’* remains one of the strongest textbooks ever written in the field of biomechanical education. The theory in the book is thorough and rigorous, and is extremely well illustrated with numerous excellent exercises. I find the chapters describing numerical implementation and finite element formulations especially useful for translating the theory of tissue mechanics to bioengineering practice. I am using this book routinely in my undergraduate and graduate courses and will continue to do so with this second edition.”

Amit Gefen, *Tel Aviv University*

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Concepts and Computation

Second Edition

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CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107163720
DOI: 10.1017/9781316681633

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First published 2010
Second edition 2018

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Oomens, C. W. J., author. | Brekelmans, Marcel, author. | Loerakker, Sandra, author. | Baaijens, Franciscus Petrus Thomas, author.

Title: Biomechanics : concepts and computation / Cees Oomens (Eindhoven University of Technology), Marcel Brekelmans (Eindhoven University of Technology), Sandra Loerakker (Eindhoven University of Technology), Frank Baaijens (Eindhoven University of Technology).

Other titles: Cambridge texts in biomedical engineering.

Description: Second edition. | Cambridge, United Kingdom; New York, NY: Cambridge University Press, 2017. | Series: Cambridge texts in biomedical engineering

Identifiers: LCCN 2017026498 | ISBN 9781107163720 | ISBN 1107163722

Subjects: LCSH: Biomechanics.

Classification: LCC QH513 .O56 2017 | DDC 571.4/3–dc23

LC record available at <https://lccn.loc.gov/2017026498>

ISBN 978-1-107-16372-0 Hardback

Additional resources for this publication at www.cambridge.org/Oomens

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About the Cover

The cover contains images reflecting biomechanics research topics at the Eindhoven University of Technology. An important aspect of mechanics is experimental work to determine material properties and to validate models. The application field ranges from microscopic structures at the level of cells to larger organs like the heart. The core of biomechanics is constituted by models formulated in terms of partial differential equations and computer models to derive approximate solutions.

- *Main image:* Myogenic precursor cells have the ability to differentiate and fuse to form multinucleated myotubes. This differentiation process can be influenced by means of mechanical as well as biochemical stimuli. To monitor this process of early differentiation, immunohistochemical analyses are performed to provide information concerning morphology and localization of characteristic structural proteins of muscle cells. In the illustration, the sarcomeric proteins actin (red), and myosin (green) are shown. Nuclei are stained blue. Image courtesy of Mrs Marloes Langelaan.
- *Left top:* To study the effect of a mechanical load on the damage evolution of skeletal tissue, an in-vitro model system using tissue engineered muscle was developed. The image shows this muscle construct in a set-up on a confocal microscope. In the device the construct can be mechanically deformed by means of an indenter. Fluorescent identification of both necrotic and apoptotic cells can be established using different staining techniques. Image courtesy of Mrs Debby Gawlitta.
- *Left middle:* A three-dimensional finite element mesh of the human heart ventricles is shown. This mesh is used to solve the equations of motion for the beating heart. The model was used to study the effect of depolarization waves and mechanics in the paced heart. Image courtesy of Mr Roy Kerckhoffs.
- *Left bottom:* The equilibrium equations are derived from Newton's laws and describe (quasi-)static force equilibrium in a three-dimensional continuum. See Eqs. (8.33), (8.34) and (8.35) in the present book.

Preface to the First Edition

In September 1997, an educational programme in Biomedical Engineering, unique in the Netherlands, started at the Eindhoven University of Technology, together with the University of Maastricht, as a logical step after almost two decades of research collaboration between both universities. This development culminated in the foundation of the Department of Biomedical Engineering in April 1999 and the creation of a graduate programme (MSc) in Biomedical Engineering in 2000 and Medical Engineering in 2002.

Already at the start of this educational programme, it was decided that a comprehensive course in biomechanics had to be part of the curriculum and that this course had to start right at the beginning of the Bachelor phase. A search for suitable material for this purpose showed that excellent biomechanics textbooks exist. But many of these books are very specialized to certain aspects of biomechanics. The more general textbooks address mechanical or civil engineers or physicists who wish to specialize in biomechanics, so these books include chapters or sections on biology and physiology. Almost all books that were found are at Masters or post-graduate level, requiring basic to sophisticated knowledge of mechanics and mathematics. At a more fundamental level, only books could be found that were written for mechanical and civil engineers.

We decided to write our own course material for the basic training in mechanics appropriate for our candidate biomedical engineers at Bachelor level, starting with the basic concepts of mechanics and ending with numerical solution procedures, based on the finite element method. The course material assembled in the current book comprises three courses for our biomedical engineering curriculum, distributed over the three years of their Bachelor studies. Chapters 1 to 6 mostly treat the basic concepts of forces, moments and equilibrium in a discrete context in the first year. Chapters 7 to 13 in the second year discuss the basis of continuum mechanics, and Chapters 14 to 18 in the third year are focussed on solving the field equations of mechanics using the finite element method.

What makes this book different from other basic mechanics or biomechanics treatises? Of course, as in standard books, there is the usual attention focussed on kinematics, equilibrium, stresses and strains. But several topics are discussed that are normally not found in one single textbook or only described briefly.

- Much attention is given to large deformations and rotations and non-linear constitutive equations (see Chapters 4, 9 and 10).
- A separate chapter is devoted to one-dimensional visco-elastic behaviour (Chapter 5).
- Special attention is given to long, slender, fibre-like structures (Chapter 4).
- The similarities and differences in describing the behaviour of solids and fluids and aspects of diffusion and filtration are discussed (Chapters 12 to 16).
- Basic concepts of mechanics and numerical solution strategies for partial differential equations are integrated in one single textbook (Chapters 14 to 18).

Because of the usually rather complex geometries (and non-linear aspects) found in biomechanical problems, hardly any relevant analytical solutions can be derived for the field equations, and approximate solutions have to be constructed. It is the opinion of the authors that, at Bachelor level, at least the basis for these numerical techniques has to be addressed.

In Chapters 14 to 18 extensive use is made of a finite element code written in MATLAB by one of the authors, which is especially developed as a tool for students. Applying this code requires that the user has a licence for the use of MATLAB, which can be obtained via MathWorks (www.mathworks.com). The finite element code, which is a set of MATLAB scripts, including manuals, is freely available and can be downloaded from the website: www.tue.nl/biomechanicsbook.

Preface to the Second Edition

Since 2009, when this book was published for the first time, we have been using it in our Biomechanics courses in the educational programme Biomedical Engineering, giving us hands-on experience with the book and the exercises. When we were investigating ideas for a second edition, we found that external reviewers were primarily asking for more examples and exercises, concurring with our own thoughts on the book. Over the years, we have assembled quite a number of examples that were often used to animate the lectures and to illustrate the theory in biomedical applications. At the same time, the number of available exercises increased considerably. Eventually, adding many of these examples and increasing the number of exercises are the most significant changes in the second edition. The major changes in the text are:

- Mathematical preliminaries are now concentrated in Chapter 1 and no longer spread over different chapters.
- At some points in the original text, explanations were terse and too concise for students. Based on our experiences over the past eight years, we have extended the text at a number of points and, most importantly, added the earlier mentioned new examples. The biggest change is in Chapter 12, including a separate section on material frame indifference of constitutive equations and a more extensive treatment of hyperelastic materials.

The objectives of the book did not change. It is still meant to be a basic training in mechanics, appropriate for our candidate biomedical engineers at Bachelor level, starting with the basic concepts of mechanics and ending with numerical solution procedures. This book differs from most books on biomechanics, which are usually aimed at students with already considerable knowledge in continuum mechanics and wishing to enter the field of biomechanics. Consequently, those books pay great attention to biology and physiology. In contrast, we assume that students start at a very basic level in terms of mechanics, but already have substantial physiological and biological background knowledge.