A Student's Guide to Dimensional Analysis

This introduction to dimensional analysis covers the methods, history, and formalization of the field, and provides physics and engineering applications. Covering topics from mechanics, hydro- and electrodynamics to thermal and quantum physics, it illustrates the possibilities and limitations of dimensional analysis. Introducing basic physics and fluid engineering topics through the mathematical methods of dimensional analysis, this book is perfect for students in physics, engineering, and mathematics. Explaining potentially unfamiliar concepts such as viscosity and diffusivity, this text includes worked examples and end-of-chapter problems with answers provided in an accompanying appendix, which help make this text ideal for self-study. Long-standing methodological problems arising in popular presentations of dimensional analysis are also identified and solved, making it a useful text for advanced students and professionals.

DON S. LEMONS is Professor Emeritus of Physics at Bethel College in North Newton, Kansas, and has served as an assistant editor of the *American Journal of Physics*. He is a member of the American Physical Society and his research focuses primarily on plasma physics.

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DON S. LEMONS
Bethel College, Kansas

CAMBRIDGE
UNIVERSITY PRESS

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University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107161153
10.1017/9781316676165

First published 2017

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data
Names: Lemons, Don S. (Don Stephen), 1949– author.
Title: A student's guide to dimensional analysis / Don Lemons.
Description: New York : Cambridge University Press, 2017. | Includes bibliographical references and index.
Identifiers: LCCN 2016040385 | ISBN 9781107161153 (hardback)
Subjects: LCSH: Dimensional analysis. | BISAC: SCIENCE / Mathematical Physics.
Classification: LCC TA347.D5 L46 2017 | DDC 530.8–dc23 LC record available at https://lccn.loc.gov/2016040385

ISBN 978-1-107-16115-3 Hardback
ISBN 978-1-316-61381-8 Paperback

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Preface

I was charmed when as a young student I watched one of my physics professors, the late Harold Daw, work a problem with dimensional analysis. The result appeared as if by magic without the effort of constructing a model, solving a differential equation, or applying boundary conditions. But the inspiration of the moment did not, until many years later, bear fruit. In the meantime my acquaintance with this important tool remained partial and superficial. Dimensional analysis seemed to promise more than it could deliver.

Dimensional analysis has charmed and disappointed others as well. Yet there is no doubt that a deep understanding of its methods is useful to researchers in a number of fields. Attesting to this fact is that dimensional analysis is the subject of several good, graduate-level monographs. Even so, dimensional analysis is often ignored at the introductory level except when teachers admonish their students to "check the units of their result" and warn them against "adding apples and oranges."

The problem for teachers and students is that dimensional analysis has no settled place in the physics curriculum. The mathematics required for its application is quite elementary – of the kind one learns in a good high school course – and its foundational principle is essentially a more precise version of the rule against "adding apples and oranges." Yet the successful application of dimensional analysis requires physical intuition – an intuition that develops only slowly with the experience of modeling and manipulating physical variables. But how much intuition is required?

I have written *A Student's Guide to Dimensional Analysis* in the belief that the simple techniques of dimensional analysis can deepen our understanding and enhance our exploration of physical situations and processes at the introductory level. Thus, this text is designed for students who are taking or have taken an entry-level, mathematically oriented, university physics course. More experienced students and professionals may also find this text useful.

One elementary application of dimensional analysis is to a simple pendulum, that is, to a compact weight swinging from the end of a string. How long does the pendulum take to complete one cycle of its motion? It seems that this interval Δt might depend on the mass of the weight m, the acceleration of gravity g, the length of the string l, and the maximum angle of its swing θ. The adjective "simple" in "simple pendulum" means that the size of the weight is negligibly small compared to the length of the pendulum l. Therefore, we search for a function of m, l, g, and θ that produces Δt. We note that the International System or SI unit of the period Δt is the second, the SI unit of mass m is the kilogram, of length l is the meter, and of acceleration g is a meter divided by a second squared. The angle θ is dimensionless. We quickly discover that $\sqrt{l/g}$ is the only combination of m, l, and g that produces a quantity whose SI unit is a second. Therefore, it must be that

$$\Delta t = \sqrt{\frac{l}{g}} \cdot f(\theta)$$

where $f(\theta)$ is an as yet undetermined, dimensionless function of θ.

This result is typical of dimensional analysis. Much is learned but something is left unlearned. The analysis teaches us that the period Δt of a simple pendulum is directly proportional to the square root of its length $\sqrt{l}$ and does not at all depend on its mass m. We could experimentally verify these findings, but dimensional analysis suggests that limited time might be better spent exploring the undetermined function $f(\theta)$. However, this example misleads as well as informs. Not many applications of dimensional analysis are this simple, nor are we usually aware, as we probably are in this case, of the result beforehand.

Dimensional analysis is most revealing when we know *something* but not *everything* about a situation or process. Then dimensional analysis builds on that *something* we already know. We might, for instance, be familiar with the equations that describe a certain process but not have the skill or the time to solve them in the usual way. Or we might wish to expand our preliminary knowledge of a solution before completely determining its form. Then again, we might know only the category of the problem (that is, mechanics, thermodynamics, or electrodynamics) and the variable we want to determine (that is, an oscillation period, a pressure drop, or an energy loss rate). In all these cases, dimensional analysis constrains how the relevant physical variables and constants work together to produce the result sought.

The techniques of dimensional analysis could be presented in one well-chosen example, occupying one or two pages of text. Then one might, with some reason, expect to apply these techniques to new problems as they arise. But this expectation is bound to remain unrealized. In order for dimensional analysis to be fruitful, it must first be cultivated. Exploring the method's motivation, its history, and its formalization are steps in this cultivation. Examining simple applications is another. Such are the aims of Chapter 1 "Introduction."

Chapters 2 through 6 take up more complex but still introductory examples of dimensional analysis grouped into several subject areas: Mechanics, Hydrodynamics, Thermal Physics, Electrodynamics, and Quantum Physics. These examples illustrate the possibilities and limitations of dimensional analysis. As appropriate, I will explain concepts (such as surface tension, viscosity, and diffusivity) that may not be familiar to all readers.

In the final chapter, Chapter 7 "Dimensional Cosmology," I use dimensional analysis to take a few steps in the direction of uncovering the dimensional structure of our world. The result, preliminary and partial as it is, brings us close to the boundary separating the known from the unknown.

Dimensional analysis has no settled place in the physics curriculum because it fits easily in any number of places. We can use it in elementary fashion to recall the shape of a formula or to reaffirm our understanding of an already solved problem. Or we can use it to push forward into territory unknown to us or uncharted by anyone.

Sometimes dimensional analysis fails us, but it is unlikely to do so without announcing its failure and suggesting a better way to proceed. Does the method produce no result? Then we have left out a crucial variable or constant. Is the result uninformative? Then we have included too many variables and constants.

A Student's Guide to Dimensional Analysis is designed to guide readers to an understanding of the motivation, methods, and exemplary applications of dimensional analysis, its scope and powers as well as its limitations. But I also hope the text will recreate for you the charm and magic that first attracted me to the subject years ago.

Acknowledgments

One of the rewards of writing a book is receiving responses to its initial drafts from an obliging set of friends. I have been very fortunate in my "first responders." Ralph Baerlein and Galen Gisler read the whole text, suggested many improvements, and kept me from making a number of mistakes. Ikram Ahmed, Christopher Earles, Rick Shanahan, and David Watkins all reviewed parts of the text and gave me good advice. Jeff Martin suggested the problem "cooking a turkey." I offer a heartfelt "thanks" to them all.

My participation in a series of seminars on similarity methods, organized by Susan G. Sterrett of Wichita State University in the academic year 2014–15, renewed a youthful interest in dimensional analysis. As usual, Cambridge University Press has provided me with excellent editorial services.