

Optical Effects in Solids

An overview of the optical effects in solids, this book addresses the physics of materials and their response to electromagnetic radiation. The discussion includes metals, superconductors, semiconductors and insulators. The book begins by introducing the dielectric function into Maxwell's macroscopic equations and finding their plane-wave solution. The physics governing the dielectric function of various materials is then covered, both classically and using basic quantum mechanics. Advanced topics covered include interacting electrons, the anomalous skin effect, anisotropy, magneto-optics, and inhomogeneous materials. Each subject begins with a connection to the basic physics of the particular solid, after which the measurable optical quantities are derived. It allows the reader to connect measurements (reflectance, optical conductivity, and dielectric function) with the underlying physics of solids. Methods of analyzing experimental data are addressed, making this an ideal resource for students and researchers interested in solid-state physics, optics, and materials science.

David B. Tanner is a distinguished professor of physics at the University of Florida. His research focuses on condensed matter physics and on astrophysics. He has previously served as department chair and chair of the Division of Condensed Matter Physics for the American Physical Society (APS). In 2016 he received the APS Frank Isakson Prize for Optical Effects in Solids and, in the same year, shared a Special Breakthrough Prize in Fundamental Physics for the discovery of gravitational waves.

Optical Effects in Solids

DAVID B. TANNER
University of Florida



CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107160149

DOI: 10.1017/9781316672778

© David B. Tanner 2019

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2019

Printed in the United Kingdom by TJ International Ltd, Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Tanner, David B., 1945– author.

Title: Optical effects in solids / David B. Tanner (University of Florida).

Description: Cambridge ; New York, NY : Cambridge University Press, [2019] |

Includes bibliographical references and index.

Identifiers: LCCN 2018046542 | ISBN 9781107160149 (hardback)

Subjects: LCSH: Solids–Optical properties. | Light. | Materials–Optical properties. |

Light–Scattering. | Reflection (Optics) | Electromagnetic waves.

Classification: LCC QC176.8.O6 T36 2019 | DDC 530.4/12–dc23

LC record available at <https://lcn.loc.gov/2018046542>

ISBN 978-1-107-16014-9 Hardback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

for Marcia

Contents

<i>Preface</i>	<i>page xi</i>
1 Introduction	1
2 Maxwell's Equations and Plane Waves in Matter	6
2.1 Optical Constants	6
2.2 Maxwell's Equations	6
2.3 Total, Free, and Bound Charges and Currents	8
2.4 Maxwell's Equations for Solids	9
2.5 Plane-Wave Solutions	10
2.6 Converting Differential Equations to Algebraic Ones	11
2.7 Vector Directions	11
2.8 Electromagnetic Waves in Vacuum	11
2.9 Five Easy Simplifications	12
2.10 Maxwell's Equations for These Easy Cases	15
3 The Complex Dielectric Function and Refractive Index	17
3.1 Conductivity and Dielectric Constant	17
3.2 The Complex Dielectric Function	18
3.3 The Optical Conductivity	18
3.4 The Complex Refractive Index	19
3.5 Energy Density, Poynting Vector, and Intensity	21
3.6 Normal-Incidence Reflectance	24
3.7 What If My Solid Is Magnetic?	27
3.8 Negative Index Materials	28
4 Classical Theories for the Dielectric Function	30
4.1 A Polarizable Medium	30
4.2 Drude Absorption by Free Carriers	31
4.3 The Plasma Frequency	42
4.4 Lorentz Model: Absorption by Bound Electrons	44
4.5 Comments on Wave Propagation	55
4.6 The Absorption Coefficient and a Not Uncommon Mistake	61

5	Phonons	62
5.1	Harmonic Oscillator	62
5.2	Lattice Dynamics	64
5.3	Interaction with Electromagnetic Waves	72
5.4	Transverse and Longitudinal Modes	73
5.5	The Lyddane–Sachs–Teller Relation	75
5.6	Other Notations	76
5.7	Multiple Modes	76
5.8	Polaritons	78
6	A Look at Real Solids	81
6.1	Silver	81
6.2	Other Noble Metals	85
6.3	Aluminum	86
6.4	Silicon	87
6.5	Sodium Chloride	90
6.6	Other Alkali Halides	93
7	Transmission and Reflection	96
7.1	Incoherent Light	96
7.2	Coherent Light	99
7.3	The Matrix Method	107
7.4	Inverting \mathcal{R} and \mathcal{T} to Find ϵ	109
8	Free-Electron Metals	115
8.1	Schrödinger Equation for Free Electrons	115
8.2	Wave Function	116
8.3	Exclusion Principle and Boundary Conditions	117
8.4	The Fermi Energy	119
8.5	The Effect of Temperature	121
8.6	The Density of States	122
8.7	Electrical Conductivity	122
8.8	Discussion of the Drude Model	125
8.9	The Boltzmann Transport Equation	130
9	Optical Excitations: Quantum Mechanics	137
9.1	The Solid with an Electromagnetic Field	138
9.2	Perturbation Expansion	139
9.3	The Matrix Element of the Perturbation	141
9.4	Fermi’s “Golden Rule”	143
9.5	Electric Dipole Transitions	145
9.6	The Oscillator Strength	146
9.7	Oscillator Strength Sum Rule	148

10	Kramers–Kronig Relations and Sum Rules	152
10.1	Absorption and Dispersion Must Be Related	152
10.2	Kramers–Kronig Integrals	157
10.3	Kramers–Kronig Analysis of Reflectance	164
10.4	Another Look at the Conductivity	169
10.5	Sum Rules	170
10.6	Partial Sum Rules	173
11	Superconductors	175
11.1	Superconducting Phenomena	175
11.2	Theoretical Background	180
11.3	The London Model	181
11.4	Length Scales for Superconductors	188
11.5	Excitations in a Superconductor	189
11.6	The Optical Conductivity of a Superconductor	195
11.7	Thin Film Superconductors	201
11.8	Unconventional Superconductors	203
12	Semiconductors and Insulators	207
12.1	Band Structure	207
12.2	Nearly Free Electrons	209
12.3	Bloch’s Theorem	212
12.4	The Brillouin Zone	213
12.5	Band Gaps of Semiconductors	215
12.6	Effective Mass	216
12.7	Direct Interband Transitions	218
12.8	The Joint Density of States and Critical Points	224
12.9	Indirect Band Gap Absorption	225
12.10	Excitons	233
12.11	Impurity-Induced Absorption	236
13	Strongly Interacting Solids	241
13.1	Notation: The Generalized Drude Model	242
13.2	Electron–Electron Interactions	247
13.3	Electron–Phonon Interactions	257
14	Nonlocal Effects	263
14.1	The Normal Skin Effect	263
14.2	The Anomalous Skin Effect	266
14.3	The Extreme Anomalous Limit	267
14.4	ω – τ Plot	268
14.5	The Surface Impedance	270

15 Anisotropic Crystals	276
15.1 Optics of Crystals	276
15.2 Polarized Light	276
15.3 Crystal Symmetry	279
15.4 The Dielectric Tensor	282
15.5 Plane-Wave Propagation in Anisotropic Materials	285
15.6 The Uniaxial Crystal	288
15.7 The Biaxial Crystal	295
15.8 Anisotropic Material with Boundaries: \mathcal{R} and \mathcal{T}	297
15.9 Polarizers and Waveplates	300
15.10 Things Not in This Chapter	303
16 Magneto-Optics	304
16.1 Charged Particle in a Magnetic Field	304
16.2 Magneto-Conductivity Tensor of a Metal	309
16.3 Electromagnetic Wave Propagation	315
16.4 Other Magneto-Optical Effects	324
16.5 Faraday Rotation in an Insulator	325
17 Inhomogeneous Materials	333
17.1 The Effective Medium	335
17.2 Response of a Single Grain	336
17.3 Effective Medium Theories	337
17.4 Other Approaches	344
<i>Appendix A Notes about Units</i>	347
<i>Appendix B Maxwell's Equations in SI</i>	350
<i>Appendix C Partial Derivatives and Vector Operators Acting on Plane Waves</i>	352
<i>Appendix D The Wave Equation</i>	353
<i>Appendix E Reflection and Transmission at Oblique Incidence</i>	354
<i>Appendix F A Field Guide to Optical "Constants"</i>	365
<i>Appendix G Software</i>	372
<i>References</i>	373
<i>Index</i>	397

Preface

This book is a discussion of optical effects in solids, addressing the physics of many types of solids (metals, superconductors, semiconductors, insulators, and others) and their response to electromagnetic radiation. I try to make a connection between what an experimenter can measure or extract from measurements (reflectance, transmittance, optical conductivity, and dielectric function) and the microscopic physics of the solid. Methods of analyzing experimental data are addressed: the optics of thin films and the Kramers–Kronig relations.

I begin with introducing the dielectric function into Maxwell’s macroscopic equations and finding their plane-wave solution. Then I discuss (first classically and then using basic quantum mechanics) the dielectric function of various materials. Other topics include interacting electrons, the anomalous skin effect, anisotropy, magneto-optics, and inhomogeneous materials.

I’ve attempted to write as a relatively complete coverage of the subject, starting with Maxwell’s equations for the electromagnetism and with the Schrödinger equation and Newton’s laws for the solid-state physics. The level of the presentation is aimed at the first- or second-year experimental graduate student. The electromagnetism assumed is undergraduate (Griffiths or Marion); the level of quantum mechanics is about the same (Griffiths or Peebles). Finally, it is helpful to have gone through solid-state physics (Kittel or Burns).

When I was a postdoc, the book by Frederick Wooten, *Optical Properties of Solids* (1972), appeared in the University Bookstore. I bought a copy and went through it during the next few weeks. It was just what I needed to read at the time: It was clear, it covered topics of interest to me, and it was at a level I could follow. I refer to it still and recommend it to others; I have also used it several times as a text in a course on optical properties for first-year graduate students in physics, materials science, electrical engineering, and chemistry. Unfortunately, the book is long out of print. I’ve attempted to write at the same level as Wooten and cover many of the topics of his book. (The exception is photoemission, which is a huge subject of its own by now.) I have included a number of things that Wooten did not address, such as phonons, superconductivity, anisotropy, magneto-optics, and inhomogeneous materials.

This book started life as the 2013 lecture notes for a graduate class in the optical properties of solids. This class has been offered to University of Florida graduate students at two- to six-year intervals over more than 25 years. That these started as lectures may be responsible for a certain informality in presentation. I’ve written in the first person, trying to avoid the royal or inclusive “we” that in much exposition attempts to co-opt the reader or the listener. I think that whenever a person hears “and now we see” or “in the future

we must,” a quite proper response is “What do you mean by *we* Kemo Sabe.” First person avoids this response. I have also tried to avoid second-person instructions: “If you look at the figure, you can see . . .” Well, a person looking at the figure may see what I want her or him to see, but may not. If not, it is my fault for not designing the right figure, not explaining it correctly, or not providing adequate background. In any event, it is better if I say what I see when I look at the figure and not presume more.

All solids may be divided into three classes based on conductivity: metals, semiconductors, and insulators. I use silver, silicon, and sodium chloride as examples of these classes. These materials are useful to persons other than solid-state physicists. The cover of the book shows one example. It shows the bowl of a toddy ladle made in England around 1760. Set into the bowl of the ladle is a silver sixpence that was minted in 1758. The coin was designed during the reign of George II by John Sigismund Tanner, Chief Engraver of the Royal Mint. The sixpence coin was called a “tanner” by the British right up until decimalization. (Six pennies may not seem a lot but the hourly wage at the time was 2–8 pence.)

I’ve had discussion with many persons over many years about the subject of optical effects in solids. I am grateful for all these discussions. A number of these discussions had direct influence on what I wrote here. For such discussions I thank Larry Carr, Jim Garland, Alan Heeger, Claus Jacobsen, Kati Kamarás, Ricardo Lobo, Frank Marsiglio, Dmitri Maslov, Jan Musfeldt, Michael Rice, Danilo Romero, Al Sievers, David Stroud, Lila Tache, Tom Timusk, Axel Zibold, the students who took PHY7097 and made comments and found typos, and the editors at Cambridge University Press for patience and accommodation.