

A PRIMER ON STRING THEORY

Since its conception in the 1960s, string theory has been hailed as one of the most promising routes we have to unify quantum mechanics and general relativity. This book provides a concise introduction to string theory, explaining central concepts and mathematical tools and covering recent developments in physics, including compactifications and gauge/string dualities. With string theory being a multidisciplinary field interfacing with high-energy physics, mathematics, and quantum field theory, this book is ideal for both students with no previous knowledge of the field and scholars from other disciplines who are looking for an introduction to basic concepts.

VOLKER SCHOMERUS holds a joint position as Senior Researcher in the DESY Theory Group and Professor of Mathematical Physics at Universität Hamburg. His work focuses on string theory at the interface of high-energy physics, mathematics, and statistical physics, for which he has received several distinctions, including the Guy-Lussac-Humboldt award in 2010. He is also co-author of the monograph *Boundary Conformal Field Theory and the World-Sheet Approach to D-Branes* (Cambridge, 2013).

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VOLKER SCHOMERUS

Deutsches Elektronen-Synchrotron (DESY) Hamburg



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Preface

About half a century has passed since string theory was born, initially to explain the observed Regge trajectories of hadronic resonances. During these past decades, it has gone through several phases of dismissal and new appreciation, it has become a promising candidate for a quantum theory of gravity, and occasionally it has been popularized as a theory of everything. What most of these periods have in common are profound and fruitful interactions with rather different areas of theoretical physics, e.g., with quantum field theory, high-energy physics, gravity, mathematics, and even statistical and condensed matter physics. Indeed, there is a long list of string-inspired developments, a few of which we will have a chance to touch upon below. Among them are, e.g., new unified extensions of the standard model that are being tested in collider experiments, insights into black holes and their entropy, contributions to knot invariants or the theory of modular forms, and our understanding in particular of 2-dimensional critical systems, or the impressive recent progress in accessing both perturbative as well as non-perturbative effects in supersymmetric gauge theories. This interdisciplinary nature of string theory is its real strength, and it can serve as a good motivation to enter or explore the field.

While this textbook addresses primarily masters' or early PhD students, it may also be useful for scholars from neighboring fields who are looking for a short exposition of basic string theory. Compared to most other textbooks in the field, the scope here is fairly modest. Rather than attempting to cover string theory from its roots to all the fascinating modern developments, the intention is merely to provide a set of concepts and tools that are common to a wide range of recent research directions. By now many books have been written on string theory, which include various advanced and topical directions, such as [27, 28, 55, 56, 6, 39, 74, 41, 1] to list just a few that can be turned to for further reading. In addition, let me also mention a few shorter introductions such as [65, 68, 2]. Finally, a beautiful book by Barton Zwiebach [80] covers much less material but addresses undergraduates.

Some students might find it useful to consult this text, in particular to fill in occasional gaps in their undergraduate education.

The restrictions imposed on the choice of topics make it possible to cover the material of these Chapters in a one or two semester introductory course. Very few prerequisites are assumed, e.g., from quantum field theory, general relativity, or supersymmetry, so that master's students should have no problem following the exposition. The book consists of two parts. Part I describes strings in flat backgrounds. It introduces most of the basic constructions and results in string theory. This part of the material can be covered in a 10 to 12 lecture crash course, including Chapters 1–4, 6–8, and 10, along with Chapters 18 and 20 from Part II, for an outlook on modern developments. In this short form, the chapters assume some basic knowledge of supersymmetry. The minimal background is collected in Chapter 9. The remaining chapters from Part I deal with light-cone quantization, D-branes, and heterotic strings. These may be considered as additional material that may be included if time permits.

Part II contains somewhat more advanced topics on strings in curved backgrounds and string compactifications. It addresses three distinct subtopics. The first one is *conformal field theory*. As an example of the free field some very basic strategies and notions are presented in Chapter 13. The concept of modular invariance is the central theme of Chapter 14. It is explored with the example of the so-called orbifold construction. The third chapter on conformal field theory techniques deals with the SU(2) WESS-ZUMINO-NOVIKOV-WITTEN model, a theory that can be used to describe strings on a space of constant curvature. It allows us to illustrate most of the key techniques and concepts of conformal field theory.

The second subtopic treats CALABI-YAU spaces and the associated string *compactifications*. Chapter 16 introduces some background material on geometric concepts starting from complex manifolds and ending with some basic examples of complete intersections of CALABI-YAU spaces such as the famous quintic. This background material is then applied to discuss basic properties of string compactifications on CALABI-YAU spaces. The focus is on a space-time analysis, but a short discussion of the relevant world-sheet models is also provided, thereby interacting with the chapters on conformal field theory.

The book concludes with three chapters on *string dualities*. These possess the character of an overview rather than a detailed exposition. Chapter 18 starts with a brief discussion of T-duality, proceeds to the self-duality of type IIB superstring theory, and finishes with strongly coupled IIA models and some evidence for the existence of M-theory. Chapters 19 and 20 finally deal with the dualities between gauge theory and string theory that appear in the context of the *AdS/CFT correspondence*. These include a short discussion of large N_c limits before stating Maldacena's duality between the maximally (N=4) supersymmetric YANG-MILLS

(SYM) theory in four dimensions and strings in an $AdS_5 \times S^5$ background. Some basic entries of the dictionary between string and gauge theory are also introduced. The final chapter carries Maldacena's duality a bit deeper by providing more detail on N=4 SYM theory and on semiclassical strings in $AdS_5 \times S^5$.

I gave the lectures that make up this book or some of them on various occasions, in particular during the “String Steilkurs” at DESY, as one or two semester courses in Hamburg University, and at the APCTP Winter School. Many of the students who attended these courses have contributed through their questions and comments. I wish to thank all of them and in particular those who spent their time to compile seemingly never ending lists of misprints. I am particularly indebted to Philipp Höffer v. Loewenfeld and Johannes Oberreuter, who initiated these notes and typed the first 10 chapters. Some of the more advanced chapters were written with the assistance of my former students Vladimir Mitev and Maike Tormählen. Finally, I am grateful to Simon Capelin for his continued encouragement to complete the notes.

I dedicate this book to my wife Elena, to our sons Jonathan and Benedict, and to Leon.