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Historical Introduction and Overview

In this first chapter, I shall review some of the history of string theory, starting with early attempts to apply strings to hadronic physics. Then, I discuss how string theory emerged as a theory of gravitational physics and sketch the picture of modern string theory that was developed mostly during the second half of the 1990s. The discussion is based on images. It is the main goal of this book to introduce all the background that is required to give precise meaning to the cartoon you are about to see (Figures 1.2–1.6). In a first approach, the present chapter just serves for some rough and qualitative orientation.

1.1 String Theory and Strong Interactions

Most insight into the physics of particles comes from the study of scattering amplitudes. To take the simplest case, let us consider an elastic scattering process of two particles. After the scattering event, two particles emerge from the scattering event.

Such a $2 \to 2$ scattering process possesses many symmetries. The amplitude can depend only on kinematic invariants of the process. A widely used choice for these invariants is given by the so-called MANDELSTAM variables s, t, u, which are defined by

$$s = -(p_1 + p_2)^2, \tag{1.1}$$

$$t = -(p_1 + p_3)^2, \tag{1.2}$$

$$u = -(p_1 + p_4)^2. \tag{1.3}$$

Here, p_1 and p_2 denote the momenta of the incoming particles, and $-p_3, -p_4$ are momenta of the outgoing ones, see Figure 1.2. Throughout this book we shall work with a MINKOWSKI signature in which

$$q^2 = -q_0^2 + q_1^2 + \cdots + q_{D-1}^2.$$

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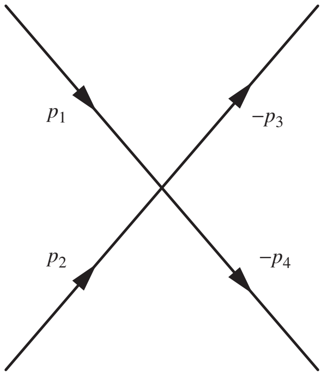


Figure 1.1 Kinematics of an elastic 2 → 2-scattering process of particles with momenta p_i.

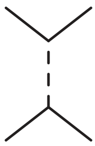
The sum of the three MANDELSTAM variables is determined by the masses m_i of the particles through $s + t + u = \sum_i m_i^2$. From now on we shall consider amplitudes as functions of s, t.

In the center of mass system, we can reexpress s and t through the energy $E = E_1 = E_2$ of the incoming particles and the center of mass scattering angle θ_s:

$$s = 4E_1^2 \,, \tag{1.4}$$

$$t = -\frac{s}{2}(1 - \cos\theta_s) \,. \tag{1.5}$$

In the late 1960s, physicists analyzed in great detail the scattering amplitudes for hadronic particles. What they found in the laboratory was an enormous number of new resonances. The longer the searches were pursued, the higher became the spins J and masses M of the observed particles. In addition, a curious linear relation $J = \alpha_0 + \alpha' M^2$ emerged that could be characterized by the so-called REGGE slope α' and intercept α_0; see, e.g. [37, 16]. Since at high energies s the t-channel exchange of a spin J resonance with mass M_J contributes a term $g_J^2(-s)^J/(t - M_J^2)$ to the total amplitude,[1] the overall contribution of the hadronic resonances takes the form

$$\mathcal{A}(s,t) \;=\; \sum_J$$

$$\sim -\sum_J g_J^2 \frac{(-s)^J}{t - M_J^2} \,, \tag{1.6}$$

where the sum is taken over all possible intermediate resonances that can be exchanged in the t channel. If there was only a finite number of such terms, i.e., if the sum stopped at some finite value of J, then the high-energy behavior

[1] The expression we work with here is valid only for large values of s. It receives lower order corrections for finite s that are well known but not relevant for our discussion.

would be dominated by the resonance with highest spin J, and the whole amplitude would diverge as we send s to infinity. If the sum is infinite, however, the outcome may be very different. As an example, let us consider the exponential function

$$\exp(-x) \;=\; 1 - x + \frac{1}{2}x^2 - \frac{1}{3!}x^3 + \dots .$$

Suppose now that we truncate this sum after the n^{th} term. Then, the finite sum diverges for $x \to \infty$. On the other hand, the entire function $\exp(-x)$ is certainly finite. In fact, it even vanishes in the limit $x \to \infty$. Could something similar happen for hadronic scattering amplitudes, i.e., could there be some analytic expression $\mathcal{A}$ that reproduces the expansion (1.6) for small s but that stays finite at large s? The answer is positive, and one such expression is given by the so-called VENEZIANO amplitude [71]

$$\mathcal{A}_{\text{Ven}}(s,t) \;=\; \frac{\Gamma(-\alpha_0 - \alpha' s)\Gamma(-\alpha_0 - \alpha' t)}{\Gamma(-2\alpha_0 - \alpha'(s+t))} \;. \tag{1.7}$$

From the pole structure and shift properties of Γ functions it is easy to deduce the following expansion of $\mathcal{A}$ at small s:

$$\mathcal{A}_{\text{Ven}}(s,t) \;\sim\; -\sum_J \frac{P^J(s)}{\alpha' t - J + \alpha_0} \;. \tag{1.8}$$

Here, P^J is a polynomial of degree J. Hence, $\mathcal{A}$ does indeed encode the exchange of resonances that lie on a REGGE trajectory $M^2 = (J - \alpha_0)/\alpha'$. In 1970, string theory seemed to provide an exciting perspective on these findings. Namely, it was shown that simple (open) string theories in flat space [47, 50, 64] naturally reproduce the VENEZIANO amplitude with $\alpha_0 = 1$. This success of string theory is not too difficult to understand. String modes in flat space are harmonic oscillators, and it is well known from basic quantum mechanics that these possess a linear spectrum with a distance between the spectral lines that is determined by the tension T_s of the string. If we choose the latter to be $T_s \sim 1/\alpha'$, then we may identify hadronic resonances with vibrational modes of a string (provided we are willing to close an eye to the first resonance with $J = 0$, which is tachyonic because $\alpha_0 = 1$).

Obviously, the formula (1.7) must not be restricted to small center of mass energies. It can also be evaluated, e.g., for fixed angle scattering at large s. Using once more some simple properties of the Γ function one can derive

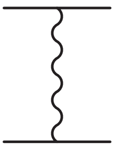
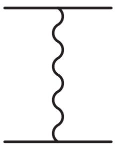
$$\mathcal{A}_{\text{Ven}}(s,t) \;\sim\; f(\theta_s)^{-1-\alpha' s} \quad (\text{large s}) \,, \tag{1.9}$$

where f is some function of the center of mass scattering angle θ_s whose precise form is not relevant for us. The result shows that fixed angle scattering amplitudes predicted by flat space string theory fall off exponentially with the energy $\sqrt{s}$. Unfortunately for early string theory, this is not at all what is found in experiments, which display much harder high-energy cross sections. The failure of string theory to produce the correct high-energy features of scattering experiments is once more easy to understand: strings are extended objects, and as such they do not interact in a single point but rather in an extended region of space-time. Consequently, their scattering amplitudes are rather soft at high energies (small distances), at least compared to point particles. In this sense, experiments clearly favored a point particle description of strongly interacting physics over fundamental GeV scale strings.

As we all know, a highly successful point particle model for strong interactions, known as Quantumchromodynamics (QCD), was established only a few years later. It belongs to the class of gauge theories that have ruled our description of nature for several decades now. Due to its asymptotic freedom, high-energy QCD is amenable to a perturbative treatment. On the other hand, low-energy (large distance) physics is strongly coupled and therefore remains difficult to address. Even though the problem to understand, e.g., confinement remains unsolved, QCD has at least never made any predictions that could be clearly falsified in a simple laboratory experiment, in contrast to what we have reviewed about early string theory. So, in spite of its intriguing success with hadronic resonances, string theory retreated from the area of strong interactions, and it even disappeared from physics before reemerging as a quantum theory of gravity.

1.2 Closed Strings and Supergravity

Before we can fully appreciate the role of closed string theory as a natural host for gravity, we would like to briefly recall the basic problem one faces when applying perturbation theory to quantize EINSTEIN's theory of gravity. In order to see the issue, let us compare the following two amplitudes:

$\mathcal{A}_0 =$  , $\mathcal{A}_1 =$  . (1.10)

The second amplitude comes weighted with NEWTON's gravitational constant G_N. Therefore, by a simple dimensional analysis, we conclude that the dimensionless quotient of $\mathcal{A}_1/\mathcal{A}_0$ must be given by

$$\frac{\mathcal{A}_1}{\mathcal{A}_0} \sim \frac{G_N E^2}{\hbar c^5} \,, \tag{1.11}$$

i.e., it is proportional to E^2, a behavior that also follows directly from the fact that the gravitational interaction is mediated by a massless particle of spin $J = 2$ ($s^2/t \sim E^2$). From this formula it is clear that quantum corrections are divergent for large energies. A similar analysis of quantum corrections shows that the divergence becomes worse and worse as we go to higher loop order. This failure of perturbative quantum gravity has led to the conclusion that EINSTEIN's theory is unlikely to be a fundamental theory of gravity. Similarly to FERMI's theory of weak interactions, it should rather be considered as an effective low-energy theory that must be deformed at high energies in order to be consistent with the principles of quantum physics.

As we mentioned before, superstring theory re-emerged in the 1980s after it had been realized that it provided a natural and consistent host for gravitons [29]. In order to be a bit more specific, we shall consider closed strings propagating in some background geometry X. It is widely known that superstrings require X to be 10-dimensional, so that contact with 4-dimensional physics is often made by rolling extra directions up on small circle or through more general compactifications. The study of such compactifications is an extremely active field of research. We will get back to the issue in the second part of this book.

Strings possess infinitely many vibrational modes that we can think of as an infinite tower of massless and massive particles propagating on X. The mass spectrum of the theory is linear, with the separation that is parametrized by the tension $T_s \sim 1/\alpha'$ or, equivalently, by the length $l_s = \sqrt{\alpha'}$ of the string. As strings propagate through X, they can interact by joining and splitting. One simple such process for a one-loop contribution to the $2 \to 2$ scattering of closed strings is depicted in Figure 1.3. On the left-hand side we have drawn a one-loop diagram in some arbitrary field theory with 3- and 4-point vertices. The latter come weighted with two independent coupling constants. Once we pass to string theory, only one fundamental coupling remains. In fact, as illustrated in the right-hand side of Figure 1.3, any string theory diagram, no matter how many external legs and loops it has, may be cut into 3-vertices. Consequently, all interactions between strings are controlled by a single coupling constant g_s that comes with the 3-vertex depicted in Figure 1.2.

String theory possesses a consistent set of rules and elaborate computational tools to calculate scattering amplitudes. These produce formulas of the form (1.7).

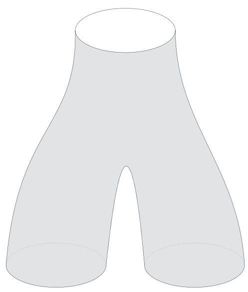


Figure 1.2 The only fundamental vertex of string theory describes the splitting and joining of strings.

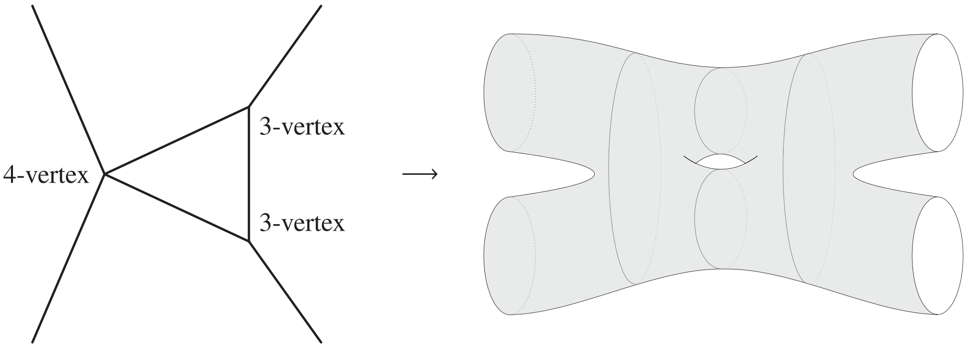


Figure 1.3 While point particles can posses many different types of vertices, all scattering amplitudes in string theory are composed of one fundamental 3-vertex.

It is of particular interest to study their low-energy properties. When $E \ll l_s^{-1}$, vibrational modes cannot be excited, and all we see are massless point-like objects. One may ask whether these behave like any of the particles we know. The answer is well known: at low energies, massless closed string modes scatter like gravitons [58, 77, 78] and a bunch of other particles that form the particle content of 10-dimensional supergravity theories. This observation is fundamental for string theory's advance into quantum gravity. In fact, it presents string theory as a consistent high-energy completion of gravitational theories.

1.3 Solitons and D-branes

For a moment, let us turn our attention to (super-)gravity theories. We are all familiar with the SCHWARZSCHILD solution of EINSTEIN's theory of gravity. It describes a black hole in our 4-dimensional world, i.e., a heavy object that is localized somewhere in space. Similar solutions certainly exist for the supergravity equations of motion. The massive (and charged) objects they describe may but need not be point-like localized in the 9-dimensional space. In fact, explicit solutions are known in which the mass density is localized along p-dimensional surfaces with $p = 1$ corresponding to strings, $p = 2$ to membranes, etc.; see [21] for a review. Such solutions were named black p-branes. Like ordinary black holes, however, most of these objects decay. But there exist certain extremal solutions, also known as solitonic p-branes, that are stable.

Now let us recall from the previous section that supergravity emerges as a low-energy description of closed string theory. Consequently, if supergravity contains massive $(p + 1)$-dimensional objects, the same should be true for closed string theory. One may therefore begin to wonder about the role p-branes could play in string theory. In order to gain some insight, let us suppose that a brane has been placed into the 9-dimensional space of our string background.

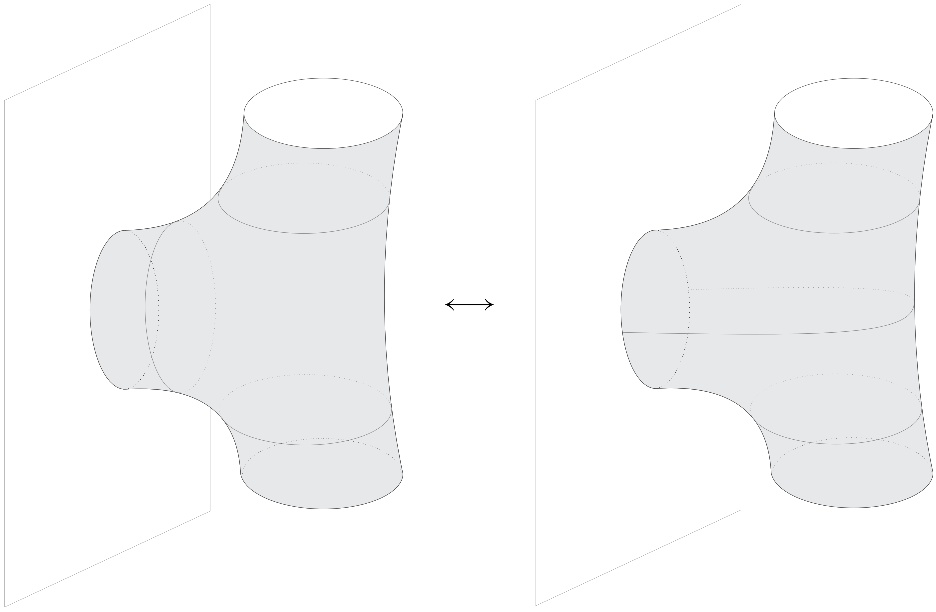


Figure 1.4 There are two ways to think about the interaction between massive closed string modes and D-branes, either in terms of closed string exchange or in terms of open string excitations.

Since it is heavy and charged, it will interact with the closed string modes in this background. In supergravity, we would describe this interaction through the exchange of gravitons or other particles mediating the relevant interaction. In string theory, a similar picture is possible only that now the interaction is mediated by exchange of closed strings as shown on the left-hand side of Figure 1.4.

But the figure suggests another way to think about the very same process. In order to allow for an unbiased view, we have re-drawn the interaction process on the right-hand side of Figure 1.4. What we see now is an infalling closed string that seems to open up when it hits the brane. For a brief period, an excited state is formed in which an open string propagates with both its ends remaining attached to the brane. Finally, this state decays again by emitting a closed string. Hence, we found two very different ways to think about exactly the same process. One of them involves an excited state of the p-brane in which an open string travels along the $p+1$-dimensional world-volume. In order for such a state to exist, branes in string theory must be objects on which open strings can end. This is indeed the defining feature of so-called DIRICHLET p-branes or for short Dp-branes in string theory (see, e.g., [54]).

1.4 D-branes and Gauge Theory

In the previous section we argued that D-brane excitations can be thought of as open strings whose endpoints move within the p-dimensional space of a brane. Therefore, branes provide us with a second set of light objects, namely the vibrational modes of open strings. One can ask again whether the massless open string modes behave like any of the known particles. The answer has been known for a long time: When $E \ll l_s^{-1}$, massless open string modes scatter like gauge bosons or certain types of matter [49].

In order to obtain non-Abelian gauge theories it is necessary to consider clusters of branes. It is a remarkable fact of supergravity that special clusters can give rise to stable configurations. This is true in particular for a stack of N parallel branes. Let us enumerate the member branes of such a cluster or stack by indices $a, b = 1, \ldots, N$. Open strings must have their end-points moving along one of these N branes (see Figure 1.5). Since an open string has two ends, modes of an open string carry a pair a, b of "color" indices. Hence, massless open string modes on a stack of N parallel branes can be arranged in a $N \times N$ matrix, just as the components of a $U(N)$ Yang-Mills field. In addition to non-Abelian gauge bosons, various matter multiplets can emerge from open strings. The precise matter content of the resulting low-energy theories depends much on the brane configuration under consideration, and we shall not make the attempt to describe it in any more detail.

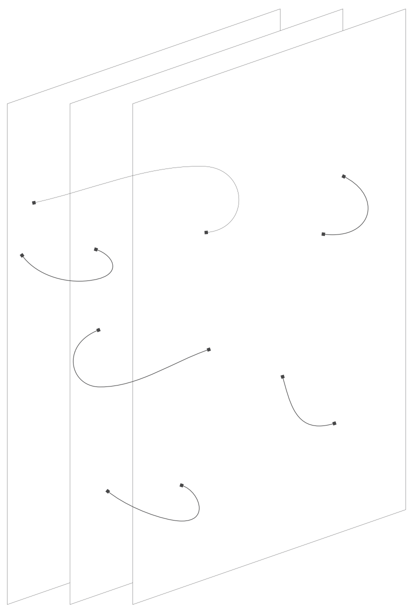


Figure 1.5 A stack of N branes with open strings stretched between them.

It is worth rehashing how $(p + 1)$-dimensional gauge theories have entered the stage through the back door. When we began this short cartoon of string theory, closed strings (and therefore gravitons) were all we had. Then we convinced ourselves that the theory contains additional heavy $(p+1)$-dimensional D-branes. Their excitations brought open strings into the picture and thereby another set of light degrees of freedom, including non-Abelian gauge bosons. Let us stress once more that the latter do not propagate in the 10-dimensional space-time but rather on the $(p+1)$-dimensional brane worlds. The dimension $p+1$ can take various values, one of them being $p + 1 = 4$. Our sketch of modern string theory has now brought us to the mid-1990s. At this point we have gathered all the ingredients that are necessary to discover a novel set of equivalences between gauge and string theory.

1.5 Gauge/String Dualities

The main origin of the novel dualities is not too difficult to grasp if we cleverly combine what we have seen in the previous section. To this end, let us suppose that we have placed two branes in our 10-dimensional background and that they are separated by some distance Δy. Since all branes are massive and charged objects, they will interact with each other. In supergravity, we would understand this interaction as an exchange of particles, such as gravitons, etc. Our branes, however, are objects in string theory, and hence there exists an infinite tower of vibrational closed string modes that mediate the interaction between them. A tree-level exchange is shown on the left-hand side of Figure 1.6. But as in our previous discussion, there exists another way to think about exactly the same process in terms of open strings. This is visualized on the right-hand side of Figure 1.6. There, the interaction appears to originate from pair creation/annihilation of open string modes with one end on each

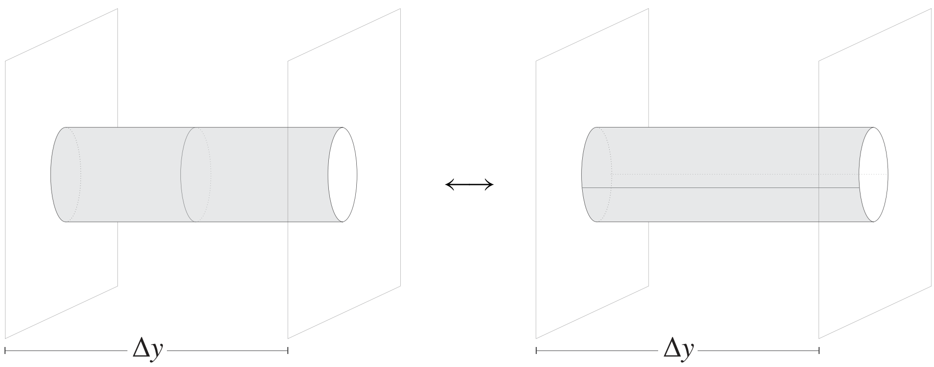


Figure 1.6 There are two ways to think about the interaction between two branes. One involves open strings, the other is mediated by closed string modes.

of the branes. In string theory, these two descriptions of the interaction give exactly the same final result for the force between the two branes.

A closer look reveals that the equivalence of our two computational schemes, one in terms of closed strings and the other in terms of open strings, is surprisingly non-trivial. Suppose, for example, that the distance Δy between the branes is very large. Then the closed string modes have to propagate very far in order to get from one brane to the other. Consequently, contributions from massive string modes may be neglected, and it is sufficient to focus on massless closed string modes, i.e., on the particles found in 10-dimensional supergravity. In the other regime in which the separation between the two branes becomes of the order of the string length l_s, such an approximation cannot give the right answer. Instead, the full tower of closed string modes must be taken into account. In other words, when $\Delta y \sim l_s$ the supergravity approximation breaks down, and we have to carry out a full string theory computation. From the point of view of open strings, the situation is reversed. When the branes are far apart, pair-created open strings propagate only briefly before they annihilate again, and hence the entire infinite tower of open string modes contributes to this computation. In the opposite regime where $\Delta y \sim l_s$, however, the interaction may be approximated by restricting ourselves to massless open string modes; i.e., all we need to perform is some gauge theory computation.

As simple as these comments on Figure 1.6 may seem, they suggest a remarkable conclusion: in the regime $\Delta y \sim l_s$, some calculation performed in the gauge theory on the world volume of our branes should lead to the same result as a full-fledged string theory calculation for closed strings propagating in the 10-dimensional background. A few aspects of this relation deserve to be stressed. In fact, we observe that it

- does preserve neither character nor difficulty of the computation,
- relates diagrams involving a different number of loops, and
- relates two theories in different dimensions, i.e., it is holographic.

These three features emerge clearly from our analysis. The first point is obvious. In fact, the two computations are so different that they would usually not be performed by members of the same scientific community. Furthermore, in our example, we related a gauge theory one-loop amplitude to a tree-level diagram of closed string theory, i.e., we showed that classical string theory encodes information on quantum gauge theory and vice versa. Finally, the gauge theory degrees of freedom are bound to the $(p+1)$-dimensional world-volume of our branes, whereas closed strings can propagate freely in 9+1 dimensions. We shall see these three features re-emerge in the concrete incarnations of the gauge/string theory dualities that we will discuss in Part II of this book.