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Excerpt
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Part I

Foundations

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Introduction

1.1 Space and Laboratory Plasma Physics

Physics has experienced several revolutions in the twentieth century that profoundly changed our understanding of nature. Quantum mechanics and (special, general) relativity are the best known and certainly the most important, but the discovery of the fourth state of matter – the state of plasma – as the most natural form of ordinary matter in the Universe, with more than 99% of visible matter being in this form, is unquestionably a revolution in physics. This discovery has led to the emergence of a new branch of physics called *plasma physics*.

Plasma physics describes the coupling between electromagnetic fields and ionized matter (electrons, ions). Thus, it is based upon one of the four foundations of physics: the electromagnetic interaction whose synthetic mathematical formulation was made by the Scottish physicist J. C. Maxwell who published in 1873 two heavy volumes entitled *A Treatise on Electricity and Magnetism*. The discovery of the electron by J.J. Thomson in 1897 and the formulation of the theory of the atom at the beginning of the twentieth century have contributed to the first development of plasma physics. It was in 1928 that the name *plasma* was proposed for the first time by I. Langmuir, referring to blood plasma in which one finds a variety of corpuscles in movement. Experimental studies of plasmas first focused essentially on the phenomenon of electrical discharge in gas at low pressure with, for example, the formation of an electric arc. These studies initiated during the second half of the twentieth century were extended to problems related to the reflection and transmission of radio waves in the Earth's upper atmosphere (this was how the first transatlantic link was established by Marconi in 1901), which led to the discovery of the ionosphere, an atmospheric layer beyond 60 km altitude with a thickness of several hundred kilometers. As explained by the astronomer S. Chapman (1931), the ionosphere consists of gas partially ionized by solar ultraviolet radiation; therefore, it is the presence of ionospheric plasma which explains why low-frequency waves can be reflected or absorbed depending on the frequency used.

With the beginning of the space age in the 1950 and 1960s, our understanding of the Earth's environment and also of the Universe significantly increased: this auspicious period saw the creation of space agencies such as the National Aeronautics and Space Administration (NASA) in 1958, the Centre National d'Études Spatiales (CNES) in 1961, and then the European Space Agency (ESA) in 1975. For example, the internal structure of the magnetosphere with the Van Allen radiation belts was discovered in 1958 by the NASA Explorer 1 probe, whereas the solar wind predicted theoretically by the American astrophysicist E. Parker[1] in 1958 (Parker, 1958) was explored for the first time in 1960 by the Russian mission Luna 2. Since these first steps in space, many other space missions have been launched to study astrophysical plasmas, such as those devoted to the Sun with the Solar & Heliospheric Observatory (SoHO) jointly launched by the ESA and NASA in 1995 and located in the vicinity of the Earth–Sun (Lagrangian) L1 point (the position in the space where the gravitational fields of the Sun and the Earth exactly balance the centrifugal force due to the rotational movement); it is to this day one of the most important solar missions to have been carried out in light of the harvest of results and its longevity. Its successor – the Solar Dynamics Observatory (SDO) – launched in 2010 currently gives the best available images of the solar corona as shown in Figure 1.1. The next space mission – Solar Orbiter – which is mainly an ESA mission, is scheduled for launch in 2018. The probe will follow an inclined orbit to study the polar regions of the Sun and will pass to its perigee at a distance of only about 45 solar radii in order to analyze, in particular, the early development of solar wind turbulence. The systematic exploration of space plasmas has allowed the investigation of many previously unknown physical phenomena, so we now distinguish this area of natural plasmas from laboratory plasmas. A distinction is also made between space plasmas accessible by *in situ* measurements and astrophysical plasmas, which are by definition more distant. In this case, the model universally chosen by astrophysicists is that of magnetohydrodynamics proposed in 1942 by the Swedish astrophysicist H. Alfvén (Alfvén, 1942). This model combines the equations of classical electrodynamics (Maxwell's equations) with fluid mechanics and describes the plasma behavior at the largest scales as a conducting mono-fluid.

Following the detonation of the first hydrogen bomb in 1952, a new area of plasma physics emerged: the so-called thermonuclear plasma physics in the context of production of energy by controlled thermonuclear fusion. When the research studies on this subject were declassified (1958), many theoretical advances had been made: the systematic mathematical treatment of plasmas

[1] The article submitted by E. Parker to *The Astrophysical Journal* was initially rejected by the two referees, who considered this solar wind model scientifically irrelevant. It was the editor – S. Chandrasekhar – who decided to ignore these opinions and published the paper.

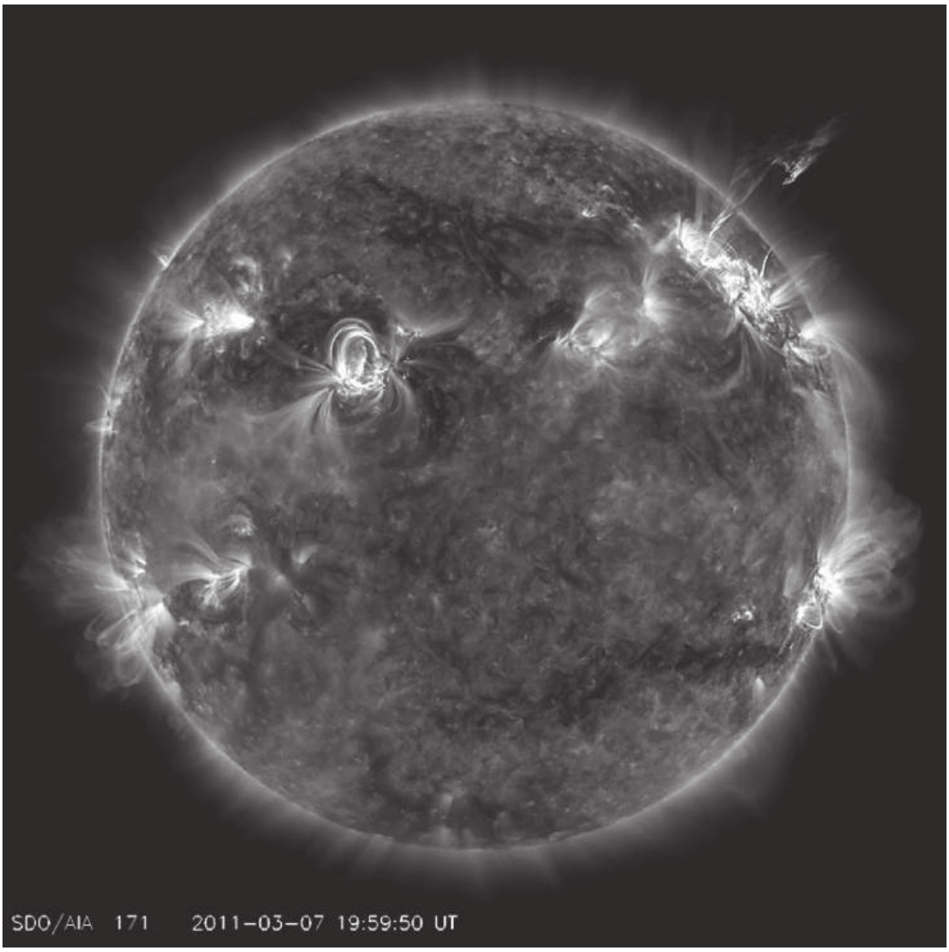

Figure 1.1 Solar corona observed at the wavelength 17.1 nm (Fe IX line). The solar plasma is at a temperature of about a million degrees. Image obtained by the AIA (SDO/NASA) imager (March 7, 2011); the spatial resolution is approximately 700 km; courtesy of NASA/SDO and the AIA science team.

completes the initial works of A. A. Vlasov (1938) on the kinetic equations, L. D. Landau (1946) on the damping of longitudinal space-charge waves, and H. Alfvén on magnetohydrodynamics. A central issue in this domain concerns the confinement of a hot plasma by a strong magnetic field; also many works have been devoted to the study of magnetohydrodynamic instabilities. Experimentally, the best-known magnetic confinement research technology is that of tokamaks, which were invented in the early 1950s by the Russians I. Tamm and A. Sakharov. A tokamak is a torus containing hydrogen (deuterium and tritium) that has been fully ionized and magnetized. Thus far, about 100 tokamaks have been built. The diversity of these experiences has led to a better physical understanding of

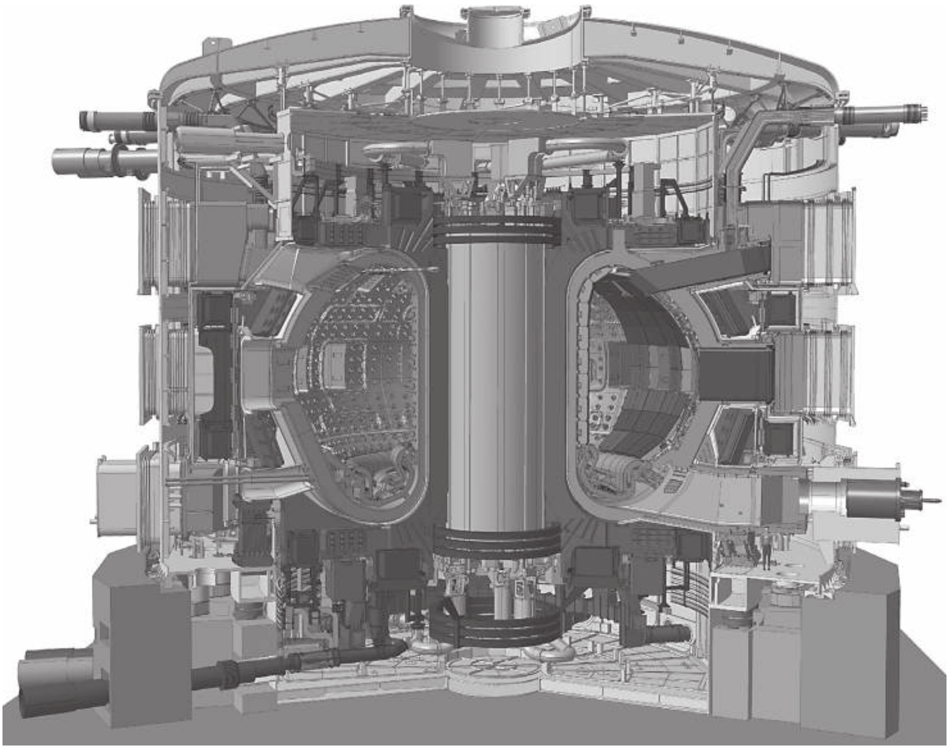

Figure 1.2 View of ITER in Cadarache. The comparison with a human (bottom right) gives an idea of the size of the tokamak. Credit: ITER Organization.

thermonuclear plasmas. It is on this basis that the International Thermonuclear Experimental Reactor (ITER) in Cadarache has been proposed (see Figure 1.2). ITER is a project to check the scientific and technical feasibility of nuclear fusion as a new source of energy for humanity; it should enter operation around the years 2025–2030. The societal issue is huge, as is its cost estimated at 20 billion euros, making it the second most expensive scientific project (after the International Space Station) ever built.

On a more modest (spatial and financial) scale, plasma technology has become invasive in our life insofar as the industrial applications are more and more numerous. Without going into detail we can cite, for example, the manufacture of small electronic components (microprocessors) in plasma reactors, or even the development of ion-propulsion engines (thrusters), which are particularly interesting for the propulsion of space probes because of the low fuel consumption (see Figure 1.3): the lunar probe SMART-1 launched in 2003 was the first ESA space mission based on the use of plasma propulsion. Eventually, the power of plasma thrusters should allow the transfer into orbit and control of the trajectory of future telecommunications satellites and should also allow control of the trajectory of planetary exploration missions.

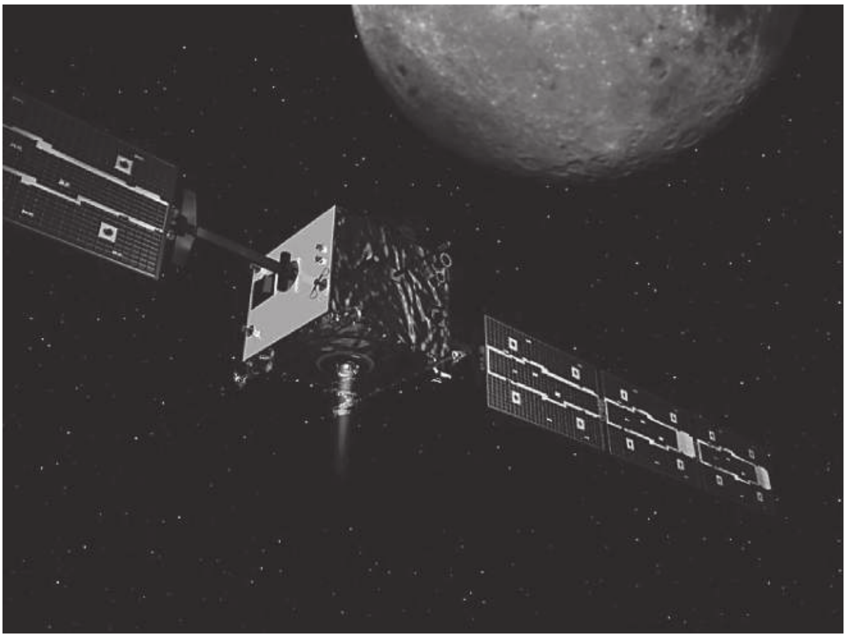

Figure 1.3 SMART-1 (Small Missions for Advanced Research in Technology) was an ESA mission (launched in 2003 and deliberately crashed into the Moon's surface in 2006) which orbited around the Moon. It was propelled by a Hall-effect thruster (visible on this image at the bottom of the satellite). Image: ESA/SMART-1.

1.2 What Is a Plasma?

In its natural state a gas is an electrical insulator. This is because it contains no free charged particles but only neutral atoms or molecules. However, if one applies, for example, a sufficiently strong electric field, it becomes conductive: the complex phenomena that occur are called gas discharges and are due to the appearance of electrons and free ions. In such a situation, the ionized gas is characterized in general by neutrality on the macroscopic scale for which $n_e = n_i$, where n_e and n_i are the electron and ion charge densities, respectively. This neutrality is a consequence of the very intense electrostatic forces that appear whenever $n_e \neq n_i$. To measure the state of ionization of a gas, one defines the degree of ionization α as the ratio

$$\alpha \equiv \frac{n}{n_0 + n}, \tag{1.1}$$

where $n = n_{\rm e} = n_{\rm i}$ and n_0 is the density of neutral (i.e. non-ionized) species. This parameter allows us to distinguish weakly ionized gases, for which, typically, $10^{-10} < \alpha < 10^{-4}$, from strongly ionized gases, where $10^{-4} < \alpha < 1$. In the first category, we essentially find industrial plasmas, whereas in the second we find astrophysical and thermonuclear plasmas. It can be surprising to speak about highly ionized gas when the degree of ionization is lower than 1%. The reason is that the potential of the Coulomb interaction between two charges has a very long range (it decays as $1/r$) so that the corresponding effective section of interaction is several orders of magnitude larger than the effective section of electron–neutral-species interaction. On the other hand, weakly ionized gases are characterized by a frequency of collisions between electrons and neutral species that is higher than those of collisions between electrons and of collisions between electrons and ions.

Having defined what a plasma is, one may wonder why an initial state consisting of positive and negative charges undergoing Coulomb interactions does not naturally lead to a simple recombination of electrons with ions and thus to the disappearance of the plasma state. In fact, the maintenance of the plasma state is due to the presence, or even the combination, of several effects such as the microscopic disorder, i.e. the thermal agitation, the low reactivity of species, or the occurrence of sources of ionization such as the radiation which is particularly important in astrophysics. In general, these effects largely counterbalance the recombination induced by the Coulomb interactions. At thermodynamic equilibrium, ionization processes, are counterbalanced by recombination processes, which leads to a thermal ionization equilibrium. For example, for a gas at sufficiently high temperatures ($T \simeq 10^4$ K, i.e. an energy $(3/2)k_B T \simeq 1$ eV), there may be ionization in a collision; at even higher temperatures atoms can be ionized several times. In many cases, the gas is not in thermodynamic equilibrium and the temperatures of electrons and ions are not the same.

The name plasma is used to describe all (partially or totally) ionized gases. In summary, we have the three following families.

- Weakly ionized gases: these are plasmas in which some ions and electrons move in a sea of neutral atoms and/or molecules. In this case, it is the binary collisions between an electron (or ion) and a neutral species that determine the dynamics of charged particles. From a theoretical point of view, these plasmas are described by the Boltzmann kinetic equation.
- Strongly ionized gases with interactions between particles: an electron can be considered to be in interaction with a large number of other charged particles (due to the long range of the Coulomb force). It is the distant cumulative collisions at low deflection that determine, among other things, the plasma dynamics. These plasmas are described by a kinetic equation of the Fokker–Planck type.
- Strongly ionized gases without interaction between particles: these are diluted plasmas in which charged particles do not suffer collision and evolve only under the effect of collective electromagnetic fields due to space-charges created by all the other charges. These plasmas are described by the Vlasov kinetic equation, which is often considered the fundamental equation of plasma physics.

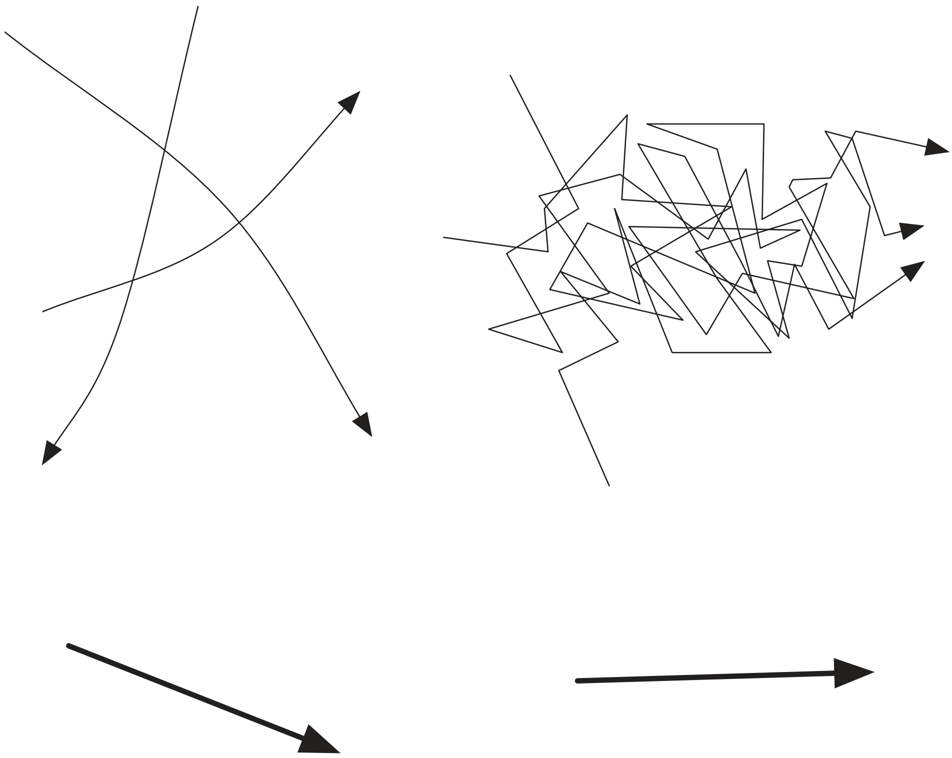

Figure 1.4 Velocities in a collisionless plasma (left) and in a collisional plasma (right), at the kinetic scale (top) and fluid scale (bottom).

Regardless of the type of plasma – collisional or collisionless – we can adopt a macroscopic (fluid) description. It is often believed that collisions are essential in order to speak about fluid models. It is even thought that a Maxwellian distribution (see Section 1.3), which is the consequence of collisions, is necessary to understand the behavior of the fluids and that it is a hidden hypothesis behind any fluid modeling. That is not true: the moment equations from which all the fluid equations derive, except the closure equation, are fully general and independent of the existence of collisions (Belmont *et al.*, 2013). However, the physical interpretation differs depending on the presence or not of collisions. In a collisional plasma, particles collide frequently and the average velocity of each particle in a given volume is equal to the local average (in this given volume) of the particle velocities. On the other hand, in a collisionless plasma each particle has an almost straight trajectory whose curvature is due to the collective field, and the average velocity can be very different from the individual velocity (see Figure 1.4).

1.3 Kinetic Description

The objective of this section is to give a rapid overview of the microscopic description of a plasma. This description is essentially based on statistical physics.

Let us define the distribution function of a particle species $f(\mathbf{r}, \mathbf{v}, t)$ such that $f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r}\, d\mathbf{v}$ is the likely number of particles in the elementary six-dimensional volume $d\mathbf{r}\, d\mathbf{v}$ centered at the point $(\mathbf{r}, \mathbf{v})$. The observable macroscopic quantities obtained from f by taking different moments are the

- particle density (moment of order 0),

$$n(\mathbf{r}, t) = \int_{\mathbf{R}^3} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}\,; \tag{1.2}$$

- particle velocity (moment of order 1),

$$\mathbf{u}(\mathbf{r}, t) = \frac{\int_{\mathbf{R}^3} \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}}{\int_{\mathbf{R}^3} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}} = \frac{1}{n} \int_{\mathbf{R}^3} \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{v}\,; \tag{1.3}$$

- pressure, temperature (moment of order 2); and
- heating flux (moment of order 3).

The equation for determining $f(\mathbf{r}, \mathbf{v}, t)$ is called the kinetic equation. In general, to derive it one has to make a distinction between macroscopic forces such as the applied fields and the microscopic forces appearing during collisions.

1.3.1 Collisionless Plasma

Let us consider a volume in the space $(d\mathbf{r},\, d\mathbf{v})$: the number of particles in this volume changes over time and the rate of change is given by the flux of particles across the surface of this given volume. In $\mathbf{r}$-space, this flow is

$$\int_{\mathbf{R}^5} f(\mathbf{r}, \mathbf{v}, t) \mathbf{v} \cdot \mathbf{n}_\mathbf{r}\, dS_r\, d\mathbf{v}\,, \tag{1.4}$$

where dS_r is the surface which delimits the volume element in the $\mathbf{r}$-space, and $\mathbf{n}_\mathbf{r}$ is the unit vector ($|\mathbf{n}_\mathbf{r}| = 1$) orthogonal to dS_r. Similarly, in the $\mathbf{v}$-space the particle flux is

$$\int_{\mathbf{R}^5} f(\mathbf{r}, \mathbf{v}, t) \mathbf{a} \cdot \mathbf{n}_\mathbf{v}\, dS_v\, d\mathbf{r}\,, \tag{1.5}$$

where $\mathbf{a}$ is the acceleration experienced by the particle at the level of the surface element dS_v which is oriented according to the unit vector $\mathbf{n}_\mathbf{v}$. We obtain

$$\frac{\partial}{\partial t} \int_{\mathbf{R}^6} f(\mathbf{r}, \mathbf{v}, t) d\mathbf{r}\, d\mathbf{v} = - \int_{\mathbf{R}^5} f(\mathbf{r}, \mathbf{v}, t) \mathbf{v} \cdot \mathbf{n}_\mathbf{r}\, dS_r\, d\mathbf{v} - \int_{\mathbf{R}^5} f(\mathbf{r}, \mathbf{v}, t) \mathbf{a} \cdot \mathbf{n}_\mathbf{v}\, dS_v\, d\mathbf{r}\,, \tag{1.6}$$

which gives, using the (generalized) Ostrogradsky relation,

$$\int_{\mathbf{R}^6} \left[\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \cdot (\mathbf{v} f) + \frac{\partial}{\partial \mathbf{v}} \cdot (\mathbf{a} f) \right] d\mathbf{r}\, d\mathbf{v} = 0\,. \tag{1.7}$$

By definition, we write

$$\frac{\partial}{\partial \boldsymbol{\xi}} \cdot \mathbf{A} = \frac{\partial A_x}{\partial \xi_x} + \frac{\partial A_y}{\partial \xi_y} + \frac{\partial A_z}{\partial \xi_z}, \tag{1.8}$$

with the elementary volume

$$d\boldsymbol{\xi} = d\xi_x \, d\xi_y \, d\xi_z \,. \tag{1.9}$$

As the volume considered is arbitrary, and $\mathbf{r}$, $\mathbf{v}$, and t are independent variables, we obtain

$$\boxed{\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0,} \tag{1.10}$$

where $\mathbf{a} = \mathbf{F}/m$. In this expression, the only force that depends on the velocity is assumed to be the Lorentz force, $q\mathbf{v} \times \mathbf{B}$; thus we can extract $\mathbf{a}$ from the derivative term (because of the cross product). Equation (1.10) is the **Vlasov equation**. We can also interpret this equation in terms of a Lagrangian and introduce the particle density conservation in a volume of the phase space which is deformed during the dynamical evolution of the plasma.

The Vlasov equation is often used to study the development of instabilities. The most remarkable study in this area is that of L. D. Landau (1946), who, from a linear analysis of the Vlasov equation and using the concept of a contour integral around singularities in complex space, was able to calculate the damping of an electrostatic Langmuir wave (see Figure 1.5). More recently, the mathematical analysis of this problem (Mouhot and Villani, 2010) and the rigorous demonstration of the Landau damping[2] in the limit of weak nonlinearities earned the French mathematician C. Villani the 2010 Fields Medal.

1.3.2 Plasma with Collisions

The effects of microscopic inter-particle forces with rapid fluctuations acting on the particles cannot be included in the force term of the Vlasov equation and must be treated separately. This leads to a new equation whose expression is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}}, \tag{1.11}$$

where $(\partial f/\partial t)_{\mathrm{c}}$ involves the collisions between all species of particle. Its form depends on the ionized gas considered. We see in this simple example that collisions lead simply to a modification of the Vlasov equation, which therefore retains a central position. Finally, notice that one of the roles of collisions in plasmas is to

[2] Note that, unlike in viscous fluid models, where the damping is an irreversible process, the Landau damping is reversible.