An Introduction to Vectors, Vector Operators and Vector Analysis

Conceived as s a supplementary text and reference book for undergraduate and graduate students of science and engineering, this book intends communicating the fundamental concepts of vectors and their applications. It is divided into three units. The first unit deals with basic formulation: both conceptual and theoretical. It discusses applications of algebraic operations, Levi-Civita notation and curvilinear coordinate systems like spherical polar and parabolic systems. Structures and analytical geometry of curves and surfaces is covered in detail.

The second unit discusses algebra of operators and their types. It explains the equivalence between the algebra of vector operators and the algebra of matrices. Formulation of eigenvectors and eigenvalues of a linear vector operator are discussed using vector algebra. Topics including Mohr's algorithm, Hamilton's theorem and Euler's theorem are discussed in detail. The unit ends with a discussion on transformation groups, rotation group, group of isometries and the Euclidean group, with applications to rigid displacements.

The third unit deals with vector analysis. It discusses important topics including vector valued functions of a scalar variable, functions of vector argument (both scalar valued and vector valued): thus covering both the scalar and vector fields and vector integration

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To Ela and Ninad
who made me write this document

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Preface

This is a textbook on vectors at the undergraduate/advanced undergraduate level. Its target readership is the undergraduate student of science and engineering. It may also be used by professional scientists and engineers to brush up on various aspects of vectors and applications of their interest. Vectors, vector operators and vector analysis form the essential background to and the skeleton of many courses in science and engineering. Therefore, the utility of a book which clearly builds up the theoretical structure and applications of vectors cannot be over-emphasized. The present book is an attempt to fulfill such a requirement. This book, for instance, can be used to give a course forming a common pre-requisite for a number of science and engineering courses. In this book, I have tried to develop the theory and applications of vectors from scratch. Although the subject is presented in a general setting, it is developed in 3-D space using basic vector algebra. A coordinate-free approach is taken throughout, so that all developments are free of any particular coordinate system and apply to all coordinate systems. This approach directly deals with vectors instead of their components or coordinates and combines these vectors using vector algebra.

A large part of this book is inspired by the geometric algebra of multivectors that originated in the 19th century, in the works of Grassmann and Clifford and which has had a powerful re-incarnation with enhanced applicability in the recent works of D. Hestenes and others [7, 10, 11]. This is one of the most general algebraic formulations of geometry of which vectors form a special case. Keeping the multivector geometric algebra at the backdrop makes the coordinate free approach for vectors emerge naturally. On a personal note, the book on classical mechanics by D. Hestenes [10], which introduced me to the multivector geometric algebra, has always been a source of joy and education for me. I have always enjoyed solving problems from this book, many of them are included here. In fact I have used Hestenes' work in various places throughout the book, without using or referring to the geometric algebra or geometric calculus.

While designing this book I was guided by two principles: A consistent development of the subject from scratch, and also showing the beauty of the whole edifice and extending the utility of the book to the largest possible cross-section of students. The book comprises three parts, one for each part of the title: First on the basic formulation, the second on vector operators and the third on vector analysis. Following is the brief description of each one of them.

The first part gives the basic formulation, both conceptual and theoretical. The first chapter builds basic concepts and tools. The first three sections are the result of my experience with students and I have found that these matters should be explicitly dealt with for the correct understanding of the subject. I hope that the first three sections will clear up the confusion and the misconceptions regarding many basic issues, in the minds of students. I have also given the applications and examples of every algebraic operation, starting from vector addition. Levi-Civita notation is introduced in detail and used to get the vector identities. The metric space structure is introduced and used to understand vectors in the context of the physical quantities they represent. Apart from the essential structures like basis, dimension, coordinate systems and the consequences of linearity, the curvilinear coordinate systems like spherical polar and parabolic systems are developed systematically. Vector fields are defined and their basic structure is given. The orientation of a linearly independent triplet of vectors is then discussed, also including the orientation of a triplet relative to a coordinate system and the related concept of the orientation of a plane, which is later used to understand the orientation of a surface. The second chapter deals with the analytical geometry of curves and surfaces emphasizing vector methods. The third chapter uses complex algebra for manipulating planar vectors and for the description and transformations of the plane curves. In this chapter I follow the treatment by Zwikker [26] which is a complete and rigorous exposition of these issues.

The second part deals with operators on vectors. Everything about vector operators is formulated using vector algebra (scalar and vector products) and matrices. The fourth chapter gives the algebra of operators and various types of operators, and proves and emphasizes the equivalence between the algebra of vector operators and the algebra of matrices representing them. The fifth chapter gives general formulation of getting eigenvectors and eigenvalues of a linear operator on vectors using vector algebra. The properties of the spectrum of a symmetric operator are also obtained using vector algebra. Thus, extremely useful and general methods are accessible to the students using elementary vector algebra. A powerful algorithm to diagonalize a positive operator acting on a 2-D space, called Mohr's algorithm, is then described. Mohr's algorithm has been routinely used by engineers via its graphical implementation, as explained in the text. The sixth chapter develops in detail orthogonal transformations as rotations or reflections. The generic forms for operators of reflection and rotation, as well as the matrices for the rotation operator are obtained. The relationship between rotation and reflection is established via Hamilton's theorem. The active and passive transformations and their connection with symmetry is discussed. The concept of broken symmetry is briefly discussed. The Euler angle construction for arbitrary rotation is then derived. The problem of finding the axis and the angle of rotation corresponding to a given orthogonal matrix is solved as the Euler's theorem. The second part ends with the seventh chapter on transformation groups and deals with the rotation group, group of isometries and the Euclidean group, with applications to rigid displacements.

The third part deals with vector analysis. This is a vast subject and a personal flavor in the choice of topics is inevitable. For me the guiding question was, what vector analysis a graduating student in science and engineering must have ? Again, the variety of answers to this question is limited only by the number of people addressing it. Thus, the third part gives my version of the answer to this question and the resulting vector analysis. I primarily develop the subject with geometric point of view, making as much contact with applications as possible. My aim is to enable the student to independently read, understand and use the literature based on vector analysis for the applications of his interest. Whether this aim is met can only be decided by the students who learn and try to use this material. This part is divided into five (Chapters 8–12). The eighth chapter outlines fundamental notions and preliminary start ups, and also sets the objectives. The ninth chapter consists of the vector valued functions of a scalar variable. Theories of space curves and of plane curves are developed from scratch with some physical applications. This chapter ends with the integration of such functions with respect to their scalar argument and their Taylor series expansion. The tenth chapter deals with the functions of vector argument, both scalar valued and vector valued, thus covering both the scalar and vector fields. Again, everything is developed from scratch, starting with the directional derivative, partial derivatives and continuity of such functions. A part of this development is inspired by the geometric calculus developed by D. Hestenes and others [7, 10, 11]. To summarize, this chapter consists of different forms of derivatives of these and inverse functions, and their geometric/physical applications. A major omission in this chapter is that of the systematic development of differential forms, which may not be required in an undergraduate course. The eleventh chapter concerns vector integration. This is done in three phases: the line, the surface and the volume integral. All the standard topics are covered, emphasizing geometric aspects and physical applications. While writing this part, I have made use of many books, especially the book by Courant and John [5] and that by Lang [15], for the simple reason that I have learnt my calculus from these books, and I have no regrets about that. In particular, my treatment of multiple integrals and matrices and determinants in Appendix A is inspired by Courant and John's book. I find in their book, the unique property of building rigorous mathematics, starting from an intuitive geometric picture. Also, I follow Griffiths while presenting the divergence and the curl of vector fields, which, I think, is possibly one of the most compact and clear treatments of this topic. The subsections 11.1.1 and 11.8.1 and a part of section 9.2 are based on ref [22]. The twelfth and last chapter of the book presents an assorted collection of applications involving rotational motion of a rigid body, projectile motion, satellites and their orbits etc, illustrating coordinate-free analysis using vector techniques. This chapter, again, is influenced by Hestenes [10].

Appendix A develops the theory of matrices and determinants emphasizing their connection with vectors, also proving all results involving matrices and determinants used in the text. Appendix B gives a brief introduction to Dirac delta function.

The whole book is interspersed with exercises, which form an integral part of the text. Most of these exercises are illustrative or they explore some real life application of the theory. Some of them point out the subtlties involved. I recommend all students to attempt all exercises, without looking at the solutions beforehand. When you read a solution after an attempt to get there, you understand it better. Also, do not be miserly about drawing figures, a figure can show you a way which thousand words may not.

I cannot end this preface without expressing my affection towards my friend and my deceased colleague Dr Narayan Rana, who re-kindled my interest in mechanics. Long evenings that I spent with him discussing mechanics and physics in general, sharing and laughing at various aspects of life from a distance, are the treasures of my life. We entered a rewarding and fruitful collaboration of writing a book on mechanics [19]. This collaboration and Hestenes' book [10] motivated me to formulate mechanics in a coordinate free way using vector methods. Apart from the book by Hestenes and his other related work, the book by V. I. Arnold on mechanics [3] has made an indelible impact on my understanding and my global view of mechanics, although its influence is not quite apparent in this book. I have always enjoyed discussing mechanics and physics in general with my colleagues Rajeev Pathak, Anil Gangal, C. V. Dharmadhikari, P. Durganandini, and Ahmad Sayeed. The present book is produced in LATEX and I thank our students, Dinesh Mali, Mukesh Khanore and Mihir Durve for their help in drawing figures and also as TEXperts.

Nomenclature

$\alpha, \beta, \gamma, \delta$ Scalars

$\angle(\mathbf{a}, \mathbf{b})$ Angle between vectors $\mathbf{a}, \mathbf{b}$

$\mathbf{a}, \mathbf{b}, \mathbf{x}, \mathbf{y}$ Vectors

θ, ϕ, ψ, χ Angles

$\mathcal{R}$ Region of 3-D space/plane

LHS Left hand side

RHS Right hand side

$\mathbb{R}^3$ Vector space comprising ordered triplets of real numbers

$\mathscr{E}_3$ 3-D vector space

$|\mathbf{a}|, a$ Magnitude of $\mathbf{a}$

$||\mathbf{a}||$ Norm of $\mathbf{a}$

A, B Matrices

$|A|, |B|$ Determinants

$\mathcal{R}(z)$, $\mathcal{I}(z)$ Real and imaginary parts of a complex number

CM Center of mass

$\boldsymbol{\mu}$ Magnetic moment

L Magnitude of angular momentum, A linear differential form

$\mathbf{h}$ Angular momentum

$\mathbf{H}$ Specific angular momentum : Angular momentum per unit mass

$\mathbf{M}$ Moment of a force, Torque

$\mathbf{B}$ Magnetic field

$\mathbf{E}$, $\mathcal{E}$ Electric field

κ Curvature

ρ Radius of curvature

p Semilatusrectum of a conic section

e Eccentricity of a conic section

$\mathbf{m}$ Moment of a line

$\mathscr{R}(\hat{\mathbf{n}}, \theta)$ Operator for rotation of vector $\mathbf{x}$ about $\hat{\mathbf{n}}$ by angle θ

$\mathscr{U}$ Canonical reflection operator, general orthogonal operator

S Similarity transformation on $\mathscr{E}_3$

$\mathscr{A}$ Affine transformation, skewsymmetric transformation

J Jacobian matrix

$|J|$, D Jacobian determinant

E, F, G Gaussian fundamental quantities of a surface

$\mathscr{I}$ Moment of Inertia operator/tensor

$\mathbf{g}(\mathbf{x}, t)$ Gravitational field of a continuous body

$\mathscr{Q}$ Gravitational quadrupole tensor

ω, $\mathbf{\Omega}$ Rotational velocity