

QUINE, NEW FOUNDATIONS, AND THE PHILOSOPHY OF SET THEORY

W. V. Quine's set theory, *New Foundations*, has often been treated as an anomaly in the history and philosophy of set theory. In this book, Sean Morris shows that it is, in fact, well motivated, emerging in a natural way from the early development of set theory. Morris introduces and explores the notion of set theory as explication: the view that there is no single correct axiomatization of set theory but rather various axiomatizations that all serve to explicate the notion of set and are judged largely according to pragmatic criteria. Morris also brings out the important interplay between *New Foundations*, Quine's philosophy of set theory, and his philosophy more generally. We see that Quine's early technical work in logic foreshadows his later famed naturalism, with his philosophy of set theory playing a crucial role in his primary philosophical project of clarifying our conceptual scheme and, specifically, its logical and mathematical components.

SEAN MORRIS is Associate Professor of Philosophy at the Metropolitan State University of Denver. He has written extensively on Quine and the history of analytic philosophy with a particular emphasis on Quine's work in the foundations of mathematics.

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Sean Morris
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This book is dedicated to Bill Hart and to Haewon and John.

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Preface

This book emerged out of my doctoral dissertation at the University of Illinois at Chicago, written some years ago under the direction of W. D. Hart. After completing the dissertation, I moved on to researching topics of a more general nature in Quine's philosophy, though always with an eye toward how his early work in logic and the foundations of mathematics shaped his general approach to philosophy. From time to time, I continued to present some of my views on Quine's philosophy of set theory and its contrasts with much contemporary philosophy of set theory. I remained surprised by – in discussions of the philosophy of set theory – how set theory was just about always and without question identified with Zermelo-Fraenkel set theory and its variants and the associated iterative conception of set. From this, it seemed to me that there might still be something to contribute to contemporary discussions of the philosophy of set theory by focusing on Quine's alternative approach – an approach that ignores the iterative conception, at least as a metaphysical view about sets, and that is a good deal more experimental and exploratory in nature, seeing set theory as a still largely unsettled area of the mathematical sciences.

In assembling the following list of acknowledgments, I could not help being reminded of the words of a recent Nobel laureate:

Some are mathematicians
Some are carpenters' wives ...
We always did feel the same
We just saw it from a different point of view.¹

The support and inspiration I drew throughout the writing of this book, teachers, friends, and colleagues supported and inspired me in ways that were as varied as they were indispensable. The paths that many of these

¹ Bob Dylan, "Tangled Up in Blue," *Blood on the Tracks* (New York, NY: Columbia Records, 1972).

people chose are very different from mine, but without their support this project never would have been finished. First of all, I would like to thank my dissertation committee Juliet Floyd, W. D. Hart, Peter Hylton, Colin Klein, and Daniel Sutherland for their input into the dissertation that led to this book. W. D. Hart and Daniel Sutherland were especially helpful in continuing to urge me to rework the dissertation into a book, and Peter Hylton provided me with very thorough and detailed comments on the original dissertation. For encouragement and comments on portions of the book manuscript, I thank Gary Ebbs and Warren Goldfarb. I also thank Timothy Bays, Patricia Blanchette, Michael Detlefsen, and Curtis Franks for allowing me to present some of this material at the 2015 Midwest PhilMath Workshop. Dean Joan Foster provided me with research funds for this project by way of multiple Metropolitan State University of Denver (MSU) Liberal Arts and Sciences (LAS) mini-grants, and Rebecca Dobbin of the Dean's office helped me to obtain research materials. I also thank Roy Cook, Thomas Forster, Craig Fox, Matthew Makley, Nick Robertson (1980–2008), Marcus Rossberg, Paul Roth, Joseph Ullian, the staff of the Houghton Library at Harvard University, and my colleagues and the staff of the MSU Denver Department of Philosophy. I was also assisted greatly by comments on an early portion of the manuscript from two anonymous referees for Cambridge University Press. I also thank Hilary Gaskin at Cambridge University Press for her interest in this project as well as for her patience and understanding as I completed my manuscript.

Although I have mentioned him already, I want to single out Bill Hart, my dissertation advisor and friend. His confidence in and support of my work have always kept me working. I thank also his wife, Faith, for her support and encouragement. My friend Kimo Quaintance deserves a special mention for the many conversations we have shared over the years and for helping me to see connections where there seemed to be none. In addition, a late-night conversation with my friend Ben Nichols was crucial to reigniting my interest in this project.

Finally, I would like to thank my family, including Bill Burke, my mother, Mary, my father, Paul (1947–1997), and my brother, Dan. They have all, in various ways, helped me to complete this book. Most of all and most important, I thank my wife, Haewon, and our son, John. They have made this work possible and have given more to me than I ever could have asked for. Now that it is done, I hope we will all be spending a lot more time at the park together.