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978-1-107-15170-3 — Probabilistic Mechanics of Quasibrittle Structures
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Excerpt
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1 Introduction

Without realistic failure mechanics, probabilistic analysis of structural safety is a fiction.

1.1 The Problem of Tail of Probability Distribution

Like most things in life, we must accept that the occurrence probability of any future event cannot be exactly zero. We must be contented with a structural failure probability that is negligible compared to other risks that people willingly take, such as car accidents. It is generally agreed that adequate safety of engineering structures is achieved by specifying a failure probability of 10^{-6} per lifetime as the maximum admissible in design [Nordic Committee for Building Structures (NKB) 1978; Melchers 1987; Duckett 2005; Ellingwood 2006]. This probability limit is generally accepted for engineering structures, whether bridges or aircraft (Duckett 2005; Department of National Defense of Canada 2007), although for some nuclear plant structures an even smaller limit is required.

The smallness of this probability limit is a source of great difficulty. To check the design merely by an experimental histogram, at least 10^8 tests of identical structures or specimens would be required. Even a direct computational verification would necessitate about 10^8 repetitions of Monte Carlo simulations with a fully realistic material model. Therefore, estimations of loads of such a small failure probability must rely on a model that is justified by a sound theory and is validated by experiments other than histogram testing.

For many years, realistic theoretical models have been available for the probability of ductile and brittle failures. The Gaussian and Weibull distributions, respectively, fit this purpose. Failures of structures made of quasibrittle materials are more difficult to predict and have been researched only recently. The difficulty is that the quasibrittle failures are transitional in nature between ductile and plastic failures.

Quasibrittle materials are heterogeneous materials with brittle constituents. At the scale of normal laboratory testing, they include concretes as the archetypical case, fiber-polymer composites, fiber-reinforced concretes, toughened ceramics, many rocks, coal, sea ice, wood, consolidated snow, particle board, rigid foams, particulate nanocomposites, biological shells, mortar, masonry, fiber-reinforced concrete, asphalt concrete (at low temperatures), stiff clays, silts, cemented sands, grouted soils, particle board, various refractories, bone, cartilage, dentine, dental ceramics, paper, carton, and cast iron.

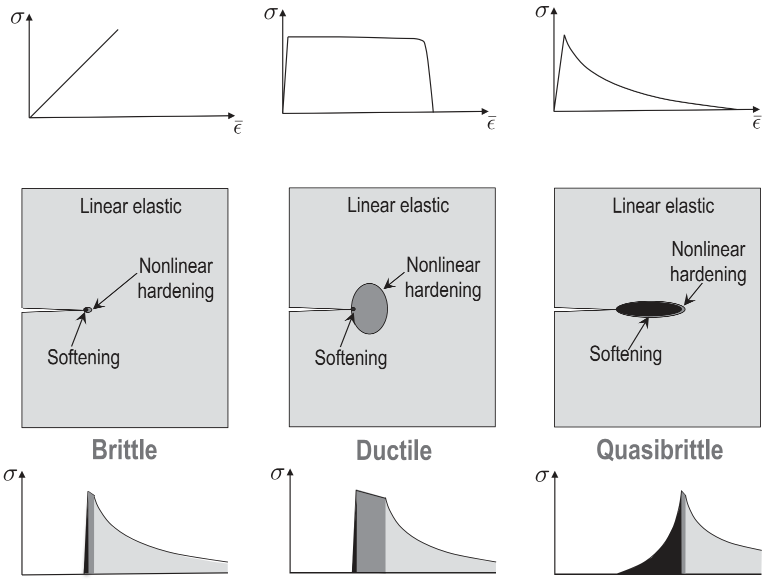

Figure 1.1. Types of fracture process. The diagrams at the top show the relationships between the cohesive stress and the averaged nominal strain across the crack extension line. The diagrams at the bottom show the stress profiles along the crack ligament.

On the nano- and micrometer scales, virtually all materials become quasibrittle, including silicon or thin metallic films.

In fracture, the degree of brittleness (or ductility, which is the opposite of brittleness) is manifested in the size of the fracture process zone (FPZ) formed at the tip of a propagating crack. In this regard, three cases, illustrated in Figure 1.1, may be distinguished:

1. In brittle failures, the FPZ is so small that it can be treated as a point (Fig. 1.1, left). Consequently, all the volume of the structure behaves elastically and the classical theory of linear elastic fracture mechanics (LEFM) is applicable.
2. In ductile (or plastic) failures, which are observed in elastoplastic materials (mainly metals) and are characterized by a stress-strain diagram with a horizontal yield plateau, there is a large nonlinear plastic (or yielding) zone. But the FPZ is still very small, micrometer size, which is almost pointwise for most applications (see Fig. 1.1, middle). Unlike the brittle case, the profile of the so-called cohesive stress transmitted in a quasibrittle material across the crack extension line is almost horizontal, with a steep stress drop at the crack tip.
3. The quintessential feature of quasibrittle failure is that the FPZ at the front of a crack is not negligible compared to the cross-sectional dimensions, and can sometimes occupy even the entire cross section of the structure. The profile of stress along the crack extension line has neither a long horizontal segment nor steep stress drop. Rather, it varies along the FPZ gradually, except for superposed statistical scatter due to heterogeneity (Fig. 1.1, right). The diagram of normal stress σ acting across the crack extension line versus the average normal strain $\bar{\epsilon}$ across the FPZ exhibits a gradual post-peak decline.

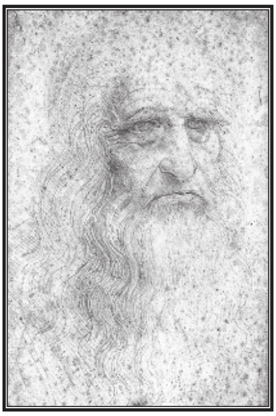

Figure 1.2. Portrait of Leonardo da Vinci (1452–1519). *Source:* Wikipedia

The FPZ length can vary enormously; it is typically about 50 cm in normal concretes, 5 cm in high-strength concretes, 10 μm to 1 mm in fine-grained ceramics, 10 nm in a silicon wafer, and 1 to 10 m in an Arctic sea ice floe. If the FPZ is negligible compared to the structure size, a quasibrittle structure becomes perfectly brittle, that is, follows LEFM. Thus concrete is quasibrittle on the scale of normal beams and columns, but perfectly brittle on the scale of a large dam. Arctic sea ice floe, fine-grained ceramic, and nanocomposites are quasibrittle on the scales of 10 m, 0.1 mm, and 0.1 μm, but brittle on the scales of 1 km, 1 cm, and 10 μm, respectively.

It is clear that the ratio between the overall structural size and the FPZ size determines the failure behavior of quasibrittle structures, transitioning from ductile to brittle as the ratio increases. Therefore, the main problem is the scaling of the failure behavior of quasibrittle structures, which has attracted significant research efforts over the last three decades. While previous research focused mainly on the scaling of the mean failure behavior, recent research has been directed toward the probabilistic aspect of this problem, in particular how such a size-dependent transitional failure behavior influences the strength and lifetime statistics of quasibrittle structures, and its consequences for the reliability-based structural design. This book presents a recently developed theory that addresses this topic, which is crucial for reliability-based analysis and design of quasibrittle structures.

1.2 History in Brief

1.2.1 Classical History

Let us start with a brief sketch of the history of the topic of size effect and scaling. Leonardo da Vinci was the pioneer in investigation of scaling law of material strength (Fig. 1.2). He speculated that “Among cords of equal thickness the longest is the least strong” (da Vinci 1500s). He also wrote that a cord “is so much stronger . . . as it is shorter”. This scaling rule implied the nominal strength of a cord to be inversely proportional to its length, which is of course a strong exaggeration of the actual size effect.

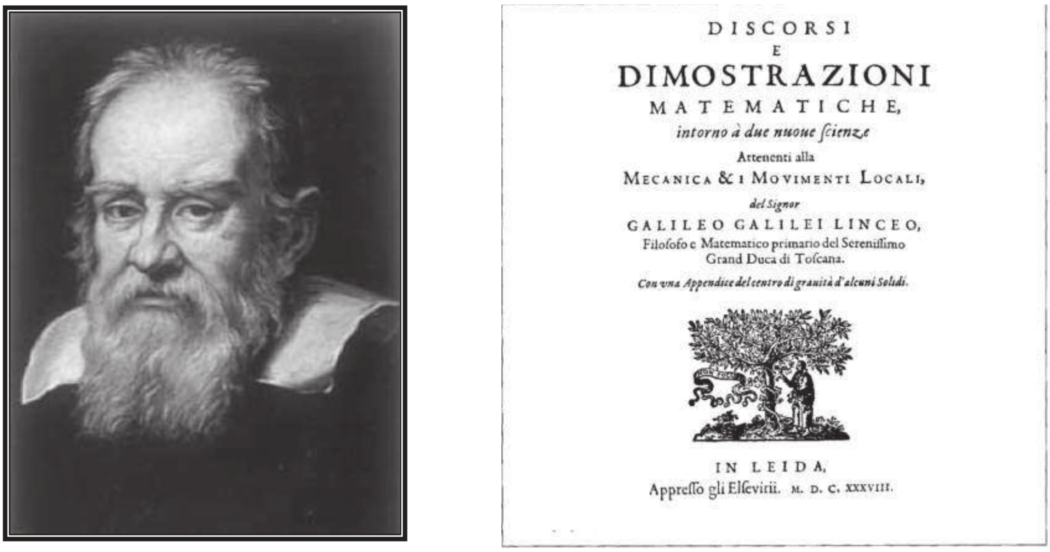

Figure 1.3. Portrait of G. Galilei (1564–1642) and the title page of *Discorsi e Dimostrazioni Matematiche Intorno a Due Nuove Scienze*. *Source:* Wikipedia

More than a century later, Leonardo’s scaling rule was rejected by Galileo Galilei (1638) in his famous book *Two New Sciences* (Fig. 1.3). Galileo argued that cutting a long cord at various points should not make the remaining part stronger (which is now known not to be true). Nevertheless, he proposed the famous “square-cube” scaling law to describe the effect of the object size on the ratio between the volume and surface area of the object. Based on this law, he explained the fact that large animals have relatively bulkier bones than small ones, which he called the “weakness of giants.”

About fifty years later, Edmé Mariotte (1686) made a major advance by reexamining da Vinci’s scaling rule (Fig. 1.4). He experimented with ropes, paper, and tin, and observed that “a long rope and a short one always support the same weight unless that in a long rope there may happen to be some faulty place in which it will break sooner than in a shorter. The same thing will happen in small slips of tin; for in a long one there may be perhaps some defect that may not be in a short one; and if you should take that part of it which did not break, it would sustain a greater weight. . . . ” Mariotte later expressed this observation more generally in terms of the variability of the material strength, which is essentially the principle of “the Inequality of the Matter whose absolute Resistance is less in one Place than another.” This statement can be considered as the first qualitative description of the statistical theory of size effect. However, at that time the probability theory was at its inception and was not yet ready to handle the problem within a formal mathematical framework. As discussed later, the mathematical formulation of statistical size effect for brittle solids was completed by Weibull (1939) almost three centuries later.

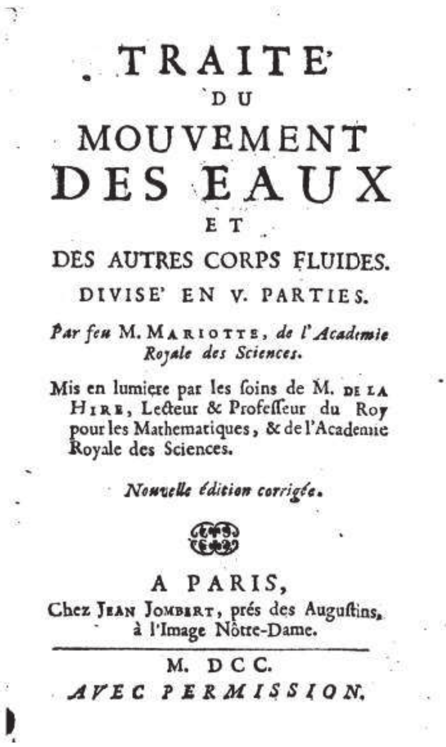
TRAITE' DU MOUVEMENT DES EAUX ET DES AUTRES CORPS FLUIDES.
DIVISE' EN V. PARTIES.
Par feu M. MARIOTTE, de l'Academie Royale des Sciences.
Mis en lumiere par les soins de M. DE LA HIRE, Lecteur & Professeur du Roy pour les Mathematiques, & de l'Academie Royale des Sciences.
Nouvelle édition corrigée.
A PARIS,
Chez JEAN JOMBERT, prés des Augustins, à l'Image Nôtre-Dame.
M. DCC.
AVEC PERMISSION.

Figure 1.4. Edmé Mariotte (1620–1684) and the title page of *Traité du mouvement des eaux*, posthumously edited by M. de la Hire. ("Principals découvertes de l'église" means "Principal discoveries of the chruch".) *Source:* Grzybowski, A, and Aydin, P. (2007) Edmé Mariotte (1620–1684) "Pioneer of Neurophysiology" Survey of Ophthalmology, 52(4), 443–451

Mariotte's conclusions were later rejected by Thomas Young (1807), who took a deterministic approach and stated that the length of a solid has no effect on its strength. This was a step backwards from Mariotte's idea on the statistical size effect. Nevertheless, Young did make a remark that the cohesive strength of a wire or bar is not always proportional to its diameter owing to some "accidental irregularities."

The second major advance was the seminal work of Griffith (1921), which laid the foundation of the theory of fracture mechanics. Meanwhile, in his paper he also pioneered the use of fracture mechanics to study the size effect. Griffith concluded that "the weakness of isotropic solids . . . is due to the presence of discontinuities or flaws . . . The effective strength of technical materials could be increased 10 or 20 times at least if these flaws could be eliminated" (Griffith 1921, page 179). He demonstrated this conclusion by an experimental investigation on the strength of glass fibers showing that the breaking stress was raised from 42,300 psi for a diameter of 0.0042 in. to 491,000 psi for a diameter of 0.00013 in. It is clear that Griffith's analysis of microscopic flaws or cracks provided a physical basis for Mariotte's statistical concept of size effect.

The mathematical description of the statistical size effect was made possible by the significant advances in probabilistic theories as well as extensive experimental investigations in the early twentieth century. Tippett (1925), Fréchet (1927), Fisher and Tippett (1928), and Peirce (1926) pioneered the mathematical formulation of extreme value statistics, which was later refined by von Mises (1936), Gnedenko (1943), and others (see also Gumbel 1958 and Freudenthal 1968). Weibull (1939) independently discovered a probability distribution function of structural strength using the weakest-link model (Weibull 1939), which is one of the three extreme value distribution functions proposed by Fisher, Tippett, and Fréchet.

The essential point of Weibull's analysis is that the structure can be statistically modeled by a chain of a large number of elements, and the tail distribution of strength of each element follows a power law. The resulting probability distribution of structural strength is now known as the Weibull distribution. The Weibull distribution of structural strength was later supported theoretically by probabilistic modeling of the distribution of microscopic flaws (see, e.g., Freudenthal 1968, 1981). It is noted that the Weibull distribution has also been widely applied to various other physical phenomena (Rinne 2009).

Another important aspect of Weibull's work is that he also derived a size effect equation on the mean structural strength based on the Weibull distribution. This is the first mathematical description of statistical size effect. Most subsequent studies until the 1980s dealt basically with refinements, justifications, and applications of Weibull's theory (Zaitsev & Wittmann 1974; Mihashi & Izumi 1977; Zech & Wittmann 1977; Mihashi & Zaitsev 1981; Mihashi 1983; Carpinteri 1986; Kittl & Diaz 1988; Carpinteri 1989; Kittl & Diaz 1989, 1990; Danzer, Supancic, Pascual, & Lube 2007; Danzer, Lube, Supancic, & Damani 2008). The essential feature of the Weibull statistical size effect is that it follows a power-law form, which implies the absence of a characteristic length (see detailed discussion in Section 1.5).

For a long time, it was generally assumed that, if a size effect was observed, it had to be of the Weibull type. Today we know this is not the case because the Weibull statistical size effect is limited to structures that (1) fail (or must be considered as failing) right at the initiation of the macroscopic fracture and (2) have at failure only a small FPZ compared to the structure size. Condition (1) is necessary for allowing the structure to be statistically modeled as a chain of elements, and condition (2) essentially implies that the number of elements in this chain is so large that it can be considered infinite. This is certainly the case for various fine-grain ceramics and for metal structures embrittled by fatigue. But this is not the case for *quasibrittle* materials, the main subject of this book.

1.2.2 Recent Developments

The most widely used quasibrittle material is concrete. The study of its fracture mechanics, initiated by Kaplan (1961), led to the discovery of a new type of size effect in quasibrittle fracture, which is fundamentally different from the statistical size effect.

Long ago it has been concluded that the classical LEFM is not applicable to concrete (Leicester 1969; Kesler, Naus, & Lott 1972; Walsh 1972). Leicester (1969) tested geometrically similar notched beams of different sizes and used a power law to fit the

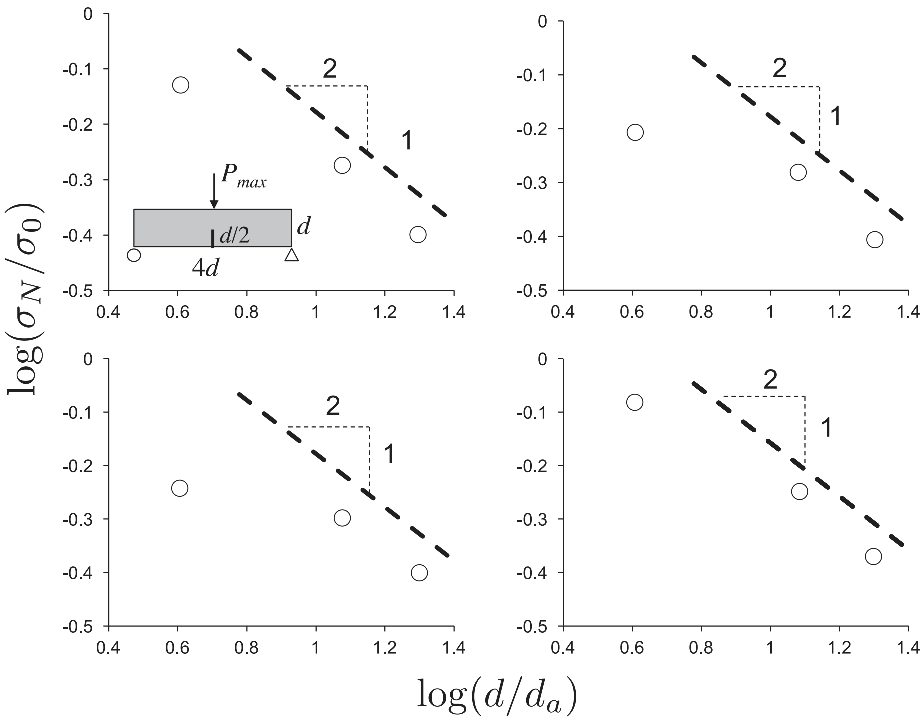

Figure 1.5. Measured size effect on the nominal strength of concrete beams.

measured size effect on the nominal strength: $\sigma_N \propto D^{-n}(n > 0)$. Based on the optimum fitting, the value of n was less than $1/2$. Such an n-value was inferred by assuming failure to be caused by a finite-angle notch, because its stress singularity exponent is weaker than that for a sharp crack. However, the optimum value of n was found to be less than the magnitude of the dominant stress singularity at the notch tip, as required by the LEFM. Furthermore, like the Weibull size effect, Leicester's power-law scaling also implied the nonexistence of a characteristic length, which cannot be the case for concrete owing to the large size of its FPZ.

The inapplicability of LEFM was further evidenced by Walsh's size effect experiments on geometrically similar notched beams (Walsh 1972, 1976). The nominal strength was plotted against the beam size in a double logarithmic diagram (Fig. 1.5), in which σ_0 is a reference strength and d_a is the average aggregate size. Without attempting a mathematical description, he made the point that this diagram deviates from a straight line of slope $-1/2$, and that this deviation must be regarded as a departure from LEFM.

The major milestone in application of fracture mechanics to concrete was the development of the fictitious crack model by Hillerborg et al. (1976). The concept of the fictitious crack model is analogous to the softening cohesive crack model of Barenblatt (1959, 1962), and is similar to the plastic fracture process zone model of Dugdale (1960) (later extended by Knauss 1973, 1974; Wnuk 1974; and Kfouri and Rice 1977). Hillerborg et al. used the fictitious crack model to simulate the failure of unnotched plane concrete beams in bending and further demonstrated a deterministic size effect, different from the Weibull statistical size effect, on the flexural strength of beams. The softening cohesive crack model by Palmer and Rice (1973) did the same for shear.

Around the same time, it was discovered (Bažant 1976) that the damage localization of strain-softening material would lead to a size effect on the post-peak deflections and energy dissipation of structures. The essential idea of crack band model was proposed as a remedy for realistic and objective finite element simulation of quasibrittle fracture (Bažant 1976, 1982; Bažant & Oh 1983). It was shown that the crack band model could accurately capture the size effect observed, by that time, on concrete structures. A more general nonlocal integral approach that can handle strain-softening damage in a more fundamental and, in some respect, more realistic manner followed soon (Bažant 1984*b*; Bažant, Belytschko, & Chang 1984; Pijaudier-Cabot & Bažant 1987; Bažant & Pijaudier-Cabot 1988; Bažant & Lin 1988*a*, 1988*b*). The nonlocal approach also led to the later development of the nonlocal implicit gradient models for quasibrittle materials (Peerlings, de Borst, Brekelmans, & de Vree 1996; Geers, Peerlings, Brekelmans, & de Borst 2000; Peerlings, Geers, de Borst, & Brekelmans 2001).

In early 1980s, Bažant (1984*b*) used an approximate energy analysis to derive a simple size effect law for the nominal strength of quasibrittle structures containing notches or traction-free (fatigued) large cracks formed prior to the peak load. This size effect law was later derived by using the equivalent LEFM with Taylor series expansion of the energy release rate function, by which the size effect curve was directly related to the fracture properties, such as the fracture energy, the softening law, and the R-curve, of quasibrittle materials (Bažant & Kazemi 1990*a*, 1991). Beginning with the mid-1980s, the interest in the size effect in quasibrittle fracture surged enormously. Besides an intensive focus on concrete (Petersson 1981; Carpinteri 1986; Planas & Elices 1988; Planas & Elices 1989; Bažant 1992*b*; Wittmann 1995), significant attention has also been paid to various other engineering materials such as ice (Bažant 1992*a*; Bažant & Li 1994; Li & Bažant 1994; Dempsey, Adamson, & Mulmule 1995; Mulmule, Dempsey, & Adamson 1995; Bažant & Kim 1998*a*, 1998*b*; Dempsey, Adamson, & Mulmule 1999); ceramics (McKinney & Rice 1981), rocks (Bažant, Gettu, & Kazemi 1991; Bažant, Lin, & Lippmann 1993; Le, Manning, & Labuz 2014), foam (Bažant, Zhou, Zi, & Daniel 2003), fiber composites (Bažant, Daniel, & Li 1996; Bažant et al. 1999, 2006; Bažant, Zhou, Novák, & Daniel 2004), braided composites (Caner et al. 2011), and bones (Bažant, Kim, & Yu 2013).

Recent research has focused back on the weakest-link model of strength statistics of quasibrittle structures that fails under controlled load at macrocrack initiation (Bažant & Pang 2006; Bažant & Pang 2007; Bažant, Le, & Bazant 2009; Le, Bažant, & Bazant 2011; Salviato, Kirane, & Bažant 2014). Different from Weibull's analysis, which is based on an infinite weakest-link model, recent studies proposed a finite weakest-link model to account for the fact that, for quasibrittle structures, the FPZ size is not negligible compared to the structure size. This model was further extended to lifetime statistics under both static and cyclic fatigue (Bažant et al. 2009; Le & Bažant 2011, 2012, 2014; Salviato et al. 2014). The finite weakest-link model predicts the size dependence of the probability distributions of structural strength and lifetime, which has been shown to have an important implication for reliability analysis of quasibrittle structures.

1.3 Safety Specifications in Concrete Design Codes and Embedded Obstacles to Probabilistic Analysis

The concept of structural reliability has penetrated into the design of many engineering structures. However, most existing design codes are not yet based on a calculation of the actual structural failure probability. Instead, they prescribe empirical safety factors of various kinds. The safety factor is understood as the ratio of the mean failure load measured by experiments to the failure load calculated deterministically. This kind of safety factor is used for some types of structures (e.g., for airframes, where it is generally considered to be 1.5). The design codes for structural concrete (as well as steel) were, in the 1970s, converted to the load factor and resistance design (LFRD), in which the safety factor is split into two types of partial factors:

1. The load factors, which account for the random variability of various kinds of load (live and dead loads, wind, earthquake, earth pressure, water pressure)
2. The resistance factors, aka strength reduction factors (popularly, “understrength” factors), applied to the calculated structural strength. These factors reflect the differences in risks in different types of failure; e.g., they are 0.9 for flexure and 0.75 for shear of concrete beams.

From the statistical viewpoint, three problems with these factors have recently been identified [for details, see Bažant & Frangopol (2002) and Bažant & Yu (2006)]:

1. While the safety factor of 1.6 for live loads seems reasonable, the safety factor (American Concrete Institute Committee 318 2011) for the self-weight acting alone, which is specified as 1.4, is excessive, by far. The value of 1.4 implies a 40% error in the self-weight, which is impossible to occur in practice. The errors in the mass density of concrete and in structural dimensions cannot justify errors of more than a few percent. Since, in a small-span bridge, the self-weight can represent 5% of the total load and in a large-span one 95%, such an excessive load factor unjustly penalizes large structures. So this penalty is equivalent to introducing in design a certain hidden size effect. But the trend of this hidden size effect is incorrect, with respect to not only the structure size, but also the structure type. For example, compared to the normal strength and non-prestressed concretes, the prestressed concrete structures and high-strength ones are lighter and thus receive less protection from the hidden size effect, but actually would need a higher protection because they are more brittle and exhibit a stronger size effect [for detailed discussion see Bažant & Yu (2006)].
2. Another problem is that various concrete design formulas have been set not to pass through the mean of experimental data but at the margin of a highly scattered data cloud, and that one must go to the original reports to find out. For example, in the case of shear failure of beams, the design formula passes about 65% below the mean

strength, which is near the bottom margin of the data cloud in which beams of different sizes, reinforcement, shear spans, aggregate sizes, etc. are commingled. This approach is necessary to make the deterministic design method adequately safe, but it implies covert understrength factors which, in many cases, lead to excessive safety, and in some cases not. The fact that the mean fit of the experimental data and the coefficient of variation (CoV) of its error (as well as the type of probability distribution) are unknown makes meaningful probabilistic analysis of structural strength impossible.

3. A similar problem occurs for the concrete strength. The design according to the code is based on the so-called required compression strength, f_c', which is set by the ACI code at about $\bar{f}_c - 1.34\delta_f$ where $\bar{f}_c$ = mean strength from cylinder tests and δ_f = their standard deviation (which is normally not known to the designer). Because the random scatter in concrete is high, f_c' can be 30% lower than $\bar{f}_c$. Again this approach is necessary to make the deterministic design approach safe, but it implies another covert understrength factor. The fact that the mean strength and its CoV are normally not reported and are unknown to the designer renders meaningful probabilistic design impossible [Model code 2010 sets $f_c' = \bar{f}_c - 8$ (units are in MPa) but the ratio of 8 MPa to δ_f is not known and varies].

Relatively sophisticated methods have been developed for the reliability analysis of civil engineering structures. They include the first-order and second-order reliability methods (FORM and SORM), where the failure risk of the structure can be estimated by using reliability indices, such as the Cornell and Hasofer–Lind indices (Benjamin & Cornell 1970; Hasofer & Lind 1974; Rackwitz & Fiessler 1976; Rackwitz & Fiessler 1978; Ang & Tang 1984; Haldar & Mahadevan 2000*a*). However, these methods are usually applied without regard to, or even in ignorance of, the aforementioned obstacles. Such an attitude limits their applicability to quasibrittle structures. Besides, these methods need restructuring to take into account the size effect on the strength distribution and its deviations from Gaussian and Weibull distributions established in this book.

Although the design code must be satisfied, it is nevertheless always prudent to also check the design of large and daring structures probabilistically, using the data from concrete strength tests conducted on the concrete chosen for the structure, or at least a reasonably estimated mean and standard deviation of such tests. This is how the probabilistic theory in this book is applicable to concrete structures, in spite of the aforementioned obstacles to probabilistic analysis. Another application should, of course, be the improvement of concrete design codes. And, of course, in the engineering practice of many other quasibrittle materials, e.g., fiber composites, tough ceramics, and rocks, the theory expounded here does not face similar obstacles. In aeronautical engineering there are no design codes, only performance requirements.

1.4 Importance of Size Effect for Strength Statistics

If the scaling of a theory is not understood, the theory itself is not understood. This motto has guided the development of fluid mechanics for more than 100 years, and recently