APPLIED CONIC FINANCE

This book introduces the brand-new theory of conic finance, also referred to as the two-price theory, which determines bid and ask prices in a consistent and fundamentally motivated manner. While theories of one price classically eliminate all risk, the concept of acceptable risks is critical to the foundations of the two-price theory, which sees risk elimination as typically unattainable in a modern financial economy.

Practical examples and case studies provide the reader with a comprehensive introduction to the fundamentals of the theory, a variety of advanced quantitative models, and numerous real-world applications including portfolio theory, option positioning, hedging and trading contexts.

This book offers a quantitative and practical approach for readers familiar with the basics of mathematical finance to allow them to boldly go where no quant has gone before.

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To our perennial supporters:

to Vimla, Maneka and Shivali
—Dilip

to Ethel, Jente and Maitzanne
—Wim

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Preface

All models are wrong, but some are a bit more useful than others.

In traditional financial mathematics the focus of derivative pricing is often solely on the so-called risk-neutral price (cfr. the law of one price), or the (equilibrium) price at which we supposedly can buy and sell. However, in real markets, one observes continuously two prices, namely the price at which the market is willing to buy (bid) and a price at which the market is willing to sell (ask). Hence, if one is going for an immediate transaction, one should take into account the direction of trade (buying or selling). This book presents a theory, referred to as the two-price theory or the conic finance theory (see later for the explanation of the word "conic"), which is about determining such bid and ask prices in a consistent and fundamentally motivated manner.

The law of one price, or (just for derivative pricers) the risk-neutral price, is a useful theoretical abstraction that serves many purposes including the concept of marking to fair market values the asset and liability positions of economic participants. Actual markets, however, recognize that the real value for positions will depend on many factors, including trade directions, the size of the trade, how fast it is conducted and who the actual counterparties turn out to be. Now, between the abstraction proposed by the law of one price and the actual realization of value in transaction lies the abstraction of the two-price theory. It provides some advantages by enabling the marking of positions to be done conservatively to higher levels (ask) for liabilities and lower ones (bid) for assets under unfavourable immediate unwinds.

Critical to the foundations of the two-price theory is an underlying concept of acceptable risks in the economy. Classically, in theories based on the law of one price, one often introduces the concept of complete markets that deliver unique prices while simultaneously eliminating all risk. Acceptable risk is then no longer an issue. The basic foundations of two-price theory is the recognition that risk elimination is typically unattainable and not available. Hence, acceptable risks must be defined.

We therefore start by recognizing that in a modern financial economy all risks cannot be eliminated and risk exposures must be tolerated. Hence, the set of acceptable risks must be defined as a financial primitive of the financial economy.

First, let us consider supposing that the non-negative random variables are acceptable at zero initial cost. Such variables should actually always be acceptable since they are in fact arbitrages: you get a non-negative, potentially positive cash flow for zero cost.

In a second step, let us elaborate on the traditional risk-neutral valuation of risks. In that traditional risk-neutral setting, the price, or value, of a financial derivative equal to the discounted risk-neutral expectation of the payoff under the risk-neutral measure. Assuming that the risk-neutral price is the "correct" price to trade on, the market will "accept", at initial zero cost, risks of non-negative risk-neutral value.

However, in reality one may recognize that the market does not trade at the risk-neutral price and that the price at which one can (immediately) trade depends on the direction of the trade, and there are typically current bid and ask quotes at which one may sell or buy. We are thus led to considering convex proper subsets of the set of risks of non-negative risk-neutral value (which forms a half-space), but containing the non-negative random variables as models for the set of potentially acceptable zero-cost cash flows by markets.

When the set of acceptable loss exposures is modelled by such a convex set of random variables that contains the always acceptable non-negative random variables, these exposures turn out to be just those that have a positive expectation, not just under the risk-neutral measure but under a whole set of different probability measures that may be called scenario probabilities. The set of acceptable loss exposures is then a convex cone of random variables, in the sense that it is closed under scaling.

The best price at which one may then sell a random outcome of cash flows is then the infimum or minimal expectation of the cash flow being priced under all the scenario probabilities. Similarly, the best price at which one may buy a random outcome of cash flows is the supremum or maximal expectation under all the scenario probabilities. These two extremal expectations are generally different, with the lower one becoming the bid price of a two-price economy while the higher one is the ask price. The former applies to assets that may eventually be sold while the latter applies to liabilities that must eventually be bought and delivered. By virtue of the infimum, the bid price is a concave function on the space of random outcomes and is suited to being maximized. The ask price, on the other hand, is a convex function of the random outcomes, suited to being minimized. In this way we obtain new market-based objectives for financial decision making. The book presents applications of these objectives to many financial decision problems, be they the design of portfolios, the valuation of credit exposure, the construction of hedges or the optimal design of derivative portfolios.

The applications require one to be quite specific about the set of scenario probabilities. Here the book makes direct connections to the literature on expectation with respect to non-additive probability that goes back to the work of Choquet. We essentially recognize the flaw embedded in expectations of treating all probabilities with an equal status, knowing full well that our uncertainty about such probabilities for extreme and rare tail events on either side of the gain–loss spectrum can be significant. Non-additive probability allows one to conservatively inflate the probabilities of large loss outcomes while simultaneously deflating those associated with large gains. When the cone of acceptable risks is defined purely by the risk distribution function, and in addition we ask for bid/ask prices of comonotone risks to be additive, it turns out that the bid/ask price is an expectation with respect to non-additive probability exactly as defined by Choquet. The book then brings to bear on financial problems the generalized Choquet expectation as the right objective for such decision making.

The context for the absence of risk elimination is essentially one of exposure to a multitude of surprise movements in markets of various directions and sizes. The complete markets context of continuous price processes eliminates such risks by assuming continuity and no surprise moves. Continuity in price processes is actually impossible, as it would require a continuum of trades at all price points between two price points at which one has a trade. With a finite and possibly large number of trades occurring over an interval of time, one typically registers a number of jumps in prices up or down that are better modelled by pure jump processes, and this requires a working knowledge of Lévy processes, their analytics and simulation. Chapters 2 and 3 provide this background. Chapter 1 introduces the two prices as they are observed in option markets and the no-arbitrage implications for them.

Conic finance is then presented in Chapter 4, with its link to Choquet expectations. Chapter 5 introduces the mechanics of conic pricing in the classical contexts of trees, Brownian motion risks and other processes. The book then takes on, in turn, the conic pricing of credit risk, design of portfolios, hedging of financial and insurance risks, the design of derivative positions and the trading of Markov processes.

Importantly, the conic pricing of credit risk shows how counterparty credit risk, when conservatively valued at the bid price, results in larger markdowns than would occur under risk-neutral pricing. On the other hand, when it comes to the debt valuation adjustment, since it is a liability, it must be priced at the ask. This mitigates and can even eliminate this adjustment.

Conic portfolio theory teaches us that the risk in a demeaned portfolio is the cost of acquiring its negative to get back to zero. Hence, the right measure for risk to offset the reward related to the mean return is the ask price for the return on the negative demeaned portfolio. The risk–reward frontier is then between the mean and this ask price. Both are measured in dollars and trade one to one. One may contrast with mean variance theory that measures reward in dollars and risk in squared dollars and then attempts unrealistically to compensate with a trade-off coefficient.

With respect to hedging liabilities, we gain the ability, in a conic setting, to compare different hedges, with the better ones being those that reduce the ask price for the hedged liability. By contrast, in a complete markets setting, two hedges that both eliminate risk are equivalent. Since, in reality, one is often unlikely to be able to eliminate risk, it is important to know which are the better hedges.

The design of derivative positions from a conic perspective leads to the selection of distribution functions or their inverses. Examples illustrate a variety of ways of organizing such distribution functions in closed form. The solutions involve the interactions between the physical and risk-neutral distribution functions.

In the final chapter, we show how the conic finance machinery can be put to work to construct optimal trading policies. We work in a discrete time Markovian context. It turns out to be critical here that the valuation is conducted by an expectation with respect to non-additive probability, for, with a classical conditional expectation operator, both the value and policy iterations embedded in the associated Markov decision process fail.

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