

Index

Analytic Hierarchy Process/rank reversal, 36 Archimedean combinations of functions, 653-57 in two dimensions, 655-57 univariate functions, 653-55 attribute dominance. See under preference functions; preference functions, additive; utility copulas; utility functions; utility functions, constructing from preference functions; utility functions, independence conditions for twoattribute, utility independence; utility functions, single attribute attributes converting into monetary equivalents, 76–77, 80 decomposition of, 62, 70, 73, 74, 76, 77 direct value attributes, 62, 63-64 clear characterization of, 83-84 confusion with indirect values, 61-62, 65-67, 647, 649 grounding condition, 401 indirect values, 62-64 meaningful and measurable scales, 27, 69-73, 83, 98 reluctance to quantify, 104 mischaracterization of, 79 utility-dominant, 401

Basic Expansion Theorem. See utility functions, Basic Expansion Theorem for multiattribute; utility functions, Basic Expansion Theorem for two-attribute; utility functions, conditional, higher-order utility assessments boundary conditions, 428, 637, 638, 641, 642 boundary independence, xx, 399, 461, 462, 469–70, 471, 473, 474, 476, 478, 479–81, 483, 488, 489, 491, 493, 498, 507, 508, 509, 511, 513, 514, 516, 518, 521, 534, 619

Cauchy equation, 263
Chain Rule, 158, 368–71, 375, 387
Choice Rule, 23, 24, 27
composite functions, 651
consequences vs. prospects, xiii, 3–4
copula functions. See preference copulas;
probability copulas; utility
copulas; utility functions, attribute
dominance, independence
conditions, nested combinations of
utility copulas; value copulas
correlation aversion, 384–86

671



> 672 Index

Equivalence Rule, 22-23, 24, 26, 28, 54, 181, 202, 204, 244, 397, 439, 442, 665 expected utility behavior prediction and, 32-33 calculator analogy, 31–32 complexity of, 33 indifference probability and, 25 original axioms of, 33-34 preference statements and, 25 rationality and, 32

five rules for preference order, 20, 23-24, 646 good outcome not guaranteed, 33 limitation of, 27 multiattribute prospects, 27 with value measure, 28–29 without value measure, 28 probability assignment, 29-30 flawed decision-making methods approximate vs. fundamentally flawed, 36,648 arbitrary assessment of linear function weights, 107-08 arbitrary metrics, 81, 157, 649 confusion of direct and indirect values,

double counting, 62 exclusive monetary value fallacy, 18 false appeal of simplicity, 6, 35-36, 44, 646, 648 majority votes on individual

61-62, 67, 152, 647, 649 constructed scale fallacy, 18-19, 79, 83

attributes, 38-41 min-max regret criterion, 41-44, 80, 175 misapplication of discount factors for time, 175, 177

non-validation of preference functions, 121, 150, 177

pairwise comparisons, 36-38 probability

arbitrary formulations excluding new information, 157

treated as direct value, 156

probability of success as direct value, 69, 80 rank reversal, 36, 43, 44, 79

ratios, 80

relative scales, 79-80

requirements without specifying trade-offs, 121-25, 649

risk aversion misunderstood, 290 risk scenario plots on two axes, 192 risk tolerance misunderstood, 290 treating monetary equivalents differentially, 80 unnecessary preference functions, 95–96 value functions that ignore domain

knowledge, 167, 176 weight-and-rate scoring system, 67-69, 73, 154-56, 648, 649

arbitrary functional form, 151-52 arbitrary scores, 151 arbitrary weights, 151 constructed with additive utility function, 536 manipulation of results, 153

metrics used as direct values, 152-53 Franklin, Benjamin, 58

generating function. See also under utility copulas, Archimedean by iteration, 595, 596, 597, 601, 602–03 power generating function, 428-29

Howard, Ronald. See "Introduction to Decision Analysis"

indifference assessment, 23, 28, 184, 186, 216, 230, 397–400, 404, 406, 439, 444, 522, 530, 536, 549, 568, 569, 592, 595, 606, 620, 665-66, 667

"Introduction to Decision Analysis", 205-06, 207-08

risk preference model, 207 structural model, 206-07 time preference model, 207 value model, 207

inverse functions, 651–52

isopreference contours, definition of, 100

Jensen's equality and inequality, 252, 253, 254, 255, 267, 339–40, 658–59

Matheson, James. See "Introduction to Decision Analysis" Monte Carlo simulation, 438, 585–86, 621–22

Order Rule, 22, 23, 25, 28, 47, 53, 62, 64, 80, 82, 152, 156, 197, 397, 646



Index 673

ordinal one-switch independence, 639-41 assessing weights of, 107 satisfied by cardinal one-switch utility limitations of, 108 independence, 641 more preferred to less, 101-02, 104 sufficient conditions for, 640-41 non-linear, 108-12 power equation, 112 personal indifference selling price. See reciprocal equation, 109-10 under value scaled reciprocal equation, 110-11 piecewise cubic polynomial interpolation, level sets, 100 multiattribute cases 594, 610-11 power transformations of lotteries, 313 complexity of, 98, 99 preference, xiii, 7-8, 646 definition of, xiv, 99 misuse of term, 17 not always sufficient, 161 preference statements, insufficiency of, 8 not always useful, 95 preference copulas, 635 attribute is indirect, 96 cases of few prospects, 95, 97 double-sided, 638 single-sided, 636-37 visualization sometimes sufficient, 95-96 preference functions, 82 notation, 98-99 additive, 126-28, 647 preference dependence, 119 Archimedean combination of individual sign of cross-derivative of, 380-81 functions, 135-37, 215, 647 single attribute cases, 84-88 elasticity of substitution, 139, 157-59 categorical attributes, 93-94 attribute dominance condition, 140 changes in preference, 91 grounded constant, 139-44 one-switch preferences, 92-93 grounding property, 140 zero-switch preferences, 91-92 ungrounded constant, 144-49 structural model sub-components, 90 isopreference contours, 128 structural model use, 102-03 constant fractional trade-offs, 129-30 trade-offs, 98, 647 uses of, 96 constant slope, 129 power function slope, 130 validation of, 121, 650 Priestley, Joseph, 58 more of an attribute not preferred to less, 130 priorities, 67 notation, 127 probability copulas, 571-72, 628 preferential independence, 131-32 Probability Rule, 22, 23 mutual, 133-35 pros/cons analysis, 58-60 three or more attributes, 133-35 prospects, 4 two attributes, 132-33 characterization of, 5, 6 weight-and-rate, 150-56. See also flawed deterministic, 5, 49 decision-making methods, weight-Probability Rule, 22 and-rate scoring system prospects vs. consequences, xiii, 3-4 visualization of, 49-56, 83, 95-96, attribute dominance conditions, 197-200 161, 646 changes in preference, 120 constructing by capturing important features of the problem, 112-19 risk aversion, 649 isopreference contours, 98, 99-101, 647. Chain Rule, 368-71 See also under preference functions, discounting value not utility, 370-71 additive historical perspective on, 370 direct assessment with, 103-04, 647 misuse of term, 290 functional forms for, 105 multivariate, 647 linear, 105-08 relating to correlation aversion, 384-86



674 Index

multiplicative sub-delta property, 297–98 risk aversion (cont.) relating to risk aversion function over exponential utility function, 298–99 value, 387–92 multiplicative super-delta property, 299-300 risk-seeking over value, 391-92 exponential utility function, 300 two-attribute, 384 separating deterministic and uncertainty risk aversion functions, 240, 244, 272-73 analysis phases, 30 absolute, 273, 335 shift transformations of lotteries, 258, 269, 293, constant, 274-76 294, 309, 310, 313, 314, 315, 316, 317, exponential utility function, 274-75 320, 321, 322, 333-34 linear utility function, 275–76 Sklar copulas. See probability copulas determining utility function from, 278-79 structural models, 62, 73-76, 77, 90, 98, inclination, 286 102-03, 208 delta property, 287-88 Substitution Rule, 23, 27, 54, 186 sub-delta property, 287, 288 super-delta property, 287, 288 uncertainty, xiii, xiv, xix, 4, 646 increasing or decreasing uses of the book Gaussian utility function, 278 engineering courses, xxi logarithmic utility function, 276 utility practitioners, xxi power utility function, 277 utility theory courses, xxi linear transformation on utility utility, xiv, 11-13, 646 function, 274 deterministic deals and, 25 magnitude of, 281 Equivalence Rule, 22–23 certain equivalent related to mean inferring preference statements from, 12 value, 281-86 misuse of term, 17 sign of, 280 notation of, 12 related to bounds of certain utility copulas, 549-50, 551-52, 553, 648 equivalent, 281 Archimedean, 424–33, 573–74, 587, 650 related to shape of utility function, 280 assessing, 588-89 Chain Rule, 373 class 0, 579-82, 585 g-risk, 321-23, 335 class 1, 574-79, 585 relative, 293, 301-02 generating functions, 575-79 exponential utility function, 302 multiplicative, 581 linear utility function, 302 comparison with mutual utility logarithmic utility function, 304 independence, 603-06, 608-09 multiplicative delta property, 306 constructing two-attribute utility power utility function, 304 function, 589-95 with a Scale Amount m > 1,305-06additional assessments to determine utility transversality, 371–74, 647 generating function, 590–92, 593–95 implications of, 374–75 boundary utility functions, 590, 592-93 risk premium, 281 skewed diagonal curve, 591-92 risk tolerance, 203, 273 convergences of successive linear risk tolerance utility functions, 182, iterations, 602-03 189, 277-78, 307-08 determining generating function on misuse of term, 289-90 interval $[k_1, 1]$, 595–602 convergence of $\eta_m(t)$ to $[k_y,1]$, scale transformations of lotteries, 293, 309, 597-98, 607 310, 313, 314, 317, 319, 320, 321, 322 determining generating function on multiplicative delta property, 293-94 interval [0,k], 599 logarithmic utility function, 295, 296 relating g(r) to $[k_y,1]$, 599–602, 607, 608 power utility function, 295-96 relating S(t) to $[k_x, 1]$, 595–97, 607



Index 675

improper generating functions, 582-84 grounded Archimedean copula function, match only one boundary assessment for 625-27, 632 each attribute, 614-23 monotonicity, 627-28, 633 properties of S(t), 609–10 utility functions, 181-82 additive, 220, 386, 392-94, 647, 649 sign of cross-derivative of, 640 two attributes sufficient to determine Archimedean combination of individual generating function, 588-89 utility assessments, 215-16, 222, 232 verifying additive ordinal attribute dominance, 401-02, 408-09 preferences, 588 Bayes' rule for utility inference, 406-07 attribute dominance, 423-33, 552 example, 410 calculating for given multiattribute utility conditional utility function, 405-06 function, 564-66 decomposition of indifference probability classes, 559-61 assessment, 406 class 0, 557-59 multiattribute utility tree of single class 1 consequence, 670 with normalizing arguments, 553-55 two-attribute utility tree for single without normalized arguments, prospect, 667-70 556-57 diagrams, 418-22 consistency conditions, 566-69 with arrow, 420 without arrow, 419 fixing as class 0, 568 fixing as class 1, 568-69 arrow reversals, 421-22 violation of, 566-67 conditional utility double-sided, 614-23 independence, 420-21 changing the height of the utility utility independence a symmetric surface, 621-22 property, 418-19 class (0,0), 633-34 independence conditions class (0,1), 634 higher-order, 435-36 class (1,0), 634 nested combinations of utility transformation functions relating copulas, 436-37 partial utility independence, 434-35 boundary assessments, 614, 630 two-attribute double-sided class (1,1), marginal utility function, 404-05 614-15, 630-31 marginal-conditional expansion of, 416, 647 using Archimedean grounded copulas, 619-20 with multiple attributes, 417-18 using three grounded surfaces, product expansion theorem, 405 623, 631-32 properties of, 402 hybrid independence, 619 trade-offs implied by, 416-17 mutual utility independence, 619 Basic Expansion Theorem for sum product double-sided multiattribute, 483, 649-50 family, 618-19 conditional interpolation extensions to multiple attributes, 562-64 independence, 506 class 1 and class 0, 562-63 conditional utility function of first general definition for arbitrary reference attribute in expansion, 486-88 values, 563-64 conditional utility function of second attribute in expansion, 488-93, matching all boundary assessments 502, 506 Archimedean copulas match only one for each attribute, 613 conditional utility function of need for, 612 third attribute in expansion, 493-96, 503-05 relating to probability copulas, 571-72 using a single grounded surface, 623-28, 632 extensions to multiple attributes



676 Index

utility functions (cont.) characteristic transformation of utility conditional utility function of attribute function, U, and transformation, X_{\circ} , 497–98 g, 329-30 interpolation independence of logarithmic utility function with shift, attribute X, 498 scale and power transformations, 331 multiattribute expansions, 497 power transformation with exponential utility independence of attribute utility function, 330 $X_{.},498$ scale transformation with exponential reversing order of attributes in utility function, 330 conditioning statements, 499 shape of, 331–33, 341–42 three-attribute utility tree on continuous shift transformation with exponential domain, 483-86 utility function, 330 representation of, 484–85 conditional utility independence on a prior deriving at different levels, 367–68 attribute, 506 higher-order conditional utility utility tree networks for three or more assessments attributes, 499-501 conditional disutility function, 522 Basic Expansion Theorem for normalized conditional utility, 522 two-attribute, 439-41 higher-order utility assessments general expression and notation for Basic Expansion Theorem, 522 utility inference, 455 constructing from preference functions, 197 generalized notation, 449-50 attribute dominance condition not reversing order of attributes in satisfied, 200 conditioning statements, 456 attribute dominance condition satisfied, two-attribute utility trees on a continuous 197-200 domain, 441-43 constructing from value functions, constructing two-attribute utility 200-02, 204-05 functions, 448 CES functions, 204 around a single attribute, 443-45 Cobb-Douglas functions, 203-04 around two attributes, 445 discounting value not utility, 202-03, 219 utility inference for two continuous, 244 attributes, 450-51 exponential, over additive value reversing two-attribute utility tree, functions, 216-17 451-52, 457-59 exponential, over multiplicative value utility inference for two-attribute functions, 217-18 utility function, 453-55 higher-order conditional utility bidirectional utility diagrams with three or assessments, 522-23 more attributes, 509–10 higher-order utility independence, 523-25 multilinear form, 510-13 definition illustrated, 524-25 derived from utility and boundary implications for simplifying utility independence, 512-13 assessment, 524 partial utility independence, 507, mutual utility independence, 533-34, 514-15 iteratively determining utility vs. additive preference functions, 534 assessments needed for, 515-18 functional form of, 533-34 bounded, 244 relating to lower-order, 525-29 calculating indifference contours, assserting higher-order independence 192-95 from lower, 529-32 change of variables, 208–12, 233–35 independence blanket, 528, 535-36



Index 677

independence conditions for two-attribute, adding a shift amount, 357 461-62, 648 characterizing by extension of invariance boundary independence, 462, 469-70 functional equation, 352-53, functional form of, 470 363-65 graphical representation, 476 expressing in terms of zero-switch utility bidirectional utility independence functions, 359-60 diagrams, 476-79 extension of property beyond wealth utility tree networks, 479-81 changes, 538-39 extensions to two attributes, 362 utility-dependence matrix, 479 interpolation independence, 462, 471 four functions with shift relating utility, boundary, and transformations, 354-57 interpolation independence conditions for, 359-60, 365-66 conditions, 471-76 linear plus exponential utility utility independence, 462 function, 356 attribute dominance utility risk aversion function implied functions, 465 by, 358-59 attribute risk aversion, 467 $U(y) = \sin(y)$ utility function, 356 implications of, 467 power transformations, 361 invariant transformation, 465-66 scale transformations, 360-61 linear relation between two curves on over a two-attribute Archimedean preference or value function, 220-22 surface, 463-64 normalized conditional utility over an additive preference or value function, 223-29 function, 462-63 not necessarily symmetric, 467-69 product of two (or more) normalized utility preferences for lotteries, 464-65 functions, 412-16 independence relations with multiattribute required when decision uncertain, 181 utility trees, 507-09 sign of cross-derivative of, 381-84, 389 three-attribute tree with utility single attribute, 182 and boundary independence assessing over value measures, 183, 239conditions, 508 40, 241, 647 invariant transformation of, 311-14, 324-25, certain equivalent, 243, 245-48 538, 648 concave transformation of characterizing the utility function, function, 255-56 317, 325-26 concave utility functions, 252-54 invariance with k and d on an convex utility functions, 254 interval, 320-21 delta property, 258-65 exponential utility function, 261 exponential utility function, 315-16, 318-19 linear transformation of function, 249-50, 251-52 linear utility function, 314 logarithmic utility function, 319–20 linear utility function, 254, 261 logarithmic utility function, power utility function, 320 inverse function, 243 261, 267-69 logarithmic, over additive value monotone transformation of functions, 219 function, 250-51 logarithmic, over multiplicative value notation, 242 sub-delta property, 266-68 functions, 220 non-decreasing, 244 super-delta property, 268-69 n-switch, 362 assuming a functional form, one-switch, 347-48, 352 188-89, 244-45



678 Index

utility functions (cont.) value copulas, 635 attribute dominance, 404, 412 double-sided, 638-39 determining form of, 189-90 satisfying ordinal one-switch direct utility assessment, 183-85 independence, 641-44 extending the domain, 185-87 single-sided, 637 over value vs. over attribute, 190 value functions, 162 time preference, 190-91 additive, 390 twice continuously differentiable, 244, 272 benefits of, 161-62 zero-switch, 347, 537-38 isopreference contours, 167 are one-switch by definition, 356 min-max regret criterion, 175 extensions to two attributes, 362 monotone transformation from preference relating delta property to, 348-51 functions, 167, 168-71, 190 relating invariant transformations Archimedean, 172 to, 350-51 marginal property, 174 relating scale transformations to, 351 multiattribute form, 173-74 utility independence, 418-19, 462-66, 467-69, special case, 174 498, 506, 538 two-attribute form, 172-73 n-switch, 544-45 multiplicative, 390-91 one-switch, 540-42, 545-47 structural models as source of, 162 assessing functional form for, accounting, 162-63 542-44, 547-48 case study, 163-67 relation to interpolation independence, time preferences, 174–75 541-42, 547 sufficiency of form, 541 wealth equivalent, 245-48 utility transversality. See under risk aversion bounds on, 328-29 functions measure of change in, 335 exponential utility function, 335-36, value, xiii, 9, 646 absolute scale, 9, 162, 647 Jensen's inequality gap, certain equivalent, 183 341-42 approximate expression, 283, 285, linear transformations and general utility 286, 290-92 functions, 337 exclusive monetary value fallacy, 18 scale transformations and general utility misuse of term, 17 functions, 337 personal indifference selling price, 9, 161 shift transformations and general utility value measures, 14-15 functions, 336 value statements, 9-10 valuing changes in the small, 338-39, inadequacy of, 10 343-46