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# FOUNDATIONS OF MULTIATTRIBUTE UTILITY

Many of the complex problems faced by decision-makers involve uncertainty as well as multiple conflicting objectives. This book provides a complete understanding of the types of objective functions that should be used in multiattribute decision-making. By using tools such as preference, value, and utility functions, readers will learn state-of-the-art methods to analyze prospects to guide decision-making and will develop a process that guarantees a defensible analysis to rationalize choices.

Summarizing and distilling classical techniques and providing extensive coverage of recent advances in the field, the author offers practical guidance on how to make good decisions in the face of uncertainty. This text will appeal to graduate students and practitioners alike in systems engineering, operations research, business, management, government, climate change, energy, and health care.

Ali E. Abbas is Professor of Industrial and Systems Engineering and Professor of Public Policy and Governance at the University of Southern California. He directs the Center for Decisions and Ethics (DECIDE) and previously directed the Center for Risk and Economic Analysis of Terrorism Events (CREATE). His research covers all aspects of decision-making under uncertainty broadly defined. He has written two books and edited two others, and has more than 100 refereed publications in the field of decision analysis. He is editor of numerous professional journals and has extensive media coverage on all aspects of decision-making and risk.

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*To my mother and family*

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# Preface

Decisions are the only means you have to change your life. What we encounter in life is shaped by the decisions that we make. Whenever you make a decision, you get an outcome, and you live your life with the outcome of this decision, and with the decision that you have made. There is always uncertainty about what might happen following our decisions. We are not guaranteed the best outcome as we make good decisions, but by using the approach presented in this book, we are guaranteed a process and a logic that can guide us and help us make the right decision despite the uncertainties that prevail. We are also guaranteed defensible analysis that can help us rationalize our choice regardless of the outcome that we get.

When thinking about what might happen following a decision, it is useful to use the term "prospects" of a decision instead of "consequences." A prospect highlights that we are thinking not only about the result of a past decision but also about our future life following the decision and the outcomes of the uncertainties that have been revealed. A prospect is characterized by the decisions that we make, the outcomes of any uncertainties following the decisions, and our future life with the decisions that we have made and with their outcomes.

If there is no uncertainty about what will happen, then the prospect you get will be determined by the decision that you make. You can determine the best decision by simply ranking the prospects you face from most preferred to least preferred. Ranking the prospects is a simple task if the prospects are few, but if they are numerous, then we will need a structured way of ranking them. A function that helps us rank the prospects is known as a "preference function." The higher the score on the preference, the more preferred is the prospect.

Sometimes we care about more than just the ordering the prospects. We might care about the value of the prospect that we get, and how much we would need to be paid (or pay) to receive one prospect instead of another. We can then think about a function that associates a value with each prospect to help with this task. This function is known as a "value function."

Many of the prospects that we face in our decisions include more than one preference criterion (or attribute) that contributes to the value of the prospect. For example, in a medical decision, we might consider attributes such as health and wealth. In a job



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selection decision, we might consider salary, vacation days, advancement, intellectual stimulation, and the overall collegial environment. When multiple attributes are present, it is helpful to have a function that can rank the multiattribute prospects. This is known a "multiattribute preference function."

When there are multiple attributes of value in the decision, as well as uncertainty about the prospect that you will get, it becomes essential to rely on a sound decision-making process. In this case, you will need to assign a "utility" to each prospect, and then choose the alternative that has the highest expected utility. The term "utility of a prospect" is widely misused in various contexts, and we shall explain its meaning in more detail in this book. We can also think of a function that returns a utility for each prospect. We call this a "utility function."

The purpose of this book is to help you make quality decisions by thinking about your preference, value, and utility for the prospects of a decision, and particularly when prospects are characterized by multiple attributes. Characterizing preferences for prospects is one of the most fundamental steps in decision analysis. Other elements of making good decisions include choosing the appropriate frame for the decision; characterizing the information that you know through a joint probability distribution; selecting the appropriate alternatives, and having a committed decision-maker that uses sound logic to make the decision. A good reference on this general topic of decision analysis is Howard and Abbas (2015).

## WHY I WROTE THIS BOOK

Many researchers and practitioners in industry and in the government face decisions with uncertainty and multiple objectives. Many of the analyses of such decisions immediately rush into assuming that there is some weighed combination of some selected metrics that needs to be constructed, and that the best decision should be determined by this weighted combination. They immediately talk about "the weights" in the decision as if every decision requires some multi-objective function of the form of some weighted combination. Of course, this is not the case and this message will be propagated throughout this book. In fact, weighted combinations of metrics could be very remote from the decision-maker's preferences, and the resulting score based on those chosen metrics could lead to poor decision-making. One main motivation for writing this book was to provide a proper understanding of the types of objective functions that should be used in multiattribute decision-making, and to highlight that these functions need not necessarily be some weighted combination. Another motivation was to highlight the arbitrariness in several existing approaches, and to explain how to determine the errors that might result when using arbitrary forms of objective functions in multiattribute decisions.

I have also found that people confuse the terms "preference," "value," and "utility," and so I felt that it is important to have a reference that clarifies the difference between these terms, to remove ambiguity about their use in the field.

When writing this book, I also wanted to highlight that many engineering and physical phenomena should be incorporated in the construction of preference, value, and utility functions, using a "structural model." The structural model often relies on

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physical principles and domain knowledge, and it is not merely about assessing weights and univariate functions for the attributes. To illustrate, think about the trade-offs between widths of pillars in a bridge. If you want to reduce the width of one pillar at the expense of increasing the width of another, then there are structural and engineering principles that should be used to determine this new design. Simply assessing weights and preference functions over the diameters of the pillars and then combining them into some arbitrary combination is a recipe for disaster. Capturing physical and engineering phenomena into the construction of preference, value, and utility functions has often been overlooked in much of the literature, particularly in engineering design.

I included a section titled "How People Get It Wrong" in many of the chapter appendices to explain some common misuses of utility theory. I have deliberately avoided citations to such misuses. The purpose is to present this message to as broad an audience as possible, and to influence the way many applications are modelled without causing anybody to feel defensive about their work, or about learning a different approach. I also included a section titled "Dialogue between Instructor and Student" in numerous chapters to answer some of the misconceptions people have about the field of multiattribute utility in a less formal way. These conversations also help explain some of the mathematical portions of the book.

While writing this book, I interacted with many people in leadership positions in the federal government and at large enterprises. I realized that in order to have an impact on the modeling of their decisions, there is a need for a reference on preference, value, and utility functions that can present the foundational concepts in a way that is accessible to a broad audience, and one that highlights the implications of using inappropriate or overly simplistic modeling methods. The content and exposition of this book have been designed to achieve this purpose. I kept some of the mathematical portions because I also felt it is important for organizations to realize that there is a science and a skill that needs to be learned if you wish to build models at the enterprise, public policy, or societal level, and that a simple "weight and rate" analysis will not necessarily lead to good decision-making.

When writing this book, I also wanted to highlight an important distinction that relates to many engineering phenomenon: the difference between problem solving and decision-making. Problem solving is about reacting to a problem and finding a solution, such as how to decrease the mass of an engine for space applications because of limited power. Decision-making is about making a decision even if you do not have a problem, and, more importantly, it requires you to think about preferences. At what cost does decreasing the engine mass come with? How do we decide on the best alternative? Do we need to reduce the mass or are there other decision alternatives that can be considered? This difference is essential in almost all aspects of our modern era.

Another motivation for writing this book was for students and practitioners in the field: to show them that there are many methods for constructing multiattribute functions. In the past fifty years, the field of multiattribute utility theory has often been classified by what is known as either the "Stanford Approach" using utility functions over value functions, or the "Harvard Approach" constructing multiattribute utility functions by assessing individual utility functions over the attributes and then combining them using independence assertions. Both of these approaches have had tremendous impact on the field. There are, however, many other methods that have been

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developed in recent years, and my purpose was to explain these new methods to a broader audience. I hope this work will provide the foundations of multiattribute utility theory for graduate students, researchers, and practitioners who wish to understand the fundamentals and the latest developments in the field. I also hope that it will inspire new research in many different directions.

## REFERENCE

Howard, R. A. and A. E. Abbas. 2015. *Foundations of Decision Analysis*. Pearson, New York.

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# Basic Structure of the Book

This book is divided into several parts.

## PART I: INTRODUCTION

This part of the book explains the basic definitions of preference, value, and utility of a prospect of a decision, because there is much confusion about these terms. The term "utility" is used with numerous meanings. A second objective of this part is to provide the rules of decision-making under uncertainty that motivate the need for determining the preference, value, and utility of the prospects of decision. This section also presents flawed methods of decision-making that are widely used to explain why it is important to follow a rigorous approach, and that simplicity is not an excuse for using a bad decision-making method.

## PART II: DECISIONS WITH NO UNCERTAINTY

This part of the book focuses on ordering deterministic prospects of a decision and then assigning a value measure to them. In deterministic decisions, the preference and value of a prospect determine the best decision alternative because each alternative corresponds to only one prospect. This part highlights that sometimes there is no need for a model before ordering the prospects, and that the mere visualization of the prospects could be sufficient. In other cases, particularly when the prospects are numerous and are characterized by multiple measurable attributes, a preference or a value function can be assigned. The part distinguishes between prospects characterized by a single attribute, such as money, and prospects characterized by multiple attributes, such as money and health. This part also emphasizes that quite often engineering or accounting principles are needed to determine preference and value functions, and that problems might arise if they are constructed arbitrarily.



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**PART III:** DECISIONS WITH UNCERTAINTY USING VALUE MEASURES

This part of the book introduces utility functions that are needed when uncertainty is present. It first describes how to assess a utility function over a value measure using simple assessments, and then shows how multiattribute utility functions can be constructed by assigning a one-dimensional utility function over the value function or over an attribute of the preference function.

**PART IV:** PROPERTIES OF SINGLE-ATTRIBUTE UTILITY FUNCTIONS OVER VALUE MEASURES

Because of the important role that single-attribute utility functions play in the construction of multiattribute utility functions (both in assigning a utility function over value or by combining individual single-attribute utility functions to construct a multiattribute utility function), this part provides a rigorous treatment of the properties and implications of single-attribute utility functions. This section uses functional equations to derive exciting new formulations in utility theory, but no prior knowledge of functional equations is required. Readers who are unfamiliar with the concept will find a detailed explanation throughout the part, as well as an additional Appendix at the end of the book.

**PART V:** CONSTRUCTING MULTIATTRIBUTE UTILITY FUNCTIONS WITHOUT PREFERENCE OR VALUE FUNCTIONS

This part of the book presents methods to construct multiattribute utility functions without preference or value functions. A general expansion theorem of multiattribute utility functions in terms of conditional utility assessments is presented, as well as methods to simplify the expansion such as attribute dominance, utility independence, boundary independence, and interpolation independence conditions. New concepts of multiattribute utility functions and graphical representations of utility functions are also introduced.

**PART VI:** CONSTRUCTING MULTIATTRIBUTE UTILITY FUNCTIONS USING COPULA STRUCTURES

This part of the book presents methods to construct multiattribute utility functions using single-attribute utility assessments and new constructs in utility theory called utility copula functions. The main idea is to construct a utility surface that matches conditional utility assessments made at the boundary values of the domain of the attributes.

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# How to Use This Book

## ENGINEERING COURSE ON SYSTEM DESIGN

If you are interested in a course on systems design using utility theory that may be applicable to undergraduate or first-year graduate engineering courses, then appropriate chapters would be those of Parts I, II, and III, in which the focus is on explaining a rigorous approach to decision-making, identifying flaws in some widely used methods of decision-making, using structural models, and identifying the value of various engineering design parameters using appropriate value functions. The use of value functions is particularly important when physical or engineering connections relate the attributes of the problem.

## PRACTITIONER INTERESTED IN ADVANCES IN MULTIATTRIBUTE UTILITY THEORY

If you are a practitioner interested in advanced models for constructing multiattribute utility functions, then Parts I, II, III, V, and VI can be relevant, in which new approaches such as expansion theorems and graphical representations are presented.

## GRADUATE CLASS IN UTILITY THEORY

If you are interested in a course on utility theory that provides advanced topics on properties of utility functions and their characterizations, then the entire book will be relevant. Part IV deals with a rigorous treatment of the mathematical foundations of single-attribute utility functions, which may appeal to an advanced audience. Parts V and VI present new approaches for constructing multiattribute utility functions and various independence conditions.



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