PART I     FOUNDATIONS OF PREFERENCE,
VALUE, AND UTILITY

The purpose of this part is to explain the meaning of three concepts – preference, value, and utility of a prospect of a decision – and to illustrate their use in decision-making. This part also explains the rationale for using the expected utility criterion as a basis for sound decision-making. It also discusses other methods of decision-making that are motivated by their simplicity but can lead to errors in decision-making.

| CHAPTER 1 | Preference, Value, and Utility |

**Chapter Concepts**

- Decision
- Decision alternative
- Prospect of a decision
- A prospect is deterministic; there is no uncertainty about what it entails.
- When uncertainty is present, a decision alternative may have multiple prospects
- Preference, value, and utility of a prospect

## 1.1 CHARACTERIZING PROSPECTS OF A DECISION

We start with the main building blocks of a decision, and in so doing we distinguish between a decision, a decision alternative, and a prospect of a decision.

**Decision vs. Decision Alternative vs. Prospect of a Decision**

**Definition**

**Decision:** A decision is a choice between two or more alternatives that involves an irrevocable allocation of resources. The term "irrevocable" implies that you will not be in the same state after making the decision.

**Decision Alternative:** A decision alternative is one of possible feasible actions that you can do in a decision you are facing. You must own your alternatives. They must be feasible.

Whenever you make a decision, you face the consequences of the decision that you have made. If you choose to wear blue jeans to a black-tie cocktail party, then you might very well end up with the consequence of being the only person in blue jeans in the party. Think about the consequence of this decision. Imagine yourself in this situation.

There is always uncertainty about what might happen following our decisions. For example, there is a possibility that people will appreciate your courage for showing up

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this way, and there is also a possibility that they will not let you in through the front door of the cocktail party in the first place. You might then miss the party if you make this "blue jeans" choice, or you might choose to go to purchase appropriate clothes and come back to attend the remainder of the party.

Decisions are the only means you have to affect your future. From here on we shall use the term "**prospect**" of a decision instead of "consequence" of a decision because it highlights the importance of thinking not only about consequences of past decisions but also about your future life following the decisions that you have made.

> **Definition**
> **Prospect of a Decision:** A prospect of a decision is one of the possible (deterministic) states of the world that might occur following a decision alternative (or a sequence of decision alternatives) that you have chosen.

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**EXAMPLE 1.1  Uncertainty Exists Even in Simple Decisions**

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The prospects we encounter in life are shaped by the decisions that we make. Think of the simple decision of either cooking dinner at home or going out to watch a movie and eating at a restaurant.

If you decide to stay at home and cook dinner, then one possible prospect is "eating a nice dinner at home." Taking uncertainty into account, can you think of other possible prospects for this decision alternative?

If you choose to go out to watch a movie and eat dinner at a restaurant instead, then a possible prospect could be "watching a movie and having a nice dinner somewhere else." Can you think of other prospects for this alternative?

Each of the alternatives discussed above is a feasible alternative, but the uncertainties that are associated with the alternative enable a number of possible prospects. This is why we say "*one possible prospect*" of this decision is "eating a nice dinner at home" instead of asserting that there will be only one prospect of "eating a nice dinner." If you choose to stay at home and cook dinner, there is a possibility that the food will burn, and so you will have another decision to make: cook another meal, go out for dinner, order pizza, or just forget about dinner and munch on some snacks.

If you choose to go to the movie theater, then there is an uncertainty about whether or not you will enjoy the movie. If you do not enjoy it, then you might have another decision to make about whether you will leave in the middle of the movie or stay till the end. You might also have an uncertainty about whether you will be sitting in a good seat (with a good view? With a noisy neighbor?) … etc.

The uncertainties about this simple decision continue to grow. At the movie theater, you might bump into an old friend while parking, and so you might have another decision to make: whether to stay at the movie theater or to go somewhere else with your friend where you can talk and catch up on old times.

Uncertainties continue to surround us as we make decisions. The art of modeling decisions requires capturing the important uncertainties that can affect the alternative that you choose and the important aspects of your preferences.

If there is no ***uncertainty*** in the decision, then the alternative you choose determines the prospect that you will get. When uncertainty is present, each alternative can result in a number of possible prospects.

> **Note:** While there might be uncertainty about which prospect you will get, following a decision, each of these prospects is deterministic: It should be described clearly before the decision is made, and so there is no uncertainty about what a prospect entails. You just don't know which one you will get.

**Characterizing a Prospect of a Decision**
The characterization of each prospect requires us to think about:

1.  The decision(s) that we make;
2.  The outcomes of any uncertainties that may characterize this prospect, and
3.  Our future life with this prospect having made those decisions and having received their corresponding outcomes.

Because a prospect is deterministic, you should be able to visualize your life with this prospect to help you better characterize your preferences for it. If you are uncertain about a particular aspect of a prospect, such as whether or not you will like your colleagues in a new job, then you should further divide the prospect of a new job into additional prospects. For example, you might consider two prospects: one where you are in your new job and you like your colleagues, and another where you are in your new job and you do not like your colleagues. Each of these prospects should be deterministic. If they are not, then you can continue to create more prospects by thinking about the different aspects of the prospects of the job.

The main philosophy of this book is to help you think about the best decision alternative (even when uncertainty is present) by reducing the problem into thinking about the deterministic prospects of the decision, your preferences for these prospects, and their likelihood. The bulk of our discussion, therefore, will focus on preferences for prospects, while keeping in mind that the ultimate goal of this discussion is to use these prospects to determine the best decision alternative under uncertainty. We shall discuss how to use the preference, value, and utility of a prospect to determine the best decision alternative when there is uncertainty. We shall not discuss probability-encoding techniques or other aspects of a decision such as framing or generating alternatives, but refer the reader to Howard and Abbas (2015) for a detailed discussion on characterizing the uncertainty about the decision, as well as framing of a decision and other elements and applications of the foundations of decision analysis.

> **Note:** As we continue with our endeavor throughout this book, we must remember that we are not guaranteed to get our most preferred prospect using decision analysis, but we are guaranteed a process and a logic that can guide us and help us make the right decision even when uncertainty prevails.

> This observation often leads people to question the validity of decision analysis, saying things like "Well, if I am not guaranteed a good outcome, then why should I use it?" As a result many people might use arbitrary methods of decision-making justifying them by their simplicity. There is no method of decision-making that will guarantee you a good outcome when faced with uncertainty that is out of your control. Everybody wants a good outcome, but using an arbitrary method of decision-making is not the way to get there. It is because you are not guaranteed a good outcome that you need to rely on a sound decision-making method, especially if you are asked to justify your decision in the case of a bad outcome.

Before we can think about the best decision alternative, it is important to characterize the prospects of the decision alternatives that we are facing. The following examples illustrate how to characterize such prospects.

### Example 1.2  Characterizing Prospects of Purchasing a Lottery Ticket

The characterization of the prospects of a decision to purchase a lottery ticket may include the decision to purchase the ticket (and the price that you will pay for it) in addition to the uncertainty about whether or not you will win the lottery. At first, you might consider only three prospects:

> Prospect A: (Buy the lottery ticket, Win the lottery)
> Prospect B: (Buy the lottery ticket, Do not win the lottery)
> Prospect C: (Do not buy the lottery ticket)

Figure 1.1. depicts these prospects graphically using a decision tree. A square (or rectangular) node represents a decision among alternatives and a circle (or oval) represents an uncertainty.

While these three prospects might be sufficient to provide clarity about this decision for some people, others might wish to further characterize them before they can decide. Winning the lottery might affect your lifestyle, the relations that you have with friends and family, as well as many other factors that you might not have thought about at first. If you have uncertainty about how the prospect of winning the lottery could affect your future life, and if this characterization can affect your decision, then you should create distinctions to help you think about each possible scenario.

The prospect of buying the lottery ticket and winning the lottery can be further divided into two prospects:

> (Buying the lottery ticket, winning the lottery, Friendships remain the same)
> (Buying the lottery ticket, winning the lottery, Friends only care about my money)

Now we have a new characterization of the decision situation as shown in Figure 1.2.

This process can go on. For example, you might add another distinction about whether you will enjoy the publicity or not, as well as many other distinctions. The process ends when you have sufficient clarity to help you make a decision visualize your life with these prospects without the need for further classification.
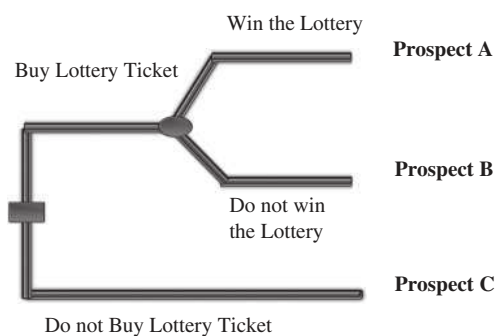
**Figure 1.1.** A decision tree characterizing three prospects of a lottery ticket purchase decision. The square node represents a decision, and the circle node represents an uncertainty.
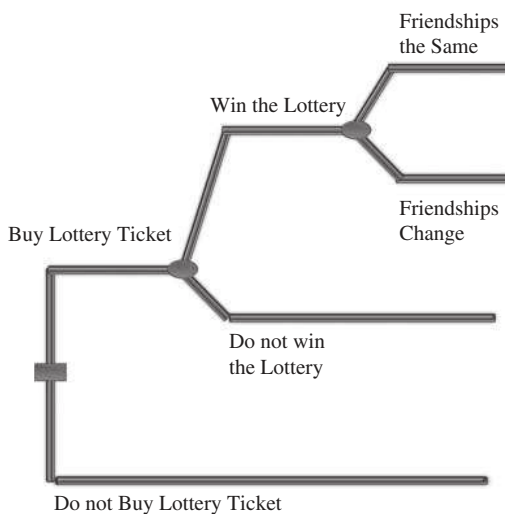


**Figure 1.2.** Further characterization of the prospects of the lottery ticket purchase decision.

We have not defined what we mean by preference, value, and utility of a deterministic prospect, so let us define them in more detail.

## 1.2 PREFERENCE FOR A DETERMINISTIC PROSPECT

Let us now clarify what we mean by a preference statement.

> **Definition**
> **Preference for deterministic prospects:** Preference is a statement about the order of a set of deterministic prospects. This preference order is determined by the decision-maker. Making a preference statement for a set of prospects requires you to order

the prospects in a list (with ties allowed) such that the higher the prospect is in the list, the more preferred it is. The preference statement is also referred to as an ordinal rank of the prospects.

For example, you might be considering three different houses for purchase and you consider several factors such as the view, the cost, the size of the house, and the location. After some thought, you might then say:

> *I prefer to purchase house A to house B to house C to not purchasing any of these houses.*

This statement takes into account the whole purchase prospect for each house and your future life when you own it. In other words, it might express something like "given what I have to pay for the house; the view I will get; my use of the house, and the convenience of the location and the commute time, taking all that into account, I choose to purchase House *A* over House *B* over House *C*."

It will be convenient to use the notation "≻" to make a preference statement. For example, we can write:

> ***House A≻House B≻House C≻Not purchasing any of these houses.***

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**Note:** If there is no uncertainty about the decision or the prospect that you will get, then this statement is sufficient to help you determine the best decision alternative: you simply purchase House A.

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We have not discussed how you come up with a preference statement. We shall provide more discussion on making such preference statements in future chapters. If you give this preference statement to an agent, he can make this purchase decision on your behalf if the price of each house remains the same. But sometimes we need more than just preference statements. We cannot tell from this preference statement, for example, by how much you prefer to purchase House *A* to House *C*. If the owner of a house, say House *B*, offers a major reduction in price, then the agent would not know whether you would still prefer House *A* to House *B* at this discounted price. This is why preference statements alone are not always sufficient, and where the need for a value statement comes into play.

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### 1.3 VALUE OF A DETERMINISTIC PROSPECT

Having defined the preference for a set of prospects, and illustrated some of the limitations of having a preference statement alone, particularly when monetary purchases are made, let us now define the value of a prospect. First, we recall that alternatives under consideration must be feasible. Therefore, you own the alternatives under consideration, and a prospect is one of possible states of the world that you could own following your decision.

> **Definition**
> **Value of a deterministic prospect:** The value of a deterministic prospect is the amount that makes you just indifferent to giving up that prospect if you owned it. The value of a prospect is the personal indifference selling price of the prospect.

A value statement for a prospect requires some value measure that is defined on an absolute scale. The value statement is usually expressed in dollars, but it can also be expressed in terms of any other value measure where more is strictly preferred to less.

> **Note: Value of an Item within the Prospect:** Sometimes an outcome of a decision could be getting an item, such as a free ticket to an opera. When we talk about the personal indifference selling price, we may simply talk about the personal indifference selling price of the ticket. By that we mean the price that makes you just indifferent to the prospect of having the ticket with your current life, and the prospect of not having the ticket with your current life, in addition to some monetary amount that is equal to the personal indifference selling price of the ticket. The personal indifference buying price of the ticket is the amount of money that makes you just indifferent to buying back the ticket given your current life situation.
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> Around a cycle of ownership (where you buy and sell an item at your indifference buying and selling prices), the personal indifference buying price is equal to the personal indifference selling price of a prospect.

An example of a value statement for an item is

*I value prospect A, of owning a mountain bike at my current life situation, by $800.*

This does not mean that you value the prospect of your life, health, and wealth with the bike by $800. It means that if you made a decision, and you ended up with prospect *A* of having the mountain bike at your current state, then you would be just indifferent to keeping it or giving it up and receiving $800 with all else being the same except that you have no bike. Furthermore, if you sold the bike at $800, then you would be just indifferent to buying it back immediately for $800. This is referred to as the cycle of ownership.

Another example of a value statement for a house is

*I value house A by $1 million, house B by $600,000, and house C by $400,000.*

This statement is the value of each house to you at your current state. It means that if you owned house *A*, then you are just indifferent to keeping it or selling it for $1 million. It also means that if you sold house *A* for $ 1 million, then you are just indifferent to

1. Purchasing house *A* back and having $ 1 million less in your bank account, or
2. Not purchasing house *A* and keeping the $1 million.

The value statement also determines the amount by which you prefer one prospect over another. For example, this value statement also means that if you owned House *B,* then you would be just indifferent to paying an extra $400,000 and giving

up the house in exchange for getting House *A*. Similarly, if you get House *A* for $1 million, then you would be indifferent to getting House *B* and receiving $400,000 (the difference). Value statements can of course change with time, wealth, or information. For example, you might learn that House *A* is in a flood zone and so you will need to evacuate it frequently or buy excessive flood insurance. Your value for House *A* might then decrease.

> **Note:** A value statement is stronger than a preference statement, because you can infer the preference order of the prospects from their value: The higher the value, the more preferred is the prospect. But, as we discussed, you cannot infer value from a preference statement. You can merely assert that one has a higher value than another from a preference statement, and if two prospects are equally preferred, then they must have the same value.

With a value statement, the agent can now answer on your behalf deterministic questions about the house for a given price. For example, if the seller offers House *A* for $1 million and House *B* for $600,000, the agent will know that you are just indifferent between the two deals. But if the seller offers House *B* for $400,000, then the agent will know to purchase House *B* on your behalf. The situation is now clear for the agent, provided there is no uncertainty.

## 1.4 UTILITY OF A DETERMINISTIC PROSPECT

When uncertainty is present, value statements might not be sufficient to determine the best decision. For example, suppose that the agent pays $1 million on your behalf to get House *A*, and then the seller offers him an exotic deal:

"For $1 million, you can either keep House A, or you can get an investment."

With the sudden perplexed look on your agent's face, the seller explains the investment:

"The investment will pay either $3 million or $100,000 with equal probability."

The seller says that a decision needs to be made instantaneously. Should the agent choose to invest in this deal on your behalf or keep house A?

Figure 1.3 depicts this decision graphically.

The agent cannot make this decision on your behalf using only the value statement you have provided because he does not know your taste for risk (your risk attitude). He would like to win the $3 million to surprise you, but he is worried that he might get the $100,000 instead (which would be an unpleasant surprise). To help with this decision, we need to determine the utility values of the prospects involved.

The utility of a prospect only has a meaningful interpretation when it is expressed in terms of two other prospects, one that is more preferred and another that is less preferred than the prospect under consideration.