

Index of Categories and Functors

Categories are printed in typewriter font

Categories

BDLat, bounded distributive lattices, **586**
 BDSLat, bounded distributive
 join-semilattices, **99**
 BiFr, bi-frames, **588**
 BiLocSp, bi-localic spaces, **313**
 BoolAlg, Boolean algebras, **19**
 BoolSp, Boolean spaces, **16**
 CReg, completely regular spaces, **289**
 Comp, compact spaces, **289**
 Fr^{op}, opposite category of Fr, **299**
 Fr, frames, **588**
 HeytAlg, Heyting algebras, **278**
 ISprings, category of indexed springs, **483**
 InvFr, inverse frames, **588**
 InvLocSp, inverse localic spaces, **313**
 LocSp, localic spaces, **313**
 PoSets, posets = partially ordered sets, **580**
 Priestley, Priestley spaces, **33**
 QoSets, quasi-ordered sets, **580**
 RedRings, reduced rings, **439**
 Rings, rings (commutative, unital), **67**
 Sets, sets, **55**
 Spec, spectral spaces, **11**
 Springs, category of springs, **483**
 T₀Sob, sober T₀-spaces, **384**
 T₀Top, T₀-spaces, **138**
 ToSets, totally ordered sets, **580**
 Top, topological spaces, **11**
 U, category of spaces with indeterminates,
 473
 semiSpec, semi-spectral spaces, **99**

Functors

λ : Rings \rightarrow BDLat, reticulation functor,
 431
 \mathcal{O} : Top \rightarrow BDLat, **588**
 \mathcal{O} : Top \rightarrow Fr, **588**
 \mathcal{K} : Spec \rightarrow BDLat, **11**
 con: Spec \rightarrow BoolSp, **22**
 inv: PoSets \rightarrow PoSets, BDLat \rightarrow BDLat,
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 inv: Spec \rightarrow Spec, **25**
 red: Rings \rightarrow RedRings, **439**
 PrimF: BDLat \rightarrow Spec, **91**
 PrimI: BDLat \rightarrow Spec, **91**
 Spec: BDLat \rightarrow Spec, **81**
 Spec: Rings \rightarrow Spec, **70**
 Sper: Rings \rightarrow Spec, **73**
 ba: BDLat \rightarrow BoolAlg, **94**
 P: T₀Top \rightarrow PoSets, **408**
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Symbol Index

The first entries contain symbols with no or ambiguous alphabetical value, sorted in order of appearance. After these, the symbols are ordered alphabetically.

Order, see Appendix: The Poset Zoo, 579

Symbols without Alphabetic Value, in Order of Appearance

- \rightsquigarrow , specialization, 3
- \rightsquigarrow_τ , specialization (in topology τ), 3
- X_{con} , constructible (= patch) topology of X , 16
- S^{con} , constructible closure of S , 16
- f_{con} , map induced by f on patch spaces, 21
- X_{inv} , inverse topology on X , 23
- $\overline{C}^{\text{inv}}$, inverse closure of C , 23
- f_{inv} , map induced by f on inverse spaces, 25
- $\llbracket \widehat{T} = k \rrbracket$, set of maps in 2^S with value k on $T \subseteq S$, 54
- $\llbracket \widehat{s} = k \rrbracket$, set of maps in 2^S with value k at $s \in S$, 54
- X^{\rightsquigarrow} , graph of specialization of X , 59
- $X \times_S Y$, fiber product (= pull-back) of X and Y over S , 61
- $X \times_S^{\rightsquigarrow} Y$, specialization fiber product of X, Y over S , 63
- $X_1 \oplus X_2$, topological sum of two spaces, 65
- $\bigoplus_{i \in I} X_i$, topological sum (= coproduct) of X_i , 65
- Δ , symmetric difference in Boolean algebra, 71
- A -isomorphism between morphisms in a category, 76
- A -morphism between morphisms in a category, 76
- $\neg = \neg_A$, complementation map in Boolean algebra A , 95
- 0-dimensional space, 122
- $[\cdot]_E$, equivalence class for E , 134
- \equiv_f , lattice filter congruence, 159
- \equiv_i , lattice ideal congruence, 159

- $[\cdot]_i, [\cdot]_f$, congruence classes in lattices, 159
- X/R , spectral quotient of X modulo R , 170
- X/T_0R , T_0 -quotient of X modulo R , 170
- $X/\text{set } E$, 170
- R^{sat} , saturation of relation R , 174
- \leq_R , quotient order on $X/\text{set } E$, 176
- \ll , way below (for subsets of poset), 207
- $\cdot \rightarrow \cdot$, implication (Heyting algebra), 269
- $\sim a$, pseudo-complement of a , 269
- $\langle S \rangle_L$, sub-frame generated by $S \subseteq L$, 301
- $\coprod_{i \in I} X_i$, spectral coproduct of X_i , 329
- Y^{clcon} , set of closed and constructible points in spectral space Y , 397
- $\mathfrak{a} \triangleleft A$, \mathfrak{a} ideal of ring A , 421
- A_S , ring of fractions with denominators in S , 435
- \mathfrak{a}_S , ideal in ring of fractions A_S generated by canonical image of ideal \mathfrak{a} , 435
- A_p , localization of A at prime ideal p , 436
- \mathfrak{a}_p , ideal generated in localization A_p by canonical image of ideal $\mathfrak{a} \triangleleft A$, 436
- $b : c$, quotient of ideals b, c in a ring, 439
- \leq_α , order determined by $\alpha \in \text{Sper}(A)$ on field $\text{qf}(A/\text{supp}(\alpha))$, 490
- $\neg\Delta = \{\neg\delta \mid \delta \in \Delta\}$, 542
- \models , 542
- X^{\leq} , graph of order relation \leq on X , 579
- P^{max} , set of maximal elements of poset (P, \leq) , 580
- P^{min} , set of minimal elements of poset (P, \leq) , 580
- P_{inv} , inverse of poset P , 580
- $X^{\text{max}} = (X, \rightsquigarrow_\tau)^{\text{max}}$, set of closed points of the topological space (X, τ) , 581
- $X^{\text{min}} = (X, \rightsquigarrow_\tau)^{\text{min}}$, set of generic points of the topological space (X, τ) , 581

\leq_{inv} , inverse of order relation \leq , 580
 \rightsquigarrow , proper specialization, 580
 $Q \leq p, \forall q \in Q : q \leq p$, 581
 $\perp = \perp_P$, smallest element of poset P , 581
 $\top = \top_P$, largest element in poset P , 581
 $p \leq Q, \forall q \in Q : p \leq q$, 581
 p^\downarrow , principal down-set of p , 581
 p^\downarrow , 581
 p^\uparrow , principal up-set of p , 581
 p^\uparrow , 581
 Q^\downarrow , down-set generated by Q , 582
 Q^\uparrow , up-set generated by Q , 582
 $\langle p \rangle$, graph component of vertex p in a graph, 582
 $[\cdot, \cdot]$, closed interval = convex hull of two points, 583
 $\vee Q = \text{sup}(Q)$, supremum of Q , 585
 $\wedge Q = \text{inf}(Q)$, infimum of Q , 585
 \vee , join operation on lattice, 585
 \vee -semilattice = join-semilattice, 585
 \vee_{inv} , join operation in inverse of a \wedge -semilattice, 585
 \wedge , meet operation on lattice, 585
 \wedge -semilattice = meet-semilattice, 585
 \wedge_{inv} , meet operation in inverse of a \vee -semilattice, 585
 \vee -irreducible = join-irreducible, 586
 \wedge -irreducible = meet-irreducible, 586

Alphabetic Symbol List

$\mathbb{1}$, one-element space, 8
2, Boolean algebra with two elements, 15
2, Sierpiński space, 8
 $\mathfrak{3}$, = \mathfrak{n} for $n = 3, 9$
 $\mathcal{A}(E; L)$, closed elements (of completion E of L), 303
 A_{red} , reduced ring associated with ring A , 439
 A^\times , group of units in ring A , 421
 $\mathcal{A}(X)$, set of closed sets of X , 2
 $\text{ba}(L)$, Boolean envelope of L , 93
 ba_L , embedding $L \rightarrow \text{ba}(L)$, 93
 $\text{ba}(\varphi)$, extension of lattice morphism φ to Boolean envelope, 94
 $\beta_X : X \rightarrow \beta X$, Stone–Čech compactification of the completely regular space X , 123
 $\text{CB}(X)$, Cantor–Bendixson rank of space X , 114
 $\text{CB}_X(x) = \text{CB}(x)$, Cantor–Bendixson rank of $x \in X$, 114

χ , characteristic functions map $\mathfrak{P}(X) \rightarrow 2^X$, 55
 $\text{Clop}(X)$, set of clopen subsets of X , 14
 $\text{Cong}(L)$, set of congruences of lattice L , 76
 $\text{conv}(Q) = \text{conv}_P(Q)$, convex hull of Q in P , 583
 con_X , identity $X_{\text{con}} \rightarrow X$, 21
 $\text{Coz}(X)$, lattice of cozero sets in topological space X , 287
 $C(X, \mathbb{R})$, ring of continuous functions on topological space X with values in \mathbb{R} , 287
 $D(\cdot)$, basic opens of Zariski topology, 67
 $D(\cdot) = D_L^F(\cdot)$, basic opens of $\text{PrimF}(L)$, 89
 $D(\cdot), D_L^I(\cdot)$, basic opens of $\text{PrimI}(L)$, 90
 $D(\delta)$, subbasic open sets of the space of Δ -types, 544
 δX , set of non-isolated points of X , 114
 $\delta^\alpha X, \delta^\infty X$, iterations of δX , 114
 $\Delta(X)$, diagonal of X , 59
 Δ_X , diagonal $X \rightarrow X \times X$, 59
 Δ_A , positive quantifier-free $\mathcal{L}(A)$ -sentences, 555
 $\text{diag}(A)$, diagram of structure A , 555
 $\text{diag}^+(A)$, positive diagram of structure A , 562
 $\mathcal{E}(S)$, set of equivalence relations on S , 75
 $\mathcal{E}(X)$, canonical extension of X , 304
 \mathbb{F}_2 , field with two elements, 71
 $\mathfrak{f}(a)$, principal filter generated by a , 83
 f_{inv} , map f between posets considered as a map between the inverse posets, 580
 $\text{Fml}(\mathcal{L})$, formulas of language \mathcal{L} , 542
 $\Gamma(f)$, graph of map f , 59
 Γ^* , finite words, 240
 $\Gamma^{\leq n}$, words of length at most n , 240
 $\Gamma^{< n}$, words of length $< n$, 240
 Γ_X , canonical map $X \rightarrow \Gamma(X)$, 199
 $\Gamma(X)$, set of connected components of X , 199
 $\text{Gen}(A)$, set of generalizations of elements of A , 3
 $\mathcal{H}_{\mathcal{L}(A)}$, Harrison topology on $\mathcal{L}(A)$, 500
 $\text{Hom}_{\text{BDLat}}(L, \mathbf{2})$, set of bounded distributive lattice homomorphisms of L to $\mathbf{2}$, 79
 $H_R^{\geq 0}(a_1, \dots, a_r)$, basic opens of real spectrum, 72
 $\text{ht}(\mathfrak{a})$, height of ideal \mathfrak{a} in a ring, 452
 $\mathcal{I}_{\text{fin}}(A)$, finitely generated ideals in ring A , 422
 $\mathcal{I}^{\text{rad}}(A)$, set of radical ideals of A , 421
 $\mathcal{I}_{\text{fin}}^{\text{rad}}(A)$, radical ideals generated by finite subsets of ring A , 422
 $\mathcal{I}(R)$, set of ideals of ring R , 74

- $I(P)$, set of ideals of poset P , 588
 $i \triangleleft P$, ideal in a poset, 588
 $i(a)$, principal ideal generated by a in a lattice, 83
 $\inf(Q) = \bigwedge Q$, infimum of Q , 585
 $\text{IntAlg}(L)$, interval algebra of L , 98
 $\text{int}_X(Y)$, interior of Y in space X , 121
 ι_p , localizing homomorphism at p , 436
 ι_S , canonical homomorphism to a ring of fractions, 435
 $k(\alpha)$, real closure of field $\kappa(\alpha)$, 515
 $\kappa(\alpha)$, ordered field $\text{qf}(A/\text{supp}(\alpha), \leq_\alpha)$, for $\alpha \in \text{Sper}(A)$, 490
 $\kappa(v)$, valued field determined by valuation v of a ring, 569
 $\text{Kdim}(A)$, Krull dimension of ring A , 114
 $\text{Kdim}(X)$, Krull dimension of space X , 113
 Ker^F , transfer map
 $\text{Hom}_{\text{BDLat}}(L, \mathbf{2}) \rightarrow \text{PrimF}(L)$, 89
 Ker^I , transfer map
 $\text{Hom}_{\text{BDLat}}(L, \mathbf{2}) \rightarrow \text{PrimI}(L)$, 89
 $\mathcal{K}(X)$, set of compact elements in a poset X , 207
 $\overset{\circ}{\mathcal{K}}(X)$, set of quasi-compact opens of X , 3
 $\overset{\circ}{\mathcal{K}}(\tau)$, set of quasi-compact opens (in topology τ), 3
 $\overset{\circ}{\mathcal{K}}(f)$, restriction of $\mathfrak{P}(f)$ to $\overset{\circ}{\mathcal{K}}(\cdot)$, 11
 $\mathcal{K}(X)$, set of constructible sets of X (clopens of X_{con}), 16
 $\mathcal{K}(f)$, map $\mathcal{K}(Y) \rightarrow \mathcal{K}(X)$ induced by $f: X \rightarrow Y$, 21
 $\overline{\mathcal{K}}(X)$, set of closed constructible sets of X , 16
 $\overline{\mathcal{K}}(f)$, map $\overline{\mathcal{K}}(Y) \rightarrow \overline{\mathcal{K}}(X)$ induced by $f: X \rightarrow Y$, 21
 $\overline{\overline{\mathcal{K}}}(f)$, map $\overline{\overline{\mathcal{K}}}(Y) \rightarrow \overline{\overline{\mathcal{K}}}(X)$ induced by $f: X \rightarrow Y$, 25
 $\mathcal{L}(A)$, denotes either $\text{Qmod}(A)$, $\text{Preord}(A)$, or $\text{Satpre}(A)$, 500
 λ_A , reticulation map of ring A , 429
 λ_i , homomorphism $L \rightarrow \mathbf{2}$ defined by prime filter \mathfrak{f} , 89
 λ_i , homomorphism $L \rightarrow \mathbf{2}$ defined by prime ideal i , 90
 Λ , empty word, 240
 $\overset{\circ}{\Lambda}_X$, Stone representation, 85
 $\overline{\Lambda}_X$, Stone representation, 85
 $\text{Lb}(Q)$, set of lower bounds of Q , 581
 $\mathcal{L}(C)$, language \mathcal{L} extended by new constants, 542
 L/\mathfrak{f} , factor lattice modulo filter $\mathfrak{f} \subseteq L$, 159
 L/i , factor lattice modulo ideal $i \subseteq L$, 159
 $\ell\Delta$, closure of set Δ of sentences under conjunction and disjunction, 542
 $\ell(S)$, length of word s , 240
 $\varinjlim X_i$, projective limit of X_i , 63
 $\overline{\text{LocCl}}(X)$, set of locally closed points of X , 135
 \mathcal{L}_{or} , language for ordered rings, 519
 (M, α) , expansion of structure A determined by homomorphism α , 562
 $M \cdot A = (M)$, ideal generated by subset M in ring A , 421
 $(M) = (M)_A = M \cdot A$, ideal generated by subset M in ring A , 421
 $\mu(M)$, multiplicative set generated by M , 421
 $\mu_s(M)$, saturated multiplicative set generated by M , 421
 N_* , inclusion of the image of nucleus N in the frame, 302
 N^* , nucleus N with restricted codomain, 302
 $\mathcal{N}(G)$, set of normal subgroups of group G , 75
 $\text{nil}(A)$, nilradical in ring A , 421
 $\mathbb{N} = \{1, 2, \dots\}$, natural numbers, 583
 $\mathbb{N}_0 = \{0\} \cup \mathbb{N}$, non-negative integers, 583
 $N(O)$, open regularization of $O \in \mathcal{O}(X)$, 129
 \mathbf{n} , set $\{0, 1, \dots, n-1\}$, naturally ordered, 583
 \mathfrak{n} , spectral space with elements $0, 1, \dots, n-1$ in natural order, 9
 $N_{Y, X}$, nucleus associated with localic subspace Y of X , 319
 $\mathcal{O}(E; L)$, open elements (of completion E of L), 303
 $\mathcal{O}(f)$, restriction of $\mathfrak{P}(f)$ to $\mathcal{O}(\cdot)$, 11
 $\omega = \mathbb{N}_0$, smallest infinite ordinal, 584
 $\overline{N}(A)$, closed regularization of $A \in \mathcal{A}(X)$, 130
 $\mathcal{O}(X)$, set of open subsets of X , 2
 $\mathcal{O}_x(X)$, filter of open neighborhoods of $x \in X$, 171
 $\mathfrak{P}_{\text{fin}}(I)$, set of finite subsets of I , 63
 $\mathfrak{P}(f)$, power set dual of f , 11
 π_α , canonical map $A \rightarrow \kappa(\alpha)$ for $\alpha \in \text{Sper}(A)$, 490
 π_α , canonical homomorphism from a ring to the factor ring modulo α , 435
 $\pi_{\mathfrak{f}}: L \rightarrow L/\mathfrak{f}$, canonical quotient homomorphism, 159
 $\pi_i: L \rightarrow L/i$, canonical quotient homomorphism, 159
 $\text{Preord}(A)$, set of preorders of ring A , 499

- $\text{PrimF}(L)$, set of prime filters of bounded distributive lattice L , 89
 $\text{PrimF}(\varphi)$, spectral dual of lattice morphism φ , 91
 $\text{PrimI}(L)$, set of prime ideals of bounded distributive lattice L , 89
 $\text{PrimI}(\varphi)$, spectral dual of lattice morphism φ , 91
 \mathbb{P} , set of prime numbers, 70
 $\text{pt}_{\text{bi}}(Y)$, set of bi-localic points of Y , 315
 $\text{pt}_{\text{inv}}(Y)$, set of inverse localic points of Y , 315
 $\text{pt}(Y)$, set of localic points of Y , 315
 $\mathfrak{P}(X)$, power set of X , 2
 $\text{qf}(A)$, quotient field of domains A , 436
 $\text{Qmod}(A)$, set of quadratic modules of ring A , 499
 qR, T_0 , canonical map $X \rightarrow X/T_0 R$ to T_0 -quotient, 170
 $qR : X \rightarrow X/R$, canonical spectral quotient, 170
 $q : X \rightarrow X/E, T_0$ -reflection of X , 171
 R^{-1} , inversion of relation R , 176
 RCF = real closed field, 518
 $RC(X)$, set of regular closed sets in topological space X , 130
 $\text{Rk}^\alpha(X)$, elements in a poset with rank $\geq \alpha$, 116
 $\text{Rk}(X)$, rank of a poset or of a spectral space, 116
 $\text{rk}(x) = \text{rk}_{(X, \leq)}(x)$, rank of point x in poset (X, \leq) , 116
 $RO(X)$, set of regular open sets in space X , 130
 $r = r_X$, retraction of normal spectral space X onto X^{\max} , 283
 \mathbf{R}_X , localic coreflection map, 404
 $\text{Satpre}(A)$, set of saturated preorders of ring A , 499
 S_{d} , S with the discrete topology, 415
 $S^\Delta(T)$, Δ -types of T , 544
 $\text{Sen}(\mathcal{L})$, sentences of language \mathcal{L} , 542
 $\mathbf{S}(f)$, spectral reflection of $f : X \rightarrow X$, 372
 $\sigma(X, \leq) = \sigma(X) = \sigma$, Scott topology on poset (X, \leq) , 212
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 $S_{\text{loc}}(Y)$, set of localic subspaces of Y , 315
 $S_n(T)$, space of n -types of theory T , 548
 \mathbf{Sob}_X , sobrification map of topological space X , 384
 $\mathbf{Sob}(X)$, sobrification of space X , 384
 $\mathbf{Sob}(f)$, sobrification of continuous map f , 385
 $\text{Spec}(L)$, spectrum of bounded distributive lattice L , 81
 $\text{Spec}_p(A)$, p -adic spectrum of ring A , 572
 $\text{Spec}(\varphi)$, spectral dual of lattice homomorphism φ , 81
 $\text{Spec}(\varphi)$, map between Zariski spectra induced by a ring homomorphism φ , 70, 434
 $\text{Spec}_{\text{re}}(\varphi)$, map between real prime ideals induced by a ring homomorphism φ , 507
 $\text{Spec}_{\text{re}}(R)$, space of real prime ideals of R , 505
 $\text{Spec}(R)$, Zariski spectrum (or prime spectrum) of ring R , 67
 $\text{Sper}(\varphi)$, real spectral dual of ring morphism φ , 73
 $\text{Sper}(R)$, real spectrum of ring R , 72, 491
 $\text{Spez}(A)$, set of specializations of elements of A , 3
 $\text{Spv}(A)$, valuation spectrum of ring A , 569
 $\sqrt{(M)}$, radical ideal generated by M , 421
 S^* , one-point compactification of discrete space S , 44
 $S(T)$, set of types of T , 544
 supp , support of (prime) cone, 71, 489
 $\text{supp}(v)$, support of valuation v , 569
 $\text{sup}(Q) = \bigvee Q$, supremum of Q , 585
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 \mathbf{S}_X , reflection $X \rightarrow \mathbf{L}(X)$, 372
 $\mathbf{S}(X)$, spectral reflection of X , 372
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 $\tau^U(P, \leq)$, fine lower topology on poset (P, \leq) , 589
 $\tau^\ell(P, \leq)$, coarse lower topology on poset (P, \leq) , 589
 $\tau''(P, \leq)$, coarse upper topology on poset (P, \leq) , 589
 τ_\leq , spectral topology with specialization \leq , 31
 T_D -space, all points are locally closed, 135
 $\text{Th}(M)$, first-order theory of structure M , 542
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 $T\text{-Spec}_{\text{ex}}(A)$, space of expansions of A , 555
 $T\text{-Spec}(A)$, T -spectrum of structure A , 562
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 $\text{Ub}(Q)$, set of upper bounds of Q , 581
 $\mathcal{U}(G)$, set of subgroups of group G , 75

Symbol Index

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|---|--|
| Up(X), family of up-sets of (X, \leq) , 75 | $V(\delta)$, subbasic closed sets for the space of Δ -types, 544 |
| $V(\cdot)$, basic closed sets of Zariski topology, 69 | $Z(X)$, lattice of zero sets in topological space X , 287 |
| $V(\cdot) = V_L^F(\cdot)$, basic closed sets of $\text{PrimF}(L)$, 89 | |
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