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Introduction to Signals

Learning Objectives

- Definition of a signal.
- CT and DT signals.
- Sampling theorem in time domain.
- Aliasing in frequency domain.
- Interpolation formula – a sync function.
- Recovery of signal from signal samples.
- Phase reversal of the recovered signal.
- Anti-aliasing filter.

We discuss the basic definitions of analog/continuous time (CT) and discrete time (DT) signals in this chapter. We need to understand the basics of discrete time signals, i.e., the sampled signals. The theory of sampled signals is introduced in this chapter. The analog signal is first interfaced to a digital computer via analog to digital converter (ADC). ADC consists of a sampler and a quantizer. We will mainly discuss the sampler in this chapter. The analog signal, when sampled, gets converted to discrete time (DT) signal. Here, the time axis is digitized with a constant sampling interval T. The inverse of T is the sampling frequency. The sampling frequency must be properly selected for faithful reconstruction of the analog signal.

1.1 Introduction to Signals

Any physical quantity that carries some information can be called a signal. The physical quantities like temperature, pressure, humidity, etc. are

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continuously monitored in a process. Usually, the information carried by a signal is a function of some independent variable, for example, time. The actual value of the signal at any instant of time is called its amplitude. These signals are normally plotted as amplitude vs. time graph. This graph is termed as the waveform of the signal. The signal can be a function of one or more independent variables. Let us now define a signal.

Definition of a signal A signal can be defined as any physical quantity that varies with one or more independent variables.

Let us consider temperature measurement in a plant. The measured value of temperature will be its amplitude. This temperature changes from one instant of time to another. Hence, it is a function of time, which is an independent variable, as it does not depend on anything else. Temperature can be measured at two different locations in a plant. The values of temperature at two different locations may be different. Hence, the temperature measured depends on the time instant and also on the location. We can say that temperature is a function of two independent variables, namely time and location in a plant. This temperature can take on any continuous value, like 30.1, 30.001, 31.3212, etc. The time axis is also continuous i.e. the temperature is noted at each time value. The signal is then called continuous time continuous amplitude (CTCA) signal.

Definition of a CTCA/Analog signal When the signal values are continuous and are noted at each continuous time instant (the independent variable) the signal is said to be a CTCA signal. Analog signal is shown in Fig. 1.1.

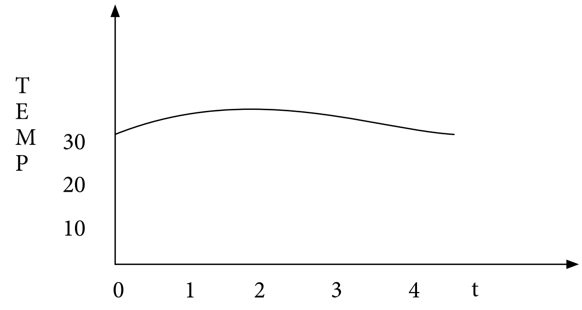

Fig. 1.1 Plot of the analog signal

If we now decide to note the temperature after one second, for instance, we will get continuous values of temperature at discrete instants. We will now define Discrete Time Continuous Amplitude (DTCA) signal.

Definition of a DTCA/DT signal When the signal values are continuous and are noted at each discrete time instant, namely the independent variable, the signal is said to be a DTCA signal. This DT signal is shown in Fig. 1.2.

Definition of a DTDA or Digital signal A digital signal, also called as DTDA signal, is obtained when amplitude axis is also discretized.

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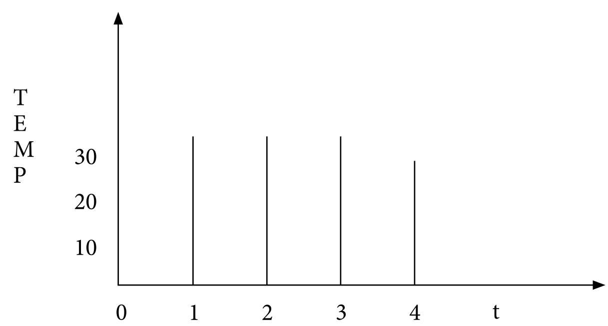

Fig. 1.2 Plot of DT signal

1.2 Sampling Theorem

When any signal is to be sampled, one must have some prior knowledge about the signal, namely, its frequency contents. This information is generally known to us. For example, a telephone grade speech signal contains useful information only up to 3.4 kHz. The signal may then be filtered to remove the frequency contents, if any, above 3.4 kHz to ensure that the signal bandwidth is within 3.4 kHz. We will now define *Sampling Theorem*. Consider any band-limited signal of bandwidth W.

Theorem When any energy signal that is band-limited with a bandwidth W is sampled using a sampling frequency $\geq 2W$, it is possible to reconstruct signal from signal samples.

The sampling rate of $2W$ is called Nyquist rate. The sampling frequency selected is denoted as F_S. We will denote the Nyquist frequency $F_S/2$ as F_N.

When an analog signal is sampled, the most important factor is the selection of the sampling frequency. In simple words, *Sampling Theorem* may be stated as, "Sampling frequency is appropriate when one can recover the analog signal back from the signal samples." If the signal cannot be faithfully recovered, then the sampling frequency needs correction. Let F_S denote the sampling frequency. We will consider two different cases to understand the meaning of proper sampling.

Concept Check

- What is sampling theorem in time domain?
- How do we find the Nyquist rate?

1.3 Sampling of Analog Signals (Case I)

Consider a signal with a bandwidth of 10 Hz. We sample it using $F_S = 100$ Hz. That is

$$F_s > 2W(2 \times 10\text{Hz})\,\text{Hz}$$

We will concentrate on the highest frequency component of the signal, namely, $F = 10$ Hz. There will be 10 samples obtained per cycle of the waveform.

To get a deeper insight into the sampling process, we introduce the theoretical concept of an impulse train. In the time domain, the sampled signal is obtained by multiplying the sinusoidal waveform with a train of impulse with separation of $T_s = 1/F_s = 1/100$, as shown in Fig. 1.3.

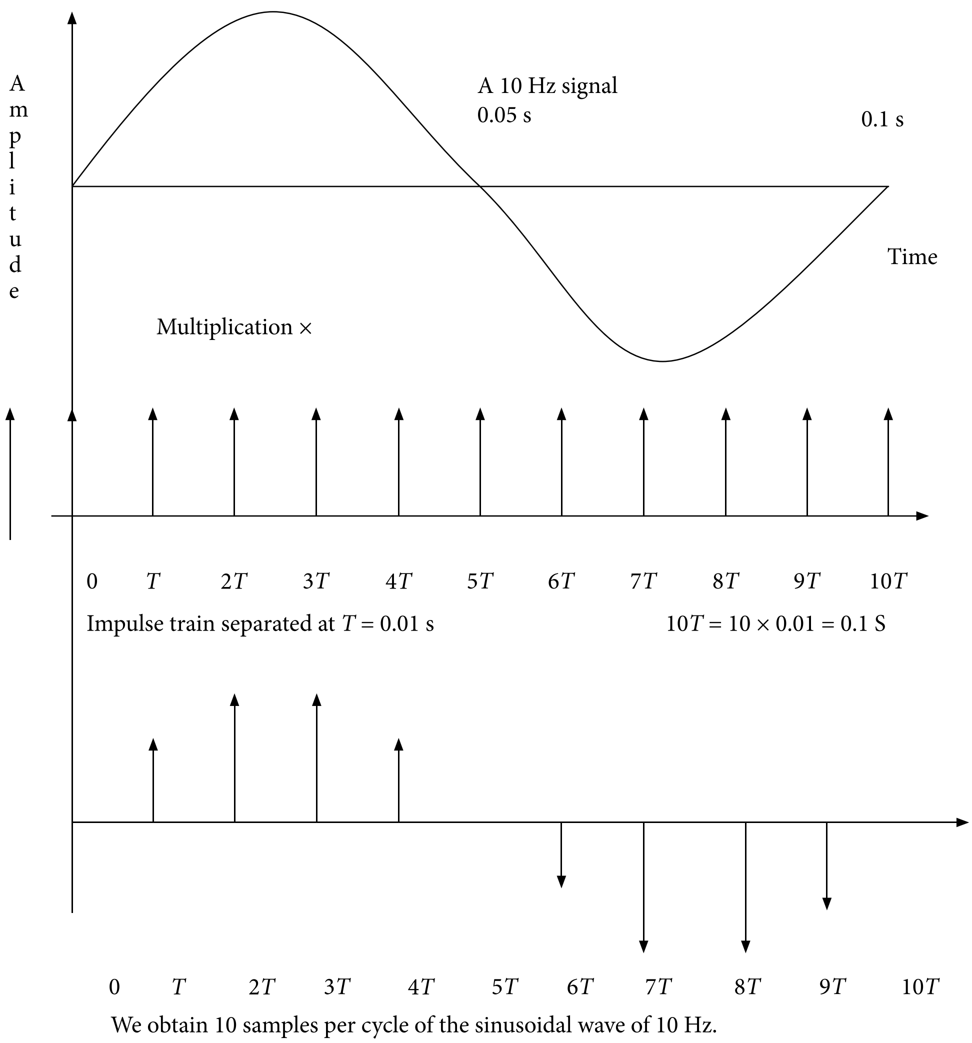

Fig. 1.3 Samples of the wave obtained by multiplication of the wave with an impulse train

Multiplication of two signals in time domain is equivalent to convolution of their transforms in the frequency domain. (This is discussed in detail in Chapter 5.) When the signal is sampled using a sampling frequency of

10F = 10*10 = 100 Hz, the signal of frequency F gets multiplied by a train of impulses with a separation interval of $T = 1/F_S$, that is, 1/100. In the frequency domain, the Fourier Transform (FT) of the signal and FT of the impulse train will convolve with each other. The FT of a sinusoid with a 10 Hz frequency is a spectral line at 10 Hz, that is, at $0.1F_S$. The FT of an impulse train is again the impulse train with a separation interval of $1/T_S$, that is, F_S (100 Hz). Hence, the convolution of two transforms results in a signal spectrum replicated around zero frequency and frequencies that are multiples of 100 Hz, as shown in Fig. 1.4.

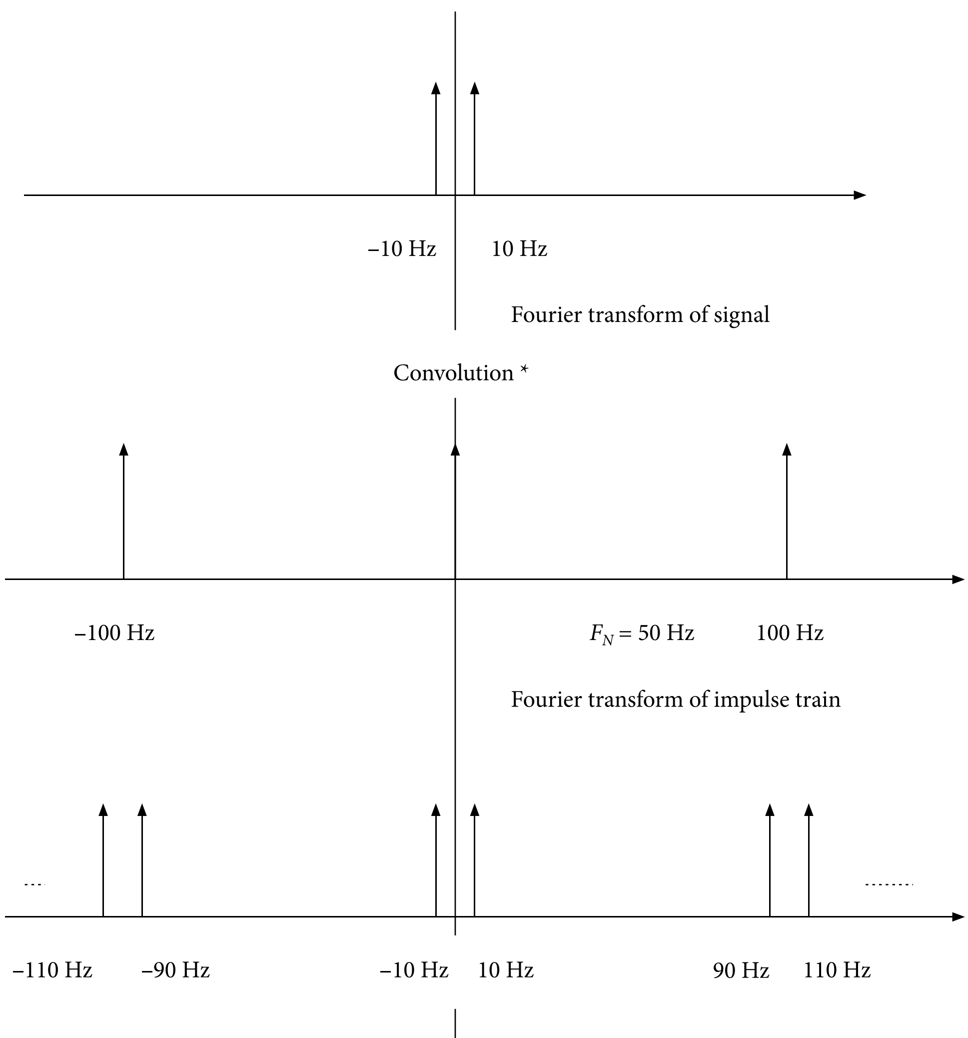

Fig. 1.4 Convolution of two transforms (FT of signal and FT of impulse train)

Concept Check

- What must be the sampling frequency if the signal bandwidth is 50 Hz?
- What is the Fourier transform of the impulse train?
- What is the Fourier transform of a sinusoid with frequency of 40 Hz?

1.4 Recovery of Analog Signals (Case I)

The convolved spectrum in Fig. 1.5 shows replicated copies of the signal spectrum around zero frequency and multiples of the sampling frequency. The replicas are well separated, that is, there is no mixing of adjacent replicas. The signal spectrum around zero frequency can be easily isolated if the convolved spectrum is passed via a rectangular window between –50 Hz and 50 Hz ($-F_N$ to $+F_N$), as shown in Fig. 1.5. The resulting isolated signal spectrum is then inverse Fourier transformed to recover the original signal of 10 Hz.

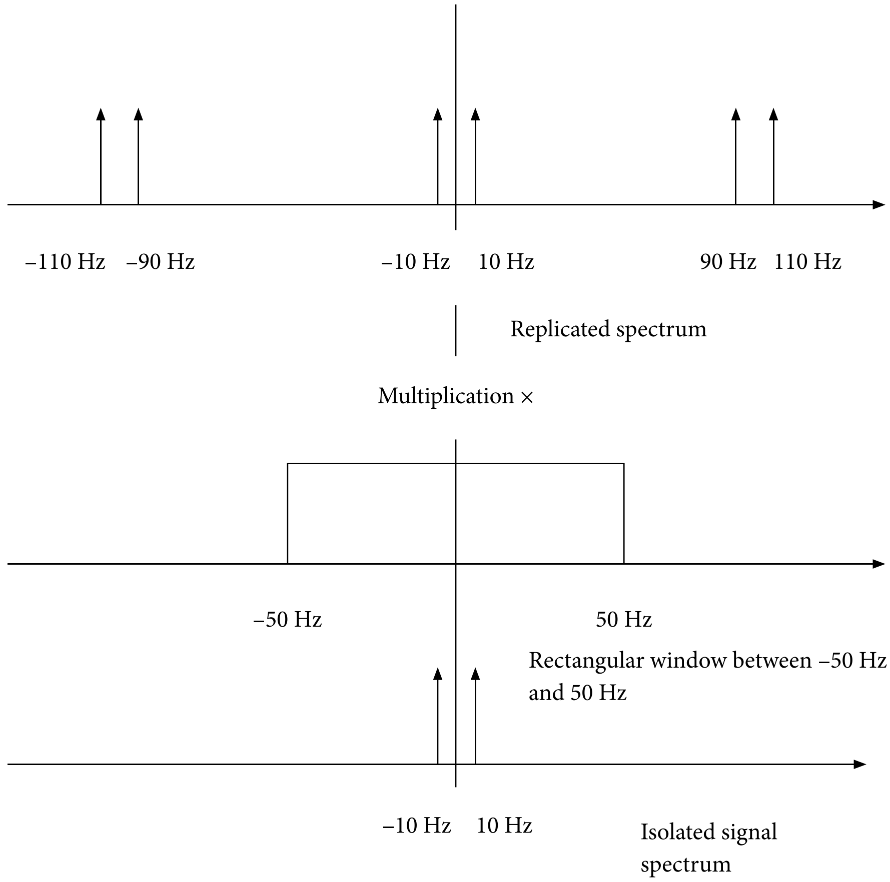

Fig. 1.5 Isolation of signal spectrum by passing the convolved spectrum via a rectangular window

There is a multiplication of convolved spectrum and a rectangular window in Fourier domain, as shown in Fig. 1.3. This is equivalently a convolution of the signal samples with the impulse response of the ideal (brick wall) low-pass filter, that is, a sync function, as shown in Fig. 1.6.

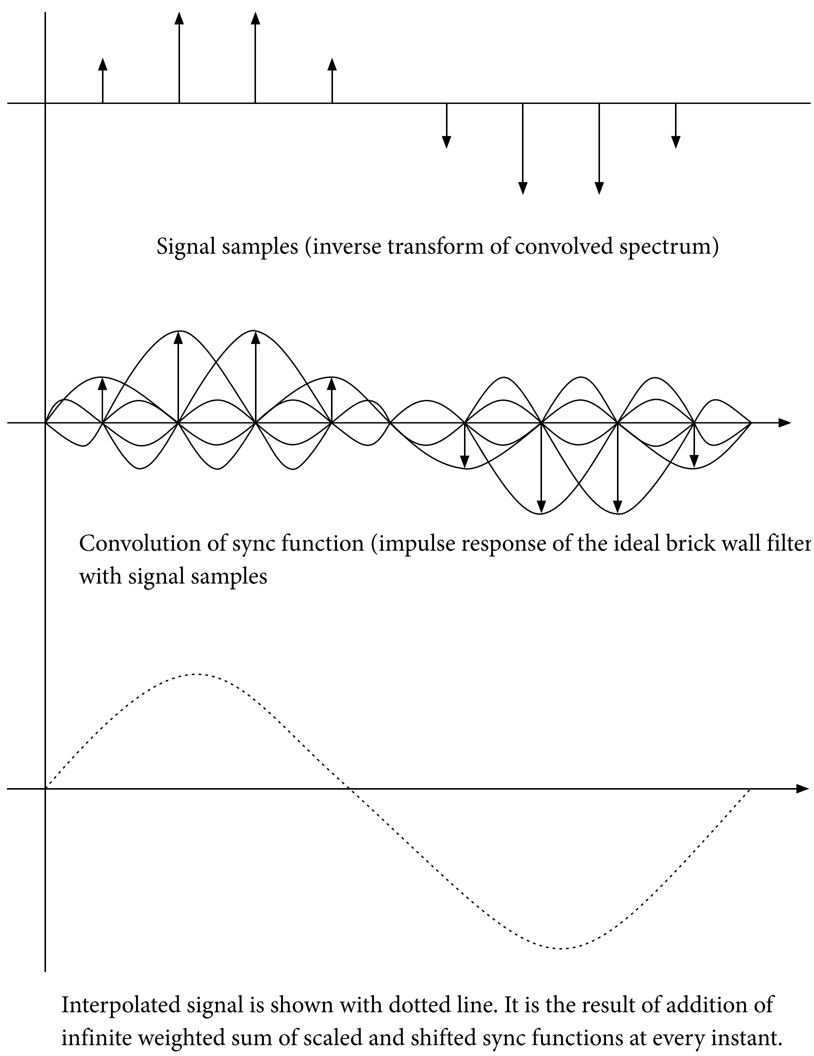

Fig. 1.6 Reconstruction of the 10 Hz signal component from signal samples using sync function as the interpolation function

We say that the sync function is used as an interpolation function to reconstruct the signal from the signal samples. The interpolation function is a sync function given by

$$g(t) = \text{sync}(2\pi Wt) = \frac{\sin(2\pi Wt)}{2\pi Wt} \tag{1.1}$$

Here W is the bandwidth of the signal. In our example, it is equal to 10 Hz. The signal can be recovered as

$$X(t) = \sum_{n=-\infty}^{\infty} X(n) g\left(t - \frac{n}{F_S}\right) \tag{1.2}$$

where F_S is the sampling frequency, equal to 100 Hz in our example.

The reconstruction process is complicated involving the infinite weighted sum of the function $g(t)$ and its shifted versions. The reconstruction formula is only of theoretical interest. Practically, the signal is just low-pass filtered with the cut-off frequency of the signal bandwidth to recover the signal.

Concept Check

- When the two signals are multiplied in Fourier domain, what happens in the time domain?
- What is a sync function?
- What is the meaning of interpolation function?

1.5 Sampling of Analog Signals (Case II)

Now, let us consider the 90 Hz signal component present in a signal with a bandwidth of 90 Hz. Let it be sampled using a sampling frequency of 100 Hz. The sampling frequency F_S is below the Nyquist rate (2 × bandwidth = 180 Hz). We will get 10 samples per 9 cycles of the waveform, as shown in Fig. 1.7. The sampled signal is obtained as a multiplication of the 90 Hz signal with a train of impulse with a period of 0.01 s.

In the transform domain, there will be convolution of the transform of the 90 Hz signal, that is, a spectral line at 90 Hz and the transform of the impulse train, that is, an impulse again with the period of F_S (100 Hz), as shown in Fig. 1.8. We can notice from the figure that the replicated spectral lines overlap or cross each other in the convolved spectrum.

Concept Check

- What will happen if the sampling frequency is below the Nyquist rate?
- What happens to the spectrum of signal sampled below the Nyquist rate?
- What is aliasing?

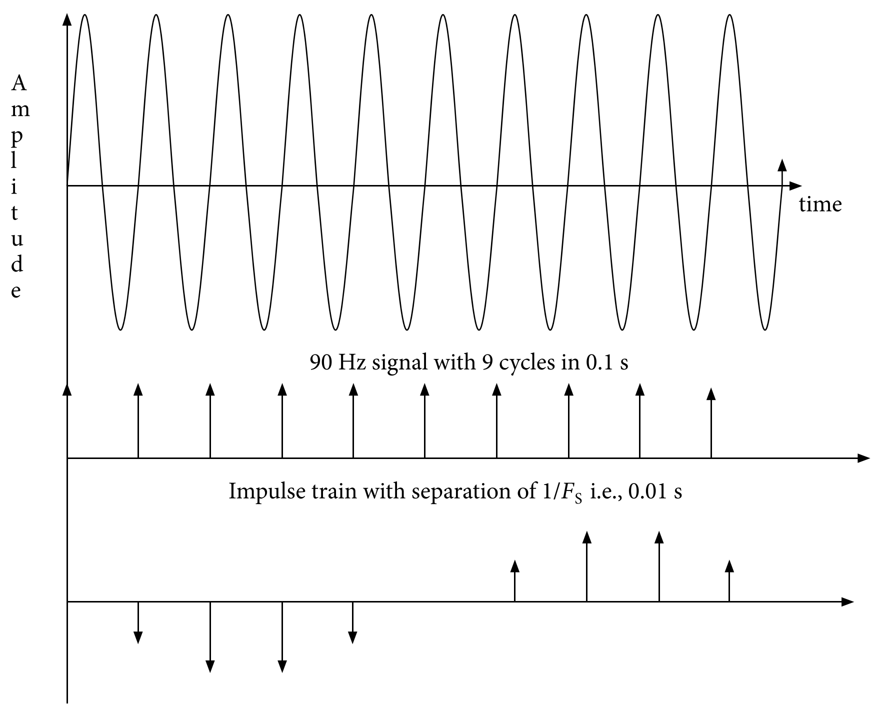

Fig. 1.7 Ten samples per 9 cycles of waveform when the 90 Hz signal is sampled with 100 Hz F_S

Note The signal that will be reconstructed from these samples is a 10 Hz signal with a 180° phase shift.

1.6 Recovery of Analog Signals (Case II)

The convolved spectrum in Fig. 1.6 shows replicated copies of the signal spectrum around zero frequency and multiples of the sampling frequency, that is, 100 Hz. The replicas are not well separated and there is mixing of adjacent replicas. The signal spectrum around zero frequency has 90 Hz and –90 Hz components, whereas the replica around 100 Hz has 10 Hz and 190 Hz components and the replica around –100 Hz has –190 Hz and –10 Hz components. The replica of 90 Hz around 100 Hz appears as a 10 Hz signal. We say that 90 Hz is aliased as 10 Hz. The overlapping of the spectrum does not allow the isolation of the 90 Hz signal. We can use a low-pass filter of 50 Hz, that is, F_N to filter the spectrum and it will then pass 10 Hz, which is an aliased version of 90 Hz.

The phenomenon of high-frequency components in the spectrum of the original signal taking the identity of low-frequency in the spectrum of its sampled version is called aliasing effect. The high frequencies are aliased as low frequencies.

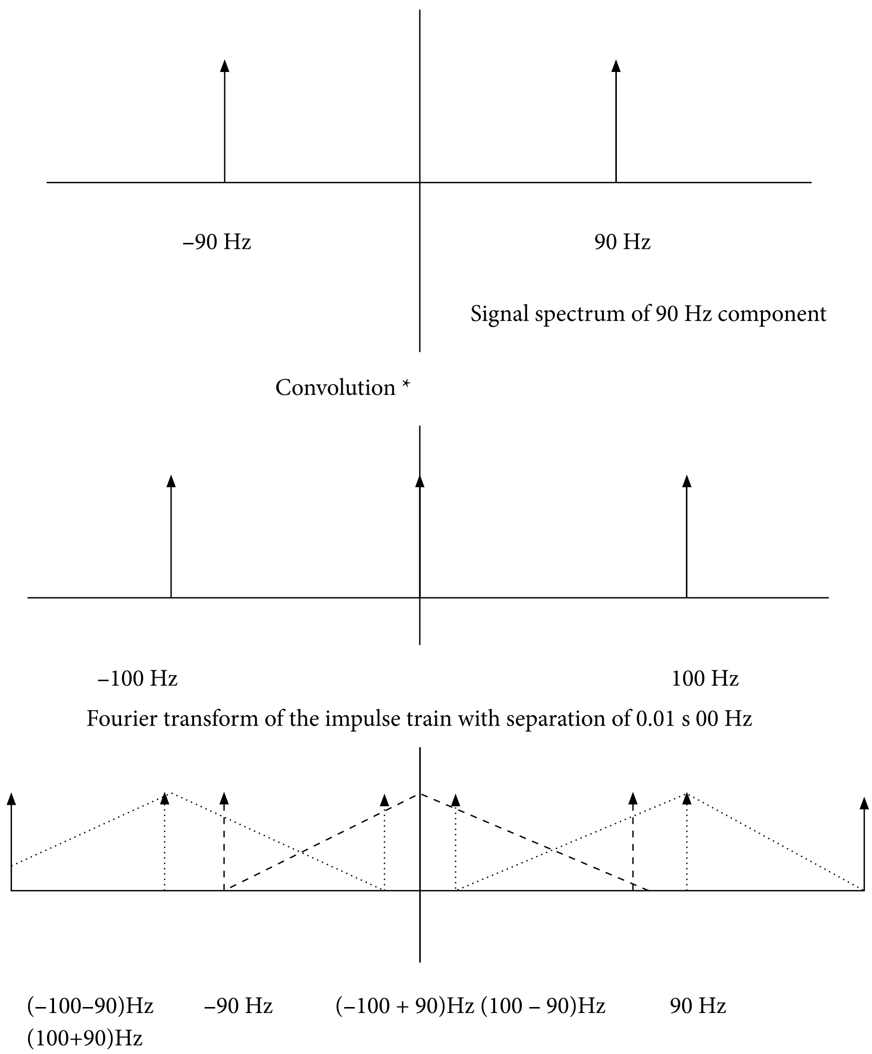

Fig. 1.8 Overlapped replicated spectrum (a replica of 90 Hz around 100 Hz) is seen as 10 Hz (dotted lines). The 90 Hz signal line is the replica of 90 Hz around 0 Hz (dashed lines). The 90 Hz signal is aliased as a 10 Hz signal when the spectrum is passed via a 50 Hz window (50 Hz being Nyquist frequency)

Concept Check

- What is aliasing effect?
- What will be the aliased frequency if a signal of 70 Hz is sampled with a 100 Hz sampling frequency?