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978-1-107-14495-8 — Logic of Statistical Inference
Ian Hacking, Preface by Jan-Willem Romeijn
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IAN HACKING



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Shaftesbury Road, Cambridge CB2 8EA, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
 a department of the University of Cambridge.

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www.cambridge.org

Information on this title: www.cambridge.org/9781107144958

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First published 1965

First paperback edition 1976

Cambridge Philosophy Classics edition 2016

A catalogue record for this publication is available from the British Library

Library of Congress Cataloging-in-Publication data

Names: Hacking, Ian, author.

Title: Logic of statistical inference / by Ian Hacking.

Description: First paperback edition. | New York : Cambridge University Press, 2016. |

Originally published: 1965. | Includes bibliographical references and index.

Identifiers: LCCN 2015049113 | ISBN 9781107144958 (Hardback : alk. paper) |

ISBN 9781316508145 (Paperback : alk. paper)

Subjects: LCSH: Mathematical statistics.

Classification: LCC QA276 .H2 2016 | DDC 519.5/4—dc23 LC record available at
<http://lcn.loc.gov/2015049113>

ISBN 978-1-107-14495-8 Hardback

ISBN 978-1-316-50814-5 Paperback

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Some connexions must be established between chance and support. The simplest is as follows: given that on some kind of trial an event of kind *A* happens more often than one of kind *B*, then the proposition that *A* occurs on some individual trial is, lacking other data, better supported than the proposition that *B* occurs at that trial. Something like this feeble connexion is widely held to follow from its long run success when used as the basis for a guessing policy. But no such long run defence is valid, and the connexion, if it exists at all, must have an entirely different foundation.

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A principle named the *law of likelihood* is proposed as an explication of the banal connexion stated in the preceding chapter, and then is used as a postulate to be added to Koopman's and Kolmogoroff's axioms as part of the postulational definition of chance. It relies on mere comparisons of support, justifying conclusions of the form, 'the data support this hypothesis better than that'. But it is relevant not only to guessing what will happen on the basis of known frequencies, but also to guessing frequencies on the basis of observed experimental results. Any such suggested law must be regarded as a conjecture to be tested in terms of its consequences, and this work is commenced.

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 The traditional theory of point estimation is developed, so far as is possible, along the lines of the preceding chapter, and relying on the theory of statistical support. Unfortunately the very concept of estimation seems ill adapted to statistics, and unless other notions are imported, it is impossible to define a ‘best estimate’ for typical problems. At most *admissible estimates* can be defined, which are not

demonstrably worse than other possible estimates. Usually the currently popular theories of estimation provide estimates which are admissible in the sense of the present chapter, but where popular estimates diverge, there is seldom any way of settling which is best without some element of arbitrary convention or whim.

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Bayes’ ideas have naturally led to a subjective theory of statistical inference, which is here explained sympathetically, and shown to be in principle consistent with our theory of statistical support. Neo-Bayesian work claims to analyse a much wider range of inferences than our theory attempts, but ours gives the more detailed account of the inferences within its domain, and hence it has the virtue of being more readily open to refutation and subsequent improvement.

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Preface to this edition

JAN-WILLEM ROMEIJN



The first sentences of the previous Preface to this book could have been written very recently: 'Logic has traditionally been the science of inference, but [...] few have studied the actual inferences made by statisticians, or considered the problems specific to statistics.' That this holds for mathematical logicians is no surprise. But it also holds for philosophers concerned with the logic of science. Studies of induction, confirmation, experimentation and hypotheses formation abound, but the role of statistics in these scientific activities is not always apparent. Considering the centrality of statistics in scientific practice, and the focus on this practice within contemporary philosophy of science, one would expect interest in statistics to be far more pronounced.

When Ian Hacking wrote those sentences, he may have been referring to the research programme of inductive logic, as carried out by Carnap and his co-workers. Since then much has been done to bring inductive logic and the philosophy of statistics closer together. Over the past decades we have seen the rise of probabilistic, mostly Bayesian, epistemology. And in statistics, due to conceptual advantages but possibly also due to improved computing power, the use of Bayesian statistics has become more mainstream. These developments have invited research activity on the intersection of statistics, logic and epistemology, albeit over mostly Bayesian communication channels, and in line with an already impressive literature on the foundations of Bayesian statistics.

Philosophers of science are aware that many scientists use classical rather than Bayesian statistical methods, associated with the names of Fisher, Neyman and Pearson. But philosophical attempts to clear up classical methods are far less developed than those pertaining to Bayesian ones. Now that many scientists adopt a pragmatic attitude towards statistical methods, and many statisticians and methodologists take a nuanced stance towards the choice of methods, it seems high time that philosophers provide conceptual clarity for the full spectrum of methods. It is precisely for this reason that this reprint is remarkably well-timed. The *Logic of Statistical Inference* offers a broad and open-minded investigation into the nature and justification of statistical methods. It develops a middling

position, centred on the notion of likelihoods as measuring support. It thereby provides an epistemic yet objectivist interpretation to classical statistics, which still deserves our serious consideration.

There are many ways in which one might sing this book's praises but I would like to mention two things in particular. Firstly, while the book reviews debates that were ongoing at the time of writing (e.g., the fiducial argument), it focuses on aspects of those debates that are still relevant today. This makes the book a true classic and shows its philosophical depth. Secondly, all of this is achieved without introducing any of the tedious notation and formal clutter that sometimes obscures the writings of our predecessors. The book is a joy to read, for scientists and statisticians which have an interest in the foundations of their methods, and for philosophers of science who seek an introduction into one of their core subjects: the philosophy of statistics.

Preface



This book analyses, from the point of view of a philosophical logician, the patterns of statistical inference which have become possible in this century. Logic has traditionally been the science of inference, but although a number of distinguished logicians have contributed under the head of probability, few have studied the actual inferences made by statisticians, or considered the problems specific to statistics. Much recent work has seemed unrelated to practical issues, and is sometimes veiled in a symbolism inscrutable to anyone not educated in the art of reading it. The present study is, in contrast, very much tied to current problems in statistics; it has avoided abstract symbolic systems because the subject seems too young and unstable to make them profitable. I have tried to discover the simple principles which underlie modern work in statistics, and to test them both at a philosophical level and in terms of their practical consequences. Technicalities are kept to a minimum.

It will be evident how many of my ideas come from Sir Ronald Fisher. Since much discussion of statistics has been coloured by purely personal loyalties, it may be worth recording that in my ignorance I knew nothing of Fisher before his death and have been persuaded to the truth of some of his more controversial doctrines only by piecing together the thought in his elliptic publications. My next debt is to Sir Harold Jeffreys, whose *Theory of Probability* remains the finest application of a philosophical understanding to the inferences made in statistics. At a more personal level, it is pleasant to thank the Master and Fellows of Peterhouse, Cambridge, who have provided and guarded the leisure in which to write. I have also been glad of a seminar consisting of Peter Bell, Jonathan Bennett, James Cargile and Timothy Smiley, who, jointly and individually, have helped to correct a great many errors. Finally I am grateful to R. B. Braithwaite for his careful study of the penultimate manuscript, and to David Miller for proof-reading.

Much of chapter 4 has appeared in the *Proceedings of the Aristotelian Society* for 1963–4, and is reprinted by kind permission of the Committee. The editor of the *British Journal for the Philosophy of Science* has authorized republication of some parts of my paper ‘On the Foundations of Statistics’, from volume xv.

I.M.H.

Vancouver
September 1964