

Contents

<i>Preface</i>	<i>page</i>	ix
1 Mathematical Preliminaries		1
1.1 Integral Transforms		1
1.2 Special Functions of Fractional Calculus		17
1.3 Integral Transforms of Special Functions		43
2 A Survey of Fractional Calculus		46
2.1 The Origins of Fractional Calculus		46
2.2 The Grünwald–Letnikov Operator		57
2.3 The Caputo Operator		61
2.4 The Riesz–Weyl Operator		62
2.5 Integral Transforms of Fractional Operators		63
2.6 A Generalised Fourier Transform		68
3 From Normal to Anomalous Diffusion		71
3.1 Historical Perspectives on Diffusion Problems		71
3.2 Continuous-Time Random Walk		90
3.3 Diffusion Equation		95
4 Fractional Diffusion Equations		101
4.1 Fractional Time Derivative: Simple Situations		101
4.2 Fractional Spatial Derivative: Simple Situations		111
4.3 Sorption and Desorption Processes		114
4.4 Reaction Terms		124
4.5 Reaction and CTRW Formalism		134
5 Fractional Diffusion Equations		139
5.1 1D and 2D Cases: Different Diffusive Regimes		139
5.2 3D Case: External Force and Reaction Term		145

viii	<i>Contents</i>	
5.3	Reaction on a Solid Surface: Anomalous Mass Transfer	151
5.4	Heterogeneous Media and Transport through a Membrane	158
6	Fractional Nonlinear Diffusion Equations	169
6.1	Nonlinear Diffusion Equations	170
6.2	Nonlinear Diffusion Equations: Intermittent Motion	173
6.3	Fractional Spatial Derivatives	182
6.4	d -Dimensional Fractional Diffusion Equations	188
7	Anomalous Diffusion	200
7.1	The Adsorption–Desorption Process in Anisotropic Media	200
7.2	Fractional Diffusion Equations in Anisotropic Media	209
7.3	The Comb Model	220
8	Fractional Schrödinger Equations	234
8.1	The Schrödinger Equation and Anomalous Behaviour	234
8.2	Time-Dependent Solutions	242
8.3	CTRW and the Fractional Schrödinger Equation	249
8.4	Memory and Nonlocal Effects	254
8.5	Nonlocal Effects on the Energy Spectra	264
9	Anomalous Diffusion and Impedance Spectroscopy	271
9.1	Impedance Spectroscopy: Preliminaries	271
9.2	The PNP Time Fractional Model	280
9.3	Anomalous Diffusion and Memory Effects	286
9.4	Anomalous Interfacial Conditions	292
10	The Poisson–Nernst–Planck Anomalous Models	306
10.1	PNPA Models and Equivalent Circuits	306
10.2	PNPA Models: A Framework	313
	<i>References</i>	323
	<i>Index</i>	341