

INTRODUCTION TO NUMERICAL GEODYNAMIC MODELLING

This hands-on introduction to numerical geodynamic modelling provides a solid grounding in the necessary mathematical theory and techniques, including continuum mechanics and partial differential equations, before introducing key numerical modelling methods and applications. Fully updated, this second edition includes four completely new chapters covering the most recent advances in modelling inertial processes, seismic cycles and fluid-solid interactions, and the development of adaptive mesh refinement algorithms. Many well-documented, state-of-the-art visco-elasto-plastic 2D models are presented, which allow robust modelling of key geodynamic processes. Requiring only minimal prerequisite mathematical training, and featuring over 60 practical exercises and 90 MATLAB examples, this user-friendly resource encourages experimentation with geodynamic models. It is an ideal introduction for advanced courses and can be used as a self-study aid for graduates seeking to master geodynamic modelling for their own research projects.

TARAS GERYA is Professor in the Department of Earth Sciences at the Swiss Federal Institute of Technology (ETH-Zürich). He is an expert in numerical geodynamic modelling, with his current research focusing on subduction and collision processes, ridge-transform oceanic spreading patterns, intrusion emplacement into the crust, generation of earthquakes, fluid and melt transport in the lithosphere, Precambrian geodynamics, formation of terrestrial planets and evolution of life. In 2008 he was awarded the Golden Owl Prize by ETH students for his teaching on continuum mechanics and numerical modelling.

“A great introduction to computational geodynamics with vivid examples, hands-on exercises and step-by-step derivations of formulas. Even better than the first edition.”

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“This book is so much more than an introduction to geodynamic modelling. Taras Gerya opens the world of geodynamic experiments by taking the reader through a carefully designed set of hands-on programming exercises that will convince you that modelling is not terribly complicated, but a process to logically follow through. Go ahead and get started!”

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– Professor Thorsten Becker, *University of Texas at Austin*

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SECOND EDITION

TARAS GERYA

Swiss Federal Institute of Technology (ETH), Zürich



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Preface to the second edition

The main reason for writing this second edition is the rapid recent progress in the field of numerical geodynamic modelling, which is one of the most dynamic and fast growing fields of the modern Earth sciences. Since the publication of the first edition in 2010 (almost a decade ago . . .), several important research directions have become very prominent and advanced in computational geodynamics, such as investigation of coupled solid-fluid processes, coupling of geodynamic evolution to surface processes, modelling of seismic cycles at plate boundaries, development of adaptive grid refinement methods and free surface stabilization approaches, elaboration of more accurate continuity-based Lagrangian advection algorithms, development and broad application of new efficient 3D visco-elasto-plastic highly parallelized numerical modelling tools etc. In order to account for some of these exciting novelties, I both significantly revised some of the previously published chapters (especially numerical modelling of advection processes in Chapter 8 and numerical treatment of visco-elasto-plastic materials in Chapters 12 and 13) and added four new chapters focusing on recent numerical advances in

- modelling of inertial processes (Chapter 14),
- modelling of seismic cycles (Chapter 15),
- modelling of coupled fluid-solid processes (Chapter 16) and
- development of adaptive mesh refinement algorithms (Chapter 17).

As in the first edition, a single relatively simple numerical modelling method (combination of staggered finite differences with marker-in-cell techniques, SFD+MIC) and MATLAB programming are used uniformly throughout this textbook. I hope you will enjoy this new edition!

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