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Hardy's Life

Can you name a twentieth-century mathematician who has appeared as a character in a popular novel, an acclaimed play, a feature film, and a murder mystery? Better yet, can you name one who was described by a famous author as having written "... in his own clear and unadorned fashion, some of the most perfect English of his time?" [97, p. 151].

G. H. (Godfrey Harold) Hardy was that mathematician. He left deep footprints, both as a scholar and as a colorful, eccentric example of the English intellectual from a century ago.

(By the way, the novel is David Leavitt's *The Indian Clerk*, the play is Ira Hauptman's *Partition*, the film is *The Man Who Knew Infinity*, the mystery is Randall Collins's *The Case of the Philosophers' Ring*, and the famous author is C. P. Snow, whose admiration for Hardy's literary gifts knew no bounds.)

For all that has been written about Hardy, his own memoir, *A Mathematician's Apology*, in a sense tells it all. It is an eloquent essay reflecting upon his life as a creative mathematician. It is, as well, a poignant document, written at an age when waning powers deprived him of the ability to do first-class work in his beloved mathematics. The essay is preceded by a moving remembrance from the aforementioned novelist and physicist, C. P. Snow, who knew Hardy well when they were at Cambridge together. Their friendship continued up to the time of Hardy's death in 1947.

Although *A Mathematician's Apology* has its moments of melancholy—a word Hardy used to describe it on the opening page—it stands as an unrivaled manifesto for mathematics as a creative art. In it Hardy treated "pure" mathematics, which was, for him, the only kind that mattered. He of course recognized the practical side of the subject as it applied to real-world activities, but even here his praise was faint. He conceded that basic arithmetic has its uses; after all, "... the currency system of Europe conforms to it

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approximately" [89, p. 382]. But his true feelings were clear. "It is undeniable," he wrote,

> . . . that a good deal of elementary mathematics . . . has considerable practical utility. These parts of mathematics are, on the whole, rather dull; they are just the parts which have least aesthetic value. The "real" mathematics of the "real" mathematicians, the mathematics of Fermat and Euler and Gauss and Abel and Riemann, is almost wholly "useless". . . [54, p. 119].

For someone of Hardy's sensibilities, a comparison between mathematics and the arts seemed obvious. His views were perhaps best expressed in this famous passage from the *Apology*:

> A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because his are made with *ideas* [54, p. 84].

And he went on to say,

> The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colours or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics [54, p. 85].

Hardy saw mathematical ideas as timeless, with an eternal appeal. He made a case for this view when he wrote:

> The Babylonian and Assyrian civilizations have perished; Hammurabi, Sargon, and Nebuchadnezzar are empty names; yet Babylonian mathematics is still interesting, and the Babylonian scale of 60 is still used in astronomy.

These ancient civilizations were followed by the Greeks, whose

> . . . mathematics is the real thing. The Greeks first spoke a language which modern mathematicians can understand; as Littlewood said to me once, they are not clever schoolboys or 'scholarship candidates', but 'Fellows of another college'. . . Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not [54, pp. 80–81].

Can there be any remaining doubt about the provocative and engaging nature of Hardy's *Apology*? Atle Selberg, a Norwegian number theorist and early winner of the Fields Medal, called it "a great piece of literature"

[3, p. 266]. And the novelist Graham Greene, who reviewed the work, regarded it as the best account he had ever read of what it means to be a creative artist [27, p. 682].

If you are unfamiliar with *A Mathematician's Apology*, we suggest you put down this book, rush off to find a copy, and read it. You will not be disappointed.

For whatever reason, Britain has a tradition of producing scientists who are excellent writers. Charles Darwin, certainly the greatest British scientist this side of Newton, filled his *Origin of Species* with words of true beauty. Darwin's disciple, the comparative anatomist Thomas Huxley, was more than adept at turning a phrase. And Hardy's Cambridge colleague Bertrand Russell—mathematician, philosopher, and member of the Royal Society—won the Nobel Prize for *Literature* in 1950. Hardy, as a mathematician with a gift for writing, was in good company.

A Mathematician's Apology is a masterpiece. But there is more to be said about its author. Where he has described his work and given his views on important issues, we shall provide Hardy untouched and in his own words. That will constitute the bulk of this volume. In this introduction, however, we should sketch some of the milestones of his life and provide some recollections from those who knew him well.

Childhood and School

Who was G. H. Hardy? He was born in 1877 to a family of the professional class in Cranleigh, Surrey, England. His father held various positions in the preparatory section of Cranleigh School, and his mother was on the staff of another school nearby. The boy was brought up in a religious, Victorian household. His early fascination with mathematics was evident when he tried to factor the hymn numbers that appeared on the board in church. He had a sister Gertrude Edith, called "Gertie," who never married and remained close to her brother throughout his life. Indeed, Gertie may have been the only person who dared to call him "Harold."

Their mutual affection came in spite of a horrible childhood accident. We quote from Robert Kanigel:

> Gertrude lost her eye as a child, when Harold, playing carelessly with a cricket bat, struck her; she had to wear a glass eye for the rest of her life. The incident, however, did nothing to disrupt their sibling closeness, and may even have enhanced it. They were devoted to one another all their lives, kept in close touch almost as twins are said to, and for many years shared an apartment in London [97, p. 117].

Schoolboy Hardy before a soccer match at Winchester College, 1895.

At an appropriate age Hardy was enrolled in Cranleigh School, but eventually he was sent to boarding school at the famous and ancient Winchester College. This institution, with its harsh and forbidding reputation, was not to Hardy's tastes. Kanigel wrote that Hardy ". . . lived in a sort of intellectual ghetto within the college, a fortresslike complex of medieval, gray stone buildings, worlds apart from the sunny openness of Cranleigh" [97, p. 121].

Winchester had received its charter in 1382 when it was established by the local Bishop of Winchester, William of Wykeham. The good Bishop had been educated at New College, Oxford, so Winchester's graduates were expected to follow in his footsteps. Bucking tradition (and not for the last time), Hardy instead chose Trinity College, Cambridge, for it was stronger in mathematics than anything Oxford could boast. Trinity had been Newton's college, after all!

Upon leaving Winchester, Hardy never returned. He did, however, carry with him a lifelong distaste for mutton, a residue of Winchester's bizarre statute forcing the students to be served this for dinner five days a week [121, p. 452].

To this point, Hardy's mathematical education had not been an entirely satisfactory one. At Cranleigh and Winchester, his talents were such that he never took regular classes but instead was coached privately. Cambridge, of course, was famous beyond measure, but it had a drawback of its own: an

Scholar Hardy at Winchester College, 1895.

excessive emphasis on something called the "Tripos examination." We quote Kanigel's chilling description:

> The mathematical Tripos was impossibly arduous. You sat for four days of problems, often late into the evening, took a week's break, then came back for four more days . . . Here, sometimes on problem after problem, the brightest students, destined for distinguished mathematical careers, would not even know where to begin. It was a frightful ordeal . . . The Tripos, wrote one English-born mathematician years later, "became far and away the most difficult mathematical test that the world has ever known, one to which no university of the present day can show any parallel" [97, p. 129].

Worse, the mathematical skills necessary to succeed on this terrifying exam bore no resemblance to "modern" mathematics as practiced in the wider world. The research topics that were engaging mathematicians in France and Germany and the United States were nowhere to be found on the Cambridge Tripos. Yet, Kanigel observed, ". . . as [the Tripos] grew more demanding and more important, it also, in inimitable English fashion, took on the luster—and the deadweight—of Tradition" [97, p. 129].

So, when Hardy arrived at Cambridge, he found himself among students who were forced to expend far too much of their university careers in preparation for two dreaded weeks of testing during their final year. And the stakes were high. There was a ranking scheme that designated the best student as "Senior Wrangler," followed by Second Wrangler, Third Wrangler, and so on. Rewards for the top finish were significant. Shops in Cambridge actually sold postcards featuring the images of the Senior Wrangler (mathematicians as rock stars!).

Hardy wanted none of it. He raced through his studies, took the Tripos a year early, and perhaps as a consequence ended up as only Fourth Wrangler. He was later to campaign for the abolition of this famous, or perhaps infamous, examination. His plan of action was unequivocal: "I adhere to the view that the [Tripos] system is vicious in principle, and that the vice is too radical for what is usually called reform. I do not want to reform the Tripos, but to destroy it" [32, p. 71].

The Professor's Life

Hardy survived it all. He ended his student days after having received the prestigious Smith's Prize in mathematics —a far more telling indicator of mathematical ability than any wranglerhood—and was granted a Fellowship allowing him to stay at Trinity College. This would be his academic base for the next two decades.

In 1900, Hardy published an article, "On a class of definite integrals containing hyperbolic functions," in the *Messenger of Mathematics*. This was the first in his long line of 375 mathematical papers. In 1912 he coauthored his first paper with J. E. Littlewood. It was published in the *Proceedings of the Fifth International Congress of Mathematicians*, which was held that year in Cambridge and had brought to town such luminaries as Jacques Hadamard, E.H. Moore, Vito Volterra, and E. T. Whittaker [93, pp. 223–229].

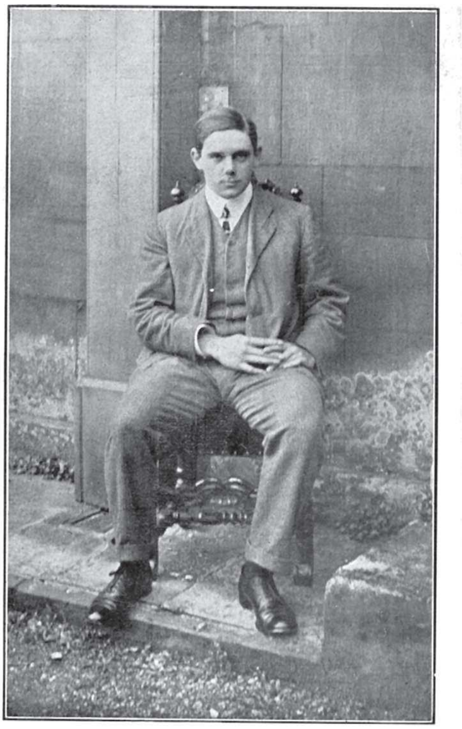
That initial Hardy-Littlewood paper was titled, "Some Problems of Diophantine Approximation." Over the years these two great mathematicians collaborated on such topics as Fourier series, Waring's problem on the representation of integers as sums of kth powers, Goldbach's conjecture, and the Riemann zeta-function. Their 93 joint papers were of an extraordinary depth and significance. C. P. Snow claimed that the Hardy-Littlewood research team was the strongest mathematical partnership in history, and one can think of few others that even come close. Hardy's friend and colleague Harald Bohr, the mathematician brother of physicist Niels Bohr, was not far off the mark when

Hardy in 1900, the year in which he was elected a Fellow Of Trinity College.

he quipped that "nowadays there are only three great English mathematicians: Hardy, Littlewood, and Hardy-Littlewood" [9, pp. xxvii–xxviii].

In many ways the Hardy-Littlewood collaboration was an odd one. The two men were scholars of roughly equal powers, but their personalities were quite different. Hardy was elegant and sophisticated, rather patrician in his English way. Littlewood had a rougher appearance and demeanor. Though they lived in the same college during much of their careers, their work was usually carried on via *letters* with the understanding, according to Bohr, that when one wrote to the other, the recipient "was under no obligation whatsoever to read it, let alone answer it." Further, it did not matter whether "one of them had not contributed the least bit to the contents of a paper under their common name; otherwise there would constantly arise quarrels and difficulties that now one, and now the other, would oppose being named co-author" [9, pp. xxvii–xxviii].

The collaboration was so peculiar that a theory developed to the effect that Hardy had created a fictional "Littlewood" in case something was published



Crisp & Co. Camb Senior Wranglers, Cambridge, 1905. *Copyright.*
Mr. J. E. Littlewood (bracketed with Mr. J. Mercer) Trinity College.

Senior Wrangler, J. E. Littlewood in 1905. Hardy and Littlewood met in 1906; their formal collaboration began in 1911. Together, they wrote more than one hundred papers.

that contained an error, which could then be blamed on his imaginary coauthor. Of course, there was no truth in this, and Hardy was more than willing to own up to a mistake. For instance, in the preface to the second edition to his Cambridge Tract, *The Integration of Functions of a Single Variable*, Hardy noted that he had earlier "reproduced a proof of Abel's which Mr. J. E. Littlewood afterwards discovered to be invalid. The correction of this error has led me to rewrite a few sections of the present edition completely." And in 1920 Hardy published an article on Waring's Problem in the *Transactions of the American Mathematical Society* that had to be corrected seven years later after "Miss G. K. Stanley, upon reading the 30-page manuscript, discovered a significant error in a formula that required a three-page set of corrections . . . to fix it up" [61, p. 845]. Miss Stanley was a Reader in Mathematics at Westfield College and had been a research student of Hardy's. One concludes from these episodes that (a) Littlewood was a real person, not a fiction upon whom to pin mathematical errors and (b) even great mathematicians can make mistakes. It is some comfort