Fluid Mechanics

SECOND EDITION

The multidisciplinary field of fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering. This textbook, fully revised and enlarged for the Second Edition, presents the minimum of what every physicist, engineer and mathematician needs to know about hydrodynamics. It includes new illustrations throughout, using examples from everyday life: from hydraulic jumps in a kitchen sink to Kelvin–Helmholtz instabilities in clouds and geophysical and astrophysical phenomena, providing readers with a better understanding of the world around them. Aimed at undergraduate and graduate students as well as researchers, the book assumes no prior knowledge of the subject and only a basic understanding of vector calculus and analysis. It contains 41 original problems with very detailed solutions, progressing from dimensional estimates and intuitive arguments to detailed computations to help readers understand fluid mechanics.

GREGORY FALKOVICH is Professor in the Department of Physics of Complex Systems at Weizmann Institute of Science in Rehovot, Israel. He has researched in plasma, condensed matter, fluid mechanics, statistical and mathematical physics and cloud physics and meteorology, and has won several awards for his work.

"This is by far the best textbook on modern Fluid Mechanics I've seen in decades. It will benefit both beginners, who wish to learn the field from scratch, and mature scientists and engineers in need of catching up on its contemporary status. Throughout the book, the emphasis is on the place of each particular equation in the "grand scheme of things", and on their internal connections. For this reason even researchers actively involved in this exciting field will find it an invaluable source of ideas and inspiration."

Professor Alexander Zamolodchikov,
Rutgers University

"This short book is elegant and poetic. It is written with passionate admiration for its subject, fluids. Mathematicians interested in the Navier Stokes model for fluids can gain intuition into its unknown mysteries. They can gain informed speculations about long time prediction and they can realize the value of knowing about the possible singularities of ideal fluids without friction. Mathematicians and physicists not primarily interested in fluids can learn why they might well profit from a familiarity with the rich panorama surveyed in this work."

Professor Dennis Sullivan,
City University of New York and Stony Brook University

"Fluid dynamics is a classic subject, and one whose importance only grows larger with time. Grisha Falkovich's text combines concision with precision as it leads physicists through a course on fluids at the depth they expect. A unique feature of this text is its treatment of dispersive wave interaction, modulations, collapse and solitons alongside traditional topics like turbulence and shocks. My students and I learn from it with pleasure during courses on both fluid and plasma physics."

Professor Patrick Diamond,
University of California San Diego

"Gregory Falkovich has written a must-read for anyone in need of a concise and basic introduction to fluid dynamics. Since fluids and gases are everywhere, from microorganisms to galactic scales, there might be more in need than realized... At the same time, advanced readers will appreciate the overall quality and some gems like the sections on quasi-momentum and dispersive waves. I have this book on my shelves and highly recommend it!"

Professor Massimo Vergassola,
University of California San Diego

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Preface to the Second Edition

Why study fluid mechanics? The primary reason is not even technical, it is cultural: A natural scientist is defined as one who looks around and understands at least part of the material world. One of the goals of this book is to let you understand how the wind blows and the water flows so that while flying or swimming you may appreciate what is actually going on. The secondary reason is to do with applications: Whether you are to engage with astrophysics or biophysics theory or build an apparatus for condensed matter research, you need the ability to make correct fluid-mechanics estimates; some of the art of doing this will be taught in the book. Yet another reason is conceptual: Mechanics is the basis of the whole of physics and engineering in terms of intuition and mathematical methods. Concepts introduced in the mechanics of particles were subsequently applied to optics, electromagnetism, quantum mechanics, etc.; here you will see the ideas and methods developed for the mechanics of fluids, which are used to analyze other systems with many degrees of freedom in statistical physics and quantum field theory. And last but not least: at present, fluid mechanics is one of the most actively developing fields of physics, mathematics and engineering, so you may wish to participate in this exciting development.

In the time of ever-increasing specialization, the universal language of fluid mechanics is one of the few remaining means of communications between people from very distant fields and disciplines. Using this simple and intuitive language, biophysicists and astrophysicists can tell each other about some of their recent achievements; meteorologists can comprehend what happens inside a collider while high-energy physicist can understand the cloud above his head; and physicists and mathematicians can appreciate and help to solve the problems that engineers face. My personal motivation in writing this book was to bring this language up-to-date and to express fluid mechanics using modern notions (like symmetry breaking and renormalization), most of which actually originated in the discipline.

Even for physicists who are not using fluid mechanics in their work, taking a one-semester course on the subject would be well worth the effort. This is one such course. It presumes no prior acquaintance with the subject and requires only basic knowledge of vector calculus and analysis. On the other hand, mathematicians and engineers may find in this book several new insights presented from a physicist's perspective. In choosing from the enormous wealth of material produced by the last four centuries of ever-accelerating research, preference was given to the ideas and concepts that teach lessons whose importance transcends the confines of one specific subject as they prove useful time and again across the whole spectrum of modern physics. To much delight, it turned out to be possible to weave the subjects into a single coherent narrative so that the book is a novel rather than a collection of short stories.

We approach every subject as physicists: start from qualitative considerations (dimensional reasoning, symmetries and conservation laws), then use back-of-the-envelope estimates and crown it with concise yet consistent derivations. Fluid mechanics is an essentially experimental science, as is any branch of physics and engineering. Experimental data guide us at each step, which is often far from trivial; for example, energy is not conserved even in the frictionless limit and other symmetries can be unexpectedly broken, which makes a profound impact on estimates and derivations.

Lecturers and students using the book for a course will find out that its 13 sections comfortably fit into 13 lectures plus, if needed, problem-solving sessions. Sections 2.3 and 3.1 each contain two extra subsections that can be treated in a problem-solving session (specifically, Sections 2.3.4, 2.3.5, 3.1.2 and 3.1.5, but the choice may be different). For second-year students, one can use a shorter version, excluding Sections 1.3.4, 3.2–3.4 and the small-font parts in Sections 2.2.1, 2.2.3 and 2.3.4. The lectures are supposed to be self-contained so that no references are included in the text. The epilogue and endnotes provide guidance for further reading; the references are collected in the reference list at the end. Those using the book for self-study will find out that in about two intense weeks one is able to master the basic elements of fluid mechanics. Those reading for amusement can disregard the endnotes, skip all the derivations and half of the resulting formulae and still be able to learn a lot about fluids and a bit about the world around us, helped by numerous pictures.

In many years of teaching this course at the Weizmann Institute, I have benefited from the generations of brilliant students who taught me never to stop looking for simpler explanations and deeper links between branches of physics and fields of science. Different versions of the course were taught in Lyon, Moscow, Stockholm and Stony Brook. I was also lucky to learn from many people: V. Arnold, E. Balkovsky, E. Bodenschatz, G. Boffetta, D. Budker, A. Celani, M. Chertkov, B. Chirikov, G. Eyink, U. Frisch, K. Gawedzki, V. Geshkenbein, M. Isichenko, L. Kadanoff, K. Khanin, B. Khesin, D. Khmelnitskii, I. Kolokolov, G. Kotkin, R. Kraichnan, E. Kuznetsov, A. Larkin, V. Lebedev, L. Levitov, B. Lugovtsov, S. Lukaschuk, V. L'vov, K. Moffatt, A. Newell, A. Polyakov, I. Procaccia, A. Pumir, A. Rubenchik, D. Ryutov, V. Serbo, A. Shafarenko, M. Shats, B. Shraiman, A. Shytov, E. Siggia, Ya. Sinai, M. Spektor, K. Sreenivasan, V. Steinberg, K. Turitsyn, S. Turitsyn, G. Vekshtein, M. Vergassola, P. Wiegmann, V. Zakharov, A. Zamolodchikov and Ya. Zeldovich. Special thanks to Itzhak Fouxon, Marija Vucelja and Anna Frishman, who were instructors in problem-solving sessions and helped with writing solutions for some of the exercises. I am grateful to the readers of the first English edition and the Russian edition for pointing out misprints, errors and unclear places, which I did my best to correct. Remaining errors, both of omission and of commission, are my responsibility alone. This book is dedicated to my family.

Prologue

> The water's language was a wondrous one, some narrative
> on a recurrent subject ...
>
> *(A. Tarkovsky, translated by A. Shafarenko)*

There are two protagonists in this story: inertia and friction. One meets them first in the mechanics of particles and solids where their interplay is not very complicated: Inertia tries to keep the motion while friction tries to stop it. Going from a finite to an infinite number of degrees of freedom is always a game-changer. We will see in this book how an infinitesimal viscous friction makes fluid motion infinitely more complicated than inertia alone ever could. Without friction, most incompressible flows would stay potential, i.e. essentially trivial. At solid surfaces, friction produces vorticity, which is carried away by inertia and changes the flow in the bulk. Instabilities then bring about turbulence, and statistics emerges from dynamics. Vorticity penetrating the bulk makes life interesting in ideal fluids though in a way different from superfluids and superconductors.

On the other hand, compressibility makes even potential flows non-trivial as it allows inertia to develop a finite-time singularity (shock), which friction manages to stop. It is only in a wave motion that inertia is able to have an interesting life in the absence of friction, when it is instead partnered with medium anisotropy or inhomogeneity, which cause the dispersion of waves. The soliton is a happy child of that partnership. Yet even there, a modulational instability can bring a finite-time singularity in the form of self-focusing or collapse. At the end, I discuss how inertia, friction and dispersion may act together.

On a formal level, inertia of a continuous medium is described by a non-linear term in the equation of motion. Friction and dispersion are described by linear terms, which, however, have the highest spatial derivatives so that the limit of zero friction and zero dispersion is singular. Friction is not only singular but also a symmetry-breaking perturbation, which leads to an anomaly when the effect of symmetry breaking remains finite even in the limit of vanishing viscosity.

The first chapter introduces basic notions and describes stationary flows, inviscid and viscous. Time starts to run in the second chapter, which discusses instabilities, turbulence and sound. The third chapter is devoted to dispersive waves. It progresses from linear to nonlinear waves, solitons, collapses and wave turbulence. The epilogue gives a guide to further reading and briefly describes present-day activities in fluid mechanics. Detailed solutions of the exercises are given.