978-1-107-12413-4 — From Servant to Queen: A Journey through Victorian Mathematics
John Heard
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1 SETTING THE SCENE

It is possible that the life of a mathematician is one which no perfectly reasonable man would elect to live.[1]
G H Hardy

This gloomy conclusion was arrived at in 1920 by the most renowned British pure mathematician of the twentieth century – perhaps the only one whose name is known to some non-mathematicians – notwithstanding that during the previous decade he had achieved international recognition as one of the leading practitioners of his generation. Until the Victorian era, pure mathematicians had been an unknown species in Britain, yet by the end of the nineteenth century, when Hardy was a new graduate at Cambridge, they were rapidly becoming an essential constituent of the mathematical scene. Their success did not come easily, because they had adopted a philosophy concerning mathematics that was opposed to the prevailing utilitarianism of the Victorians, and consequently they became the target of considerable criticism concerning the value of their activities. However, this criticism came not only from outside; there was also a good deal of self-criticism and soul-searching, as Hardy, with his usual directness, made so painfully apparent. In this book we investigate the genesis of pure mathematicians in Britain, the cause of their difficulties, the steps that they took to address them, and the eventual alignment of their objectives with one of the most controversial aspects of Victorian popular culture. In the process we shall touch upon some of the most interesting nineteenth-century debates concerning mathematics, which raised questions that are still significant today: What is the purpose of mathematics and why should it be studied and researched? What is the criterion of mathematical truth? What duty to society does a mathematician owe? What value for society does a mathematician have?

Pure mathematics has a long history that goes back to ancient times. It began with the properties of numbers and geometrical figures – Euclid's *Elements*, which dates from about 300 BCE, deals with both – and then expanded to include, amongst other things, algebra, coordinate geometry and the calculus. Such mathematics is described as 'pure' because its concepts do not directly refer to things in the world, or to measurable quantities; rather, these are the province of what is now referred to as 'applied mathematics', which puts pure mathematics to work in a great many scientific, technological and commercial disciplines. To take an example from Newtonian mechanics: to calculate the time taken for a body under constant acceleration to travel a given distance, it is necessary to solve a quadratic equation, and finding a general method of solving quadratic equations is an exercise in pure mathematics because it involves nothing but algebra. Determining that the relation between the four relevant quantities – initial speed, constant acceleration, time taken and distance travelled – can be expressed by a particular quadratic equation, and the use of the general method of solution in specific instances, form part of applied mathematics.[a]

Although Isaac Newton died in 1727, a veneration of his memory continued well into the nineteenth century, and supported a British tradition in which a mathematician's duty was to use pure mathematics for the advancement of scientific and technological disciplines, and to increase our knowledge of the world. Consequently, within the sciences, the value of pure mathematics was believed to lie in its being the essential component of what was then called 'mixed mathematics', a term for which the modern 'applied mathematics' is almost, but not quite, a synonym. This led to a description of pure mathematics as being both 'queen' and 'servant' of the sciences: 'queen', because it was supposed to yield truths greater and more secure than those of the sciences, and 'servant', because its function was to serve the sciences. However, some pure mathematics had no role to play in mixed

[a] The equation is $at^2 + 2ut - 2s = 0$, where a is the constant acceleration, t is the elapsed time that it is desired to find, u is the initial speed and s is the distance travelled. When a, u and s are known, the equation can normally be satisfied by two distinct values of t, only one of which will be appropriate for the physical situation.

mathematics, and beginning early in the nineteenth century a few British mathematicians came to believe that it was equally worthy of study, notwithstanding its apparent lack of utility; their commitment to the cause of mixed mathematics showed signs of weakening. Such mathematicians, whatever their personal predilections, at first still had to work in the mixed-mathematical tradition, but later in the century, as circumstances changed, they became able to dedicate themselves to pure mathematics, unconcerned as to whether their work had any relevance elsewhere. This new breed of mathematicians, who wished to devote their time and energy to what many thought of as useless mathematics, became known as ‘pure mathematicians’, and what distinguished them was not that they studied pure mathematics – which was nothing new – but that their motivation for studying it no longer lay in its current or potential applications.

Until the 1830s, the sciences were brought together as the constituents of natural philosophy, which had the aims of investigating and understanding better the workings of the world, and advancing the fortunes of humankind – all to the glory of God. In this enterprise, mathematics was an important adjunct, and the wide range of its applications meant that use of the word ‘mathematician’ to describe someone who was not necessarily a specialist, but simply had sufficient knowledge of mathematics to make regular use of it, was common and lasted well into the nineteenth century. In Continental Europe there was an alternative, more abstract approach to mathematics, but communication between British and Continental mathematicians had almost ground to a halt, not least because of differing views that arose in the seventeenth century as to whether the German philosopher Gottfried Leibniz should be regarded as a co-discoverer, along with Newton, of what is now always referred to as ‘the calculus’. The British believed not only that Newton should take all the credit, but also that neither the method of fluxions and fluents (as his version of the calculus was called), nor the academic system in which he flourished and produced his great works, could be significantly improved. The wars with France and the excesses of the French Revolution also did nothing to commend Continental ways of thought. Consequently, British mathematicians saw no need to follow developments on the Continent, and in time became almost incapable of understanding them, even had they wished to do so, because of new notations and methods of reasoning that they had ignored. Only at the beginning of the nineteenth century did relations begin to thaw as some mathematicians came to appreciate the significance of Continental advances, which by the 1820s had even begun to find their way into the university curriculum at Cambridge, then regarded as the nation's mathematical powerhouse. During the next two decades traditionalists offered some resistance to this trend, which they saw as concentrating too much on abstract symbolism, and in the 1830s and 1840s there was much disquiet concerning the direction in which mathematics was heading at the university.

Notwithstanding these debates, mathematics was rapidly gaining in importance as an explanatory and descriptive tool. Natural philosophers in all the sciences were acquiring great quantities of data and dealing with theories of ever-increasing complexity and scope, but often found it difficult to reconcile their findings with philosophical and theological beliefs that were based on the teachings of Christianity and the Bible. These circumstances began a process in which the old overarching concept of natural philosophy was eventually abandoned in favour of giving much more independence of thought to each of the sciences – a very welcome consequence for those investigators who wanted to eliminate the more speculative elements from the theories with which they worked.

To further the freeing of science from the shackles of philosophy and theology, in the early 1830s the word 'scientist' was coined and introduced into the language to replace 'natural philosopher', in the hope that it would emphasise the role of rational thinking and hard evidence. It is no coincidence that other changes in vocabulary occurred at the same time: within two decades not only 'scientist' but also the new terms 'pure mathematician', 'applied mathematician' and 'applied mathematics' had all come into regular use, with 'mixed mathematics' beginning to fall out of favour, along with 'natural philosopher'.[b] Although 'scientist' had been preceded by other similar-sounding words such as 'sciencist', 'sciencer', 'scientiate' and 'scientman' – which are now obsolete or rarely used – its invention was the outcome of a discussion between like-minded individuals at a meeting of the British Association for the Advancement of Science, who felt that the language as then constituted did not have a word that identified them collectively as adherents to the new evidence-based regime.[2] Although

[b] For the recent introduction of the term 'pure mathematician', see the chart in Figure 5.1 on p. 146.

philosophers, theologians and the clergy were not excluded as scientists, those presenting themselves under this new banner thereby proclaimed their avoidance of the philosophical and theological speculation that had previously permeated scientific enquiry; no longer was the Book of Nature to be read in parallel with the Good Book.

The desire to place the sciences on a completely rational footing gave additional value to mathematical reasoning. This became particularly so after the Great Exhibition of 1851, when a decline began in the relative superiority of the British Empire's products as against those from other countries, due to the prevalence of 'rule of thumb' methods in manufacturing that lacked the precision and efficiency of new techniques that had been developed elsewhere. To remain competitive, manufacturers needed to innovate, and mathematicians were expected to play their part by ensuring that their work supported the nation's attempt to maintain its place in the forefront of new scientific, industrial and commercial developments.

However, that view of the role of the mathematician was not always accepted in Continental Europe, where many believed that mathematics should provide a field for intellectual speculation, rather than serve the sciences. One of the great mathematicians of the age, the German Carl Jacobi, wrote in 1830 to Adrien-Marie Legendre:

> It is true that M. Fourier held the opinion that the main aim of mathematics is public utility and the explanation of natural phenomena; but such a philosopher should have known that the sole purpose of science is the honour of the human spirit, and that under that title a question about numbers is worth as much as a question about the system of the world.[3]

More than a decade later this idea was still a novelty in Britain, as can be judged by Jacobi's experience in 1842 when he was in Manchester to represent Prussia at a meeting of the British Association for the Advancement of Science; later he wrote to his brother Moritz: 'There I had the courage to declare that it is the honour of science to be of no use, which provoked an emphatic shaking of heads.'[4] This reaction should have been no surprise to Jacobi, given that British mathematics and science were principally valued for providing vital support to the country's burgeoning industry, manufacturing, trade and commerce; his audience, situated in Britain's industrial heartland, naturally found his views very unappealing.

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As this suggests, the first decades of Victoria's reign were not ones that gave a ready welcome to new, abstract, speculative and inapplicable mathematics. Nevertheless, opinions such as Jacobi's encouraged those British mathematicians who wanted the freedom to study pure mathematics without intending to advance the sciences, or having any reason to think that something useful would come of it. These were the new 'pure mathematicians', a description that soon led to their colleagues who continued working in the old tradition of mixed mathematics being separately categorised as 'applied mathematicians'. Pure mathematicians, who accepted pure mathematics as queen but not as servant, were immediately faced with two difficulties: the first was how best to deflect criticism, of which there was a good deal; the second was how to justify in their own minds their unwillingness to make meaningful contributions to anything that the world would find worthwhile. These difficulties had to be addressed in the face of the prevailing Victorian belief that those with technical skills should deploy them by doing useful work, a belief that was, of course, in direct opposition to the pure mathematicians' ideals.

Consequently pure mathematicians faced an uphill task; but as regards the first difficulty they eventually succeeded, because by the end of the First World War they were well-established in academic life, and had even obtained a kind of moral superiority over their worthy but workaday colleagues, the applied mathematicians. When Hardy wrote in 1940 that 'A mathematician need not now consider himself on the defensive',[5] he was remembering his own experience at Cambridge 40 years earlier. However, the second difficulty – that of self-doubt – remained, and some pure mathematicians found that indulging their predilections did not always sit well with their conscience. At first they were not completely divorced from applied mathematics, and certainly did not in any way seek to diminish its interest and importance; thus they were able to maintain a foot in each camp, and only towards the end of the nineteenth century did a new generation of hard-line pure mathematicians, of whom Hardy was one, became totally committed to pure mathematics. Given this commitment, the sombre comment at the head of this chapter is a surprising reflection on the life that he had chosen for himself, particularly as it formed part of his inaugural lecture as Oxford University's Savilian Professor of Geometry; the explanation is that, although his work gave him much pleasure, he was also very aware of how little he contributed to the common good, and this knowledge induced in him a profound feeling of melancholy.

There is an interesting parallel, evident to the Victorians themselves, between the situation in which pure mathematicians found themselves during the second half of the nineteenth century, and the situation of classicists. They both appeared to become increasingly irrelevant; indeed, pure mathematicians were probably the more irrelevant of the two, for although some proficiency in Latin and ancient Greek (very different from modern Greek) was no longer always expected from those who were well-educated, it was still necessary for the study of ancient history and literature, the value of which was appreciated by all. On the other hand, pure mathematicians apparently provided no benefit to society. Nevertheless, classics continued to decline as an academic subject, and with it the standing of classicists; but the standing of pure mathematicians steadily increased until, in the first decade of the twentieth century, Cambridge produced two of the world's finest, in Hardy and his frequent collaborator John Edensor Littlewood.

This unexpected elevation of the status of pure mathematicians, who finally abolished any hint of servitude from their motivations, brought about what some still consider to be unwelcome consequences that are illustrated by a comment from the French–American algebraist Serge Lang in the 1980s. When asked what algebra was good for, he replied 'It's good to give chills in the spine to a certain number of people, me included. I don't know what else it is good for, and I don't care.'[6] Such a lack of concern for useful applications led Achim Bachem, a distinguished German mathematician, to be 'very worried that mathematicians are moving further and further away from the relevant problems in the natural sciences and society'. His diagnosis was that real-world problems are too messy, and have too many complications, to be truly satisfying for a mathematician; but after adding that 'it is unfortunate that the majority of good mathematicians are interested in pure mathematics', he suggested that 'a mathematician would like to be recognized as a capable professional within his own community. But nowadays, he can only achieve this by asserting himself within the classical value system of ever deeper theorems. If he works with real problems, then he cannot present his result as a deep theorem.'[7]

In the following chapters, we shall investigate how this came about: how pure mathematicians and 'ever deeper theorems' gained an ascendancy at a time when the prevailing culture emphasised utility and pragmatism, and in the face of the well-established British tradition of mixed mathematics. We do so by treating mathematics, not as a body of knowledge, but as a practice – as something that people *did* – and that was therefore subject to public scrutiny and judgement. Accordingly, our account stresses pure mathematicians in society, rather than the content of pure mathematics itself, for they did not work in a vacuum, driven only by the remorseless logic of their theorems; whether they liked it or not, they were social animals, and therefore hoped to work in a world that, if not enthusiastic, was at least acquiescent and uncritical, letting them pursue their vocation without interference.

The actual content of mathematics is not absent from this story, but it does take a back seat. During the nineteenth century, mathematicians made a fundamental re-evaluation of the criteria by which mathematical truth was judged, and Bertrand Russell, who was a mathematician before turning to philosophy, made the point in a typically extravagant fashion when he wrote in 1901 that:

> The nineteenth century, which prided itself upon the invention of steam and evolution, might have derived a more legitimate title to fame from the discovery of pure mathematics. This science, like most others, was baptised long before it was born; and thus we find writers before the nineteenth century alluding to what they called pure mathematics. But if they had been asked what this subject was, they would only have been able to say that it consisted of Arithmetic, Algebra, Geometry, and so on. As to what these studies had in common, and as to what distinguished them from applied mathematics, our ancestors were completely in the dark.[8]

Formerly, and in accord with the tenets of natural philosophy, mathematics was true because it derived from, and aligned itself with, our perception of the world. Latterly, mathematical truth meant logical consistency, which Russell believed to be a unifying concept that created what was in effect a new discipline. However, although the criterion was now that of logic, logic alone was not enough; it needed foundations on which to build. Unfortunately, as Russell was to find out shortly afterwards, the insufficient attention that mathematicians had paid to underlying principles meant that some of those principles led to paradoxical and contradictory results, particularly when dealing with the infinite.

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This added to the already well-established abstract and speculative character of pure mathematics, and so diminished still further the ability of pure mathematicians to connect with anyone other than their own kind, for by then it was not possible to make even a spurious claim of relevance to anything that the world-at-large might find of value.

On the other hand, the speculative creations of pure mathematicians may *later* have unexpected and unintended applications. For example, after the publication in 1905 of his paper on the electrodynamics of moving bodies (usually referred to as the special relativity paper), the German physicist Albert Einstein needed a mathematical method capable of handling the ideas that he had developed for his general theory of relativity. He was not a mathematician by training, but his collaborator Marcel Grossmann was, and *he* knew about the tensor calculus, a recent speculation by a group of Italian pure mathematicians.[9] Grossmann realised that it was the ideal vehicle for expressing Einstein's ideas in a mathematical form, and since then it has also found many other applications in physics and engineering. There are other examples: the theory of matrices, which was substantially created by Arthur Cayley as an exercise in pure algebra, is now used in almost every branch of applied mathematics; and the theory of numbers, which was a special interest of Hardy's, and believed to have no conceivable use at all – that was one of its attractions – is now vital in maintaining the security of electronic data and communications; Hardy would have been very disappointed. However, although there could always be a remote possibility that, at some future time, this or that development in pure mathematics might provide benefit to the world-at-large, it was not possible to run the argument that, on those grounds and for no other reason, pure mathematicians should be allowed free rein; what we would now call the cost–benefit ratio was too great. Yet they managed to tap into an aspect of Victorian culture that worked in their favour, produced the desired result, and continues to colour contemporary attitudes towards them and their work.

In the following chapters, we shall see how, over the course of the nineteenth century, pure mathematicians came into being, how difficult it was for them to survive in a hostile environment, and how they achieved their eventual success. Yet it is not only mathematicians and mathematics that are considered, for the narrative is embedded in the world of Victorian Britain, and demonstrates how the adherents of a discipline that appeared to be so removed from the influences of contemporary culture could nevertheless use that culture to forge a role for themselves that achieved their overt ambitions, although not always their mental quietude; in this process, advances in mathematical knowledge – whatever that is taken to mean – played little part. Pure mathematicians had to justify their claim to a place in public life, and obtaining acceptance and respect, as they eventually did, was by no means straightforward. In following their progress, we shall be witnessing the emergence of a non-utilitarian discipline in a highly utilitarian age.

Notes and References

1. G H Hardy, *Some Famous Problems of the Theory of Numbers, and in particular Waring's Problem: An Inaugural Lecture Delivered Before the University of Oxford* (Oxford: Clarendon Press, 1920), p. 4; he gave the lecture on 18 May 1920.
2. [Anon.] William Whewell, 'On the Connexion of the Physical Sciences. By Mrs Somerville', *The Quarterly Review*, LI, CI (1834), 54–68, at 59.
3. 'Il est vrai que M. Fourier avait l'opinion que le but principal des mathématiques était l'utilité publique et l'explication des phénomènes naturels; mais un philosophe comme lui aurait dû savoir que le but unique de la science, c'est l'honneur de l'esprit humaine, et que sous ce titre, une question de nombres vaut autant qu'une question du système du monde.' From a letter dated 2 July 1830: C W Borchard (ed.), *C. G. J. Jacobi's Gessammelte Werke*, 1 (Berlin: G Reimer, 1881), pp. 454–455.
4. 'Ich hatte den muth dort den satz geltand zu machen es sei die ehre der wissenschaft keinen nutzen zu haben, was ein gewaltiges schütteln des kopfes hervorbrachte.' From a letter dated 25 September 1842: W Ahrens (ed.), *Briefwechsel Zwischen C. G. J. Jacobi und M. H. Jacobi: Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen*, XXII (Leipzig: Teubner, 1907), p. 90.
5. G H Hardy, *A Mathematician's Apology* (Cambridge: Cambridge University Press, 1967), p. 64; originally published 1940.
6. Serge Lang, *The Beauty of Doing Mathematics: Three Public Dialogues* (New York: Springer, 1985), p. 49.
7. Achim Bachem, 'Mathematics: From the Outside Looking In', in Björn Engquist and Wilfried Schmid (eds.), *Mathematics Unlimited: 2001 and Beyond* (Berlin: Springer, 2001), pp. 275–281.
8. Bertrand Russell, 'Mathematics and the Metaphysicians', in *Mysticism and Logic and Other Essays* (London: Longmans, 1918), p. 74; originally published 1901.
9. Alicia Dickenstein, 'About the Cover: A Hidden Praise of Mathematics', *Bulletin of the American Mathematical Society*, ns 46, 1 (2009), 125–129, at 126; she quotes a translation of Einstein's manuscript in which he records his indebtedness to Grossmann and other mathematicians.