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Challenges for Ice Age Dynamics: A Dynamical Systems Perspective

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Abstract

This chapter is dedicated to the slow dynamics of the climate system, at time scales of 1000 to 1 million years. We focus specifically on the phenomenon of ice ages that has characterised the slow evolution of climate over the Quaternary. Ice ages are a form of variability featuring interactions between different large-scale components and processes in the climate system, including ice sheet, deep-ocean and carbon cycle dynamics. This variability is also at least partly controlled by changes in the seasonal and latitudinal incoming solar radiation associated with the combined effects of changes in Earth's orbit shape, precession of equinoxes, and changes in obliquity. A number of possible mechanisms are reviewed in this chapter. We stress that the nature of the interactions between these slow dynamics and faster modes of variability, such as millennium and centennial modes of variability, are still poorly understood. For example, whether the time sequence of ice ages is robustly determined or not by the astronomical forcing is a matter of debate. These questions need to be addressed with a range of models. We propose to use stochastic parameterisations in the lower-resolution models (focusing on climate time scales) to account for weather and macro-weather dynamics simulated with higher-resolution models. We discuss challenges – including statistical challenges – and possible methods associated with this programme.

> If [...] we look in the 21st century and make an optimistic forecast on the type of computer which will be available [...] We may construct a super-model [...] When we integrate the equations, if they are correct, we shall necessarily obtain changes in climate, including the great ice ages.
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> — *Edward Lorenz, 1970*

1.1 The Ice Age Phenomenon

1.1.1 Short Summary of Observational Evidence

Glacial cycles, or ice ages, are a form of climate variability that can be characterised as the succession of interglacial (similar to today) and glacial conditions over time scales

of several tens of thousands of years. At the Last Glacial Maximum – 21,000 years ago, henceforth noted 21 ka BP – the sea-level was 120 m lower than today. This water had accumulated in the form of ice on the continents, the majority of which was located in the Northern Hemisphere, in the current locations of Canada, Scandinavia and the British Isles (Clark and Mix, 2002; Peltier, 2004).

The recurrent character of glacial conditions was first identified from inspection of European alluvial terraces (Penck and Brückner, 1909). It is nowadays documented from, among others, marine cores (Lisiecki and Raymo, 2005; Elderfield et al., 2012; Rohling et al., 2014), ice cores (Petit et al., 1999; Kawamura et al., 2007; EPICA community members, 2004), loess (Guo et al., 2000) and speleothems (Winograd et al., 1992).

The ice ages may be characterised as follows:

1. This is a global phenomenon. The waxing and waning of Northern Hemisphere ice sheets is coupled to variations of sea-level of the order of one hundred meters (Figure 1.1a); it is also associated with large and significant variations in greenhouse gases concentrations (CO_2, CH_4 and N_2O), tropical atmosphere dynamics, land cover and ocean circulation (e.g. Ruddiman, 2006). Different records provide complementary information on the different facets of the climate response, and considerable attention is being paid to the chronological sequence of this response (Imbrie et al., 1992; Shackleton, 2000; Ruddiman, 2006; Lisiecki et al., 2008).
2. Ice ages combine fast and slow fluctuations. For example, the time elapsed between the latest two interglacial conditions is of the order of 100 ka. During the latest deglaciation, CO_2 concentration increased by 50 ppm, that is about 50% of its glacial-interglacial range, in only 3 ka (Monnin et al., 2001), and by 10 ppm in 200 years (Marcott et al., 2014).
3. The frequencies and modulation patterns of the elements of the astronomical forcing (defined in section 1.1.2) are well recognised in climate records (Hays et al., 1976; Lisiecki and Raymo, 2007) (Figure 1.1b). There is also a statistically significant relationship between the timing of deglaciations, and the timing of astronomical configurations causing larger-than-average incoming solar radiation (insolation) in the Northern Hemisphere in summer (Raymo, 1997; Huybers, 2011), although not all positive insolation anomalies give rise to a deglaciation.
4. The duration and amplitude of ice ages has been varying through time. The latest four cycles are characterised by a marked saw-tooth shape pattern, with slow glaciation and relatively rapid deglaciation. They cover the last 400 ka (Broecker and van Donk, 1970). Before 900 ka BP, climate fluctuations generally followed a cycle of the order of 40 ka (Ruddiman et al., 1986). The lengthening of ice ages around 900 ka BP is called the Mid-Pleistocene Transition (Clark et al., 2006) (Figure 1.2). The 100 ka periodicity dominates the spectrum of ice age fluctuations since the Mid-Pleistocene Transition. These cycles display a pattern of modulation of amplitude and frequency that is compatible with a form of non-linear synchronisation on eccentricity (Rial, 2004; Berger

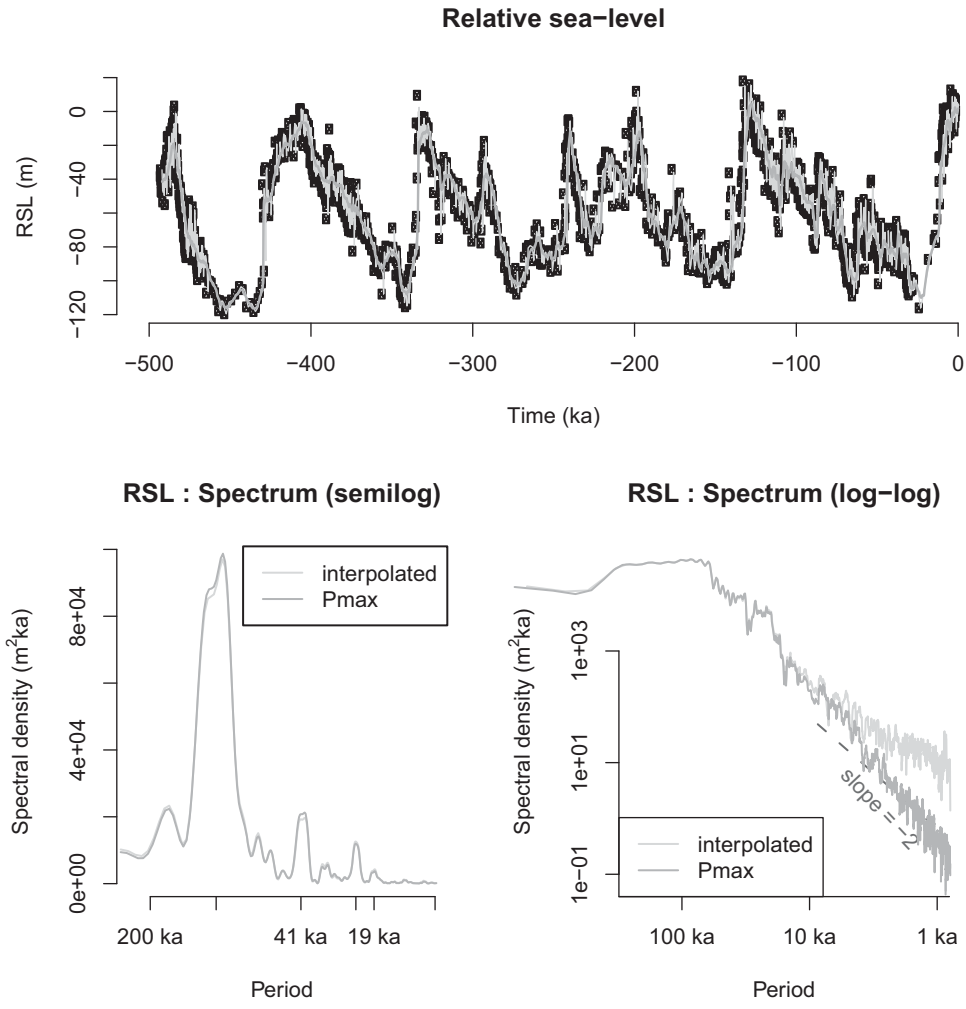

Figure 1.1 Relative Sea level (RSL) from Red Sea sediments (Grant et al., 2014) (a) Time series with raw data (black), linearly interpolated data with a 100yr time step (light gray), and maximum of probability ('Pmax') data at constant time step (dark gray). All data are supplied in (Grant et al., 2014). (b) Spectra of interpolated and Pmax data using a one 'taper'-tampering method, following Percival and Walden (1998). (c) Adaptive (weighted) multi-taper spectra of interpolated and Pmax data, with 6 tapers (analysis carried out with the pmtm Matlab function (MATLAB and Signal Processing Toolbox, 2013) after average removal). A line with slope −2 is added for visual reference. The multi-taper was developed to estimate the high-frequency tails of spectra. The multiple tapers reduce spectrum variance as well as the spectral leak, but they also flatten spectral peaks, hence the necessity, e.g. outlined by Ghil et al. (2002), to complement the analysis with other techniques focusing on variance modes.

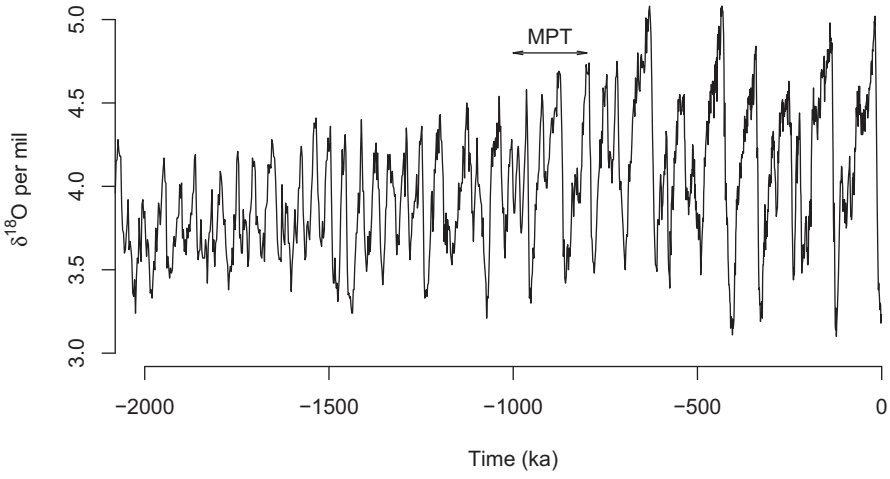

Figure 1.2 The Lisiecki and Raymo (2005) 'stack' is a composite record of 57 series of isotopic ratio of oxygen isotopes measured in shells of benthic foraminifera. It is commonly used as a proxy for global climate conditions, and shows well the increase in amplitude and length of ice age cycles over the last 2 million years.

et al., 2005; Rial and Saha, 2011) (see again section 1.1.2 for definitions of astronomical elements).

5. Ice ages represent a mode of climate variability connected to variability at shorter and longer time scales. While seemingly obvious, this statement represents a shift in our understanding of the climate spectrum. In the past (e.g. Saltzman and Maasch, 1990), it was suggested to clearly distinguish 'weather' and 'climate phenomena' as distinct modes of variability, decoupled and separated by a spectral gap (Figure 1.3). The postulated spectral gap would have acted as an efficient information barrier between climate and weather processes. Closer inspection of the data power spectra shows however that the postulated gap does not exist (Pelletier, 1997; Huybers and Curry, 2006; Lovejoy and Schertzer, 2012). The power spectrum of temperature is estimated to have approximately the typical shape of a Lorentzian spectrum, of which the power increases from the millennial scales, up to 40 ka, where it flattens (Pelletier (1997), Figure 1.1c shows a sea-level power spectrum and Figure 1.3b a temperature one). On the other hand, the spectrum flattens significantly at time scales of 1 ka and below, with only a moderate slope up to the annual time scale. This latter range of variability has been named the 'macro-weather' regime by Lovejoy and Schertzer (2012). Of course no classification is perfect. It was observed that spectra obtained from data during glacial and interglacial periods significantly differ (Shao and Ditlevsen, 2016). On the other hand, Nilsen et al. (2016) challenged the separation of 'climate' and 'macro-weather' as regimes with distinct scaling properties, at least during the Holocene. We will nevertheless keep using the phrase 'macro-weather' to designate a regime of variability which occupies an intermediate position on the spectrum between atmospheric weather and ice ages.

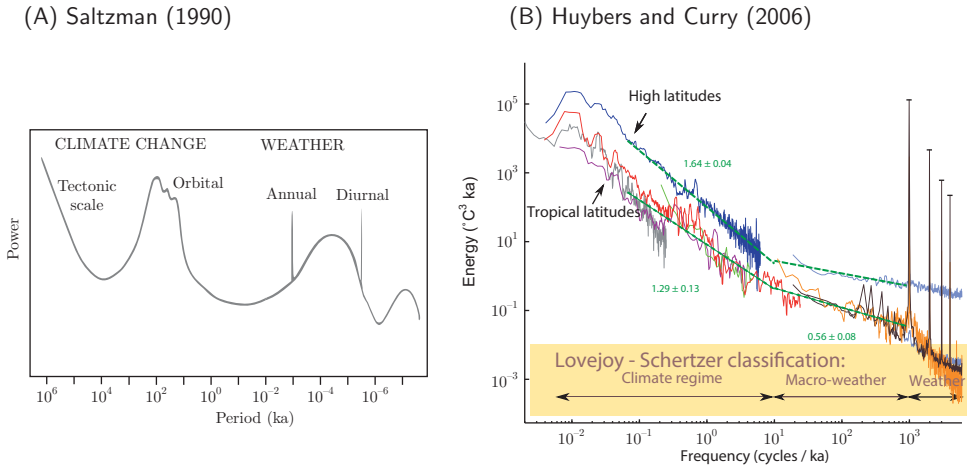

Figure 1.3 Two previously published temperature power spectra. Saltzman (1990) meant to be highly idealised, while Huybers and Curry (2006) is quantitative. Note the important structural difference: Saltzman distinguishes *climate* and *weather* regimes localised in the spectral domain and separated by a gap, giving full justification to the time-scale separation needed for the Hasselmann's stochastic theory (1976). The spectrum provided by Huybers and Curry (2006) is presented in log-log form. These authors analysed different data types and estimated spectral slopes (in green). On this plot, the more-energetic spectral estimate is from high-latitude continental records and the less-energetic estimate from tropical sea surface temperatures. The different data types are marked with the colour codes. Recent data come from instrumental records and re-analyses. Other data are from various natural archives (see the original reference for details). Note that the statistical estimation method of spectral slopes was not specifically adapted to unevenly spaced time series. The classification of regimes suggested by Lovejoy and Schertzer (2012) is added in yellow. Figures were adapted from the original publications.

1.1.2 The Astronomical Forcing

The astronomical forcing (also referred to as orbital or Milankovitch forcing) is the action of the slow changes in Earth's orbit shape, position of perihelion, and obliquity on the seasonal course of incoming solar radiation (insolation) hitting the top of the atmosphere at any point on Earth. To an excellent approximation, the great semi-axis of the Earth's orbit is constant (Lagrange, 1781; Laskar et al., 2004). Hence, the astronomical forcing may be determined from variations in Earth's orbital eccentricity e, the true solar longitude of the direction of Earth's perihelion (ϖ), and obliquity (ε).

The total amount of energy received all over the globe in one year is proportional to $1 - e^2/2 + \mathcal{O}(e^3)$. Accounting for the fact that the eccentricity varies between 0 and 0.05, this represents fluctuations of no more than 0.1% of the absolute value of insolation. On the other hand, changes in obliquity affect the seasonal contrast of insolation as well as the latitudinal distribution of annual mean insolation. Changes in the longitude of perihelion, modulated by eccentricity, produce monthly anomalies compared to the averaged seasonal cycle because the amount of insolation is inversely proportional to the squared distance to the Sun. The annual mean anomalies associated with obliquity are then of the order of 5

W/m^2, and the monthly mean variations associated of precession can be of the order of 50 W/m^2. It is therefore generally recognised that the astronomical forcing affects Earth's climate because it changes the seasonal and spatial distributions of insolation.

In low-order or conceptual models (see section 1.2.1), the astronomical forcing is often summarised by one or two well-chosen forcing functions. For example, Milankovitch (1998) used insolation over the *caloric summer*. This is the amount of insolation integrated over the half of the year experiencing the largest amount of insolation. The quantity may be used as a predictor of the amount of snow susceptible of surviving the summer season, and therefore may be used as a forcing term in ice sheet-climate models. Alternative forcing functions include insolation at 65° N at the summer solstice or in July (Imbrie and Imbrie, 1980; Saltzman and Maasch, 1990). These quantities are particular cases of a wide class of insolation forcing functions that are approximately a linear combination of $e \sin \varpi$, $e \cos \varpi$ and ε (Loutre, 1993; Crucifix, 2011). The power spectrum of these quantities is useful to know, because the astronomical forcing is believed to excite the dynamics of the climate system through this spectrum.

Nowadays, the astronomical forcing is known from numerical solutions of an eleven-body problem (Sun + 9 planets + Moon) (Laskar et al., 2004, 2011). However, the earlier solutions of Berger (1978) and Berger and Loutre (1991) are still used because their spectral decomposition is given explicitly (Figure 1.4). The dominant periods are (Berger and Loutre, 1991):

1. for precession ($e \sin \varpi$): 23708, 22394, 18964, 19116, 23149, . . . years
2. for obliquity (ε): 41090, 39719, 40392, 53864, 41811 . . . years

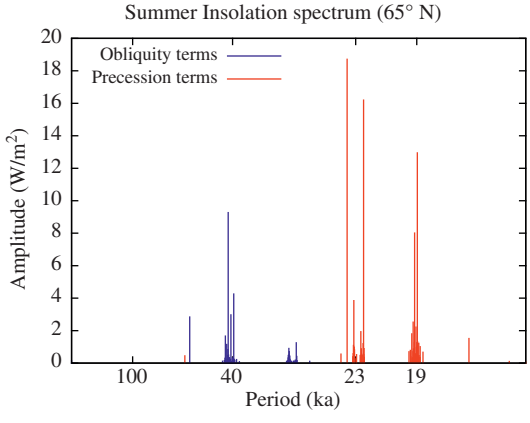

Figure 1.4 Analytical spectrum of summer solstice incoming solar radiation at 65° N, assuming an approximation of this insolation as a linear combination of $e \sin \varpi$, $e \cos \varpi$, and ε (e: eccentricity, ϖ: true solar longitude of perihelion, and ε: obliquity), and the analytical development of Berger and Loutre (1991).

The spectral decomposition of e may also be deduced from the expression $e = [(e \sin \varpi)^2 + (e \cos \varpi)^2]^{1/2}$. The dominant periods are then 404177, 94781, 123817, 130615, ... years.

1.2 Climate Models Used at the Palaeoclimate Scales

We briefly review in this section a standard classification of climate models used in palaeoclimate research, into 'conceptual', 'earth models of intermediate complexity', and 'global climate models'.

1.2.1 Conceptual Climate Models

Claussen et al. (1999) characterised conceptual climate models by the fact that the number of free parameters has the same order of magnitude as the number of state variables. Consequently these models require little computing resources and they are typically integrated on personal computers. Conceptual models represent a very broad category. One may distinguish discrete and continuous dynamical systems, deterministic and stochastic systems, and the literature provides examples of the different combinations.

Some of the early dynamical system models of ice ages were derived from ice-sheet flow equations reduced to a small number of ordinary differential equations (Weertman, 1976; Ghil and Le Treut, 1981; Le Treut and Ghil, 1983). One of the objectives of these models was to explain the emergence of 100-ka climatic cycles as a plausible consequence of ice sheet response to astronomically controlled variations of the snow line. Hence, the parameters used in these models were well identified physically, and this restricted the plausible range of these parameters. These early models then evolved towards greater realism and complexity. Advances in computing power allowed to resolve the partial differential equations on a spatial grid (Oerlemans, 1980; Birchfield et al., 1981; Hyde and Peltier, 1985). These models constitute the basis of modern state-of-the-art ice sheet models.

Since these early works, our knowledge of past CO_2 concentrations (Delmas et al., 1980; Genthon et al., 1987) has made it clear that ice age modelling requires also to consider carbon-cycle dynamics. Several dynamical system models since the eighties include prognostic variables for ocean and carbon pools, in order to account for the possibility of greenhouse gas exchanges between the oceans and the atmosphere (e.g. Saltzman and Maasch, 1988; Paillard and Parrenin, 2004).

As a general rule, the description of biogeochemical mechanisms is partly speculative. For example, several models of the same authors (Saltzman and Maasch, 1988, 1990, 1991) have slightly different equations for the carbon cycle dynamics. In fact, conceptual models have been used to support various and partially conflicting interpretations of ice ages, emphasising the roles of ocean vertical mixing and sea-ice (Gildor and Tziperman, 2001), alkalinity balance in the ocean (Omta et al., 2015), or more generically feedbacks between temperature and CO_2 (Hogg, 2008). The search for low-order models of ice ages is thus partly heuristic. It is risky to decide that one interpretation is better than another on mere inspection of the simulated sea-level (Tziperman et al., 2006). With this caveat in mind,

several investigators found it interesting to develop general theories that do not directly depend on a specific physical mechanism, but which have implications on the predictability or stability of ice ages. A couple of examples follow.

A possible starting point is to think in terms of multiple equilibria (Figure 1.5a). If we assume that feedbacks in the climate system combine to yield a stabilising response in response to a small external perturbation, at least at a certain time scale, then we come to the conclusion that the climate system is stable at that time scale. We may however speculate about some non-linear effects and imagine that a large perturbation will induce runaway effects. In this case, the climate system will drift away from its original state, towards another stable state. The co-existence of two stable equilibria appears in the energy balance model of climate proposed by Budyko (1969) and Sellers (1969), as a consequence of a non-linearity introduced with the albedo feedback (Ditlevsen, in this volume). Nowadays, we know that the Budyko-Sellers theory has some relevance to study ancient glaciations such as those in the Ordovician but it is not immediately applicable to ice ages. However, if we consider a thought experiment in which astronomical forcing *and* CO_2 concentration would be constant, then, for certain values of such parameters, simulations with ice-sheet-atmosphere models suggest to us that two stable states may effectively coexist (Oerlemans (1981), Calov and Ganopolski (2005), and Abe-Ouchi et al. (2013), Figures 1.5b and c). Having multiple equilibria is, however, not enough to explain ice ages: A mechanism is needed to jump from one state to the next. Historically, it was proposed to involve a mechanism of stochastic excitation causing transitions between the two states. More specifically, two stochastic mechanisms leading to ice ages were proposed: stochastic resonance (Benzi et al., 1982; Nicolis, 1982; Matteucci, 1989) and coherence resonance (Pelletier, 2003).

Stochastic processes can be justified as a representation of fast internal variability such as chaotic variability associated with atmosphere and ocean dynamics. In *stochastic resonance*, state transitions are induced by these additive fluctuations but their timing is controlled by a weak external forcing. This theory requires forcing at 100 ka, but this is not a strong frequency in the astronomical forcing. Furthermore, it produces ice age curves with rapid and broadly symmetric transitions between glacial and interglacial states. This conflicts with observations.

MacAyeal (1979) observed that the asymmetric shape of ice ages suggests a "catastrophic nature" of the deglaciation process. Wunsch (2003) illustrates this point with a very simple model. The drift from interglacial towards glacial conditions is modelled as a random walk, which terminates by an abrupt flush towards the interglacial state when glaciation exceeds a threshold (Figure 1.5d). The physical nature of the rapid deglaciation is itself uncertain. Pelletier (2003) theorised on the ice sheet mechanical collapse, a possibility supported by ice-sheet experts (Pollard, 1983; Abe-Ouchi et al., 2013). A flush may also be induced by the dynamics of

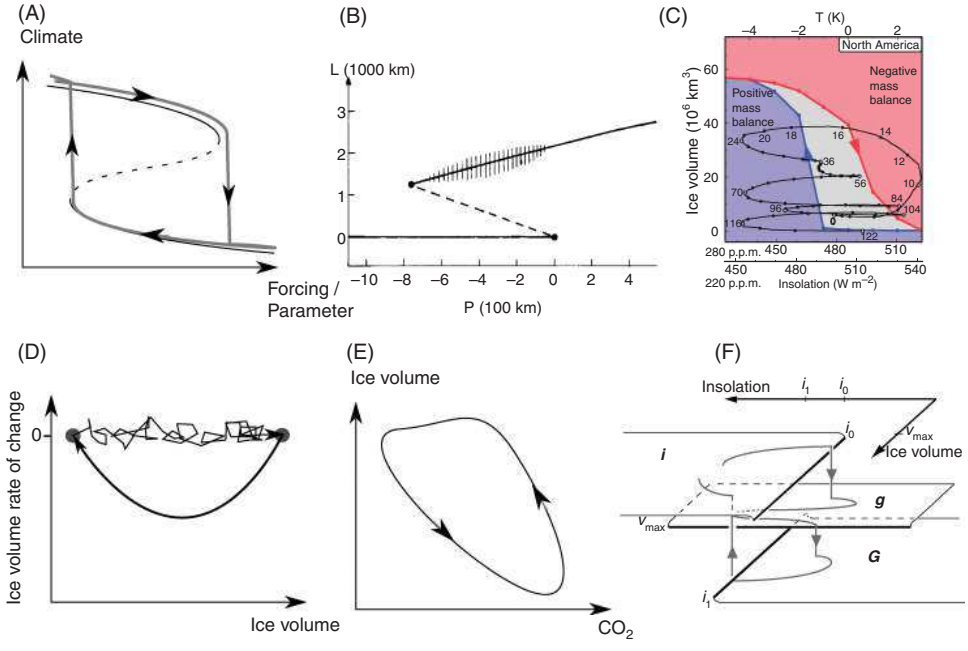

Figure 1.5 (A): Generic representation of a system with two stable states (separated by an unstable state, dashed) and twofold bifurcations. In a more complex model, the two state branches can be estimated by means of a hysteresis experiment (arrows, see also Section 1.2.2). (B): Experiments with a simple ice sheet model (Oerlemans, 1981) broadly support this scenario. Hashes on (B) indicate an 'almost intransitive state', with sluggish dynamics; *P* refers to the position of the snow line (free parameter) and *L* the extent of the Northern Hemisphere Ice sheet. (C): same but with a state-of-the-art model ice sheet atmosphere (Abe-Ouchi et al., 2013). Thick coloured full lines are the estimated system steady-states deduced from a hysteresis experiment, and the thin black curve with numbers represents an actual simulation over the last ice age, numbers denoting time, in thousands of years before present. Figures (D, E, F) are possible mechanisms for transitions between glacial and interglacial states. (D): Stochastic accumulation, with a flush mechanism to restore interglacial conditions; (E): deterministic limit cycle, caused by interactions between different system components, (F): transitions forced by changes in insolation, in this case with the postulate of an intermediate state and asymmetric transition rules (Paillard, 1998). Figures (C) and (F) reproduced by permission of Nature.

the southern ocean/carbon cycle dynamics. This second possibility is favoured by Paillard and Parrenin (2004) and Paillard (2015), though these authors favour a deterministic framework.

It is also possible to obtain alternation of glacial and interglacial climates as a result of non-linear interactions between different components of the climate system, without the requirement of an external forcing. In dynamical systems language, this amounts to view ice ages as a deterministic limit cycle (Figure 1.5e). Saltzman et al. developed this theory in a series of articles, including Saltzman et al.

(1981); Saltzman and Maasch (1988, 1990, 1991). One of the advantages of the limit cycle concept is that it provides a natural way to explain the saw-tooth shape of ice ages (Gildor and Tziperman, 2001; Ashkenazy, 2006; Paillard and Parrenin, 2004; Ashwin and Ditlevsen, 2015). It also provides a promising starting point to think about the dynamics of the Mid-Pleistocene Transition because this transition may then be interpreted as a bifurcation (Saltzman, 1990; Crucifix, 2012; Ashwin and Ditlevsen, 2015; Mitsui et al., 2015). The effect of astronomical forcing may be accounted in limit cycle models as a small additive forcing, which controls the timing of glaciation and deglaciation events, through a phenomenon of synchronisation (Tziperman et al., 2006). De Saedeleer et al. (2013), Crucifix (2013), and Mitsui and Aihara (2014) studied such synchronisation mechanisms in detail and they found that, with the astronomical forcing, the attractors of oscillator-type models of ice ages may become strange and non-chaotic. In simple terms, the modelled trajectories are synchronised to the astronomical forcing, and their greatest Lyapunov exponent (conditioned on the forcing) is slightly negative. In this rather unusual regime, the dynamics are not chaotic, and different initial conditions will typically converge to the same trajectory. However, the dynamics associated with strange non-chaotic attractors are such that the exact ice age trajectory is fragile with respect to small changes in the parameters, or to the addition of a weak stochastic process (Figure 1.6). The nature of the bifurcations from quasiperiodic to strange nonchaotic attractors are studied in more details in Mitsui et al. (2015) on the basis of a phase oscillator.

Finally, ice ages may be viewed as a forced oscillation *driven* by the astronomical forcing. A particularly clear example is provided by Paillard (1998), who defines three states (glacial, semi-glacial, interglacial) and transition rules involving astronomical forcing and ice volume thresholds (Figure 1.5f). The idea was reformulated in terms of continuous dynamics in Ditlevsen (2009), and then associated to a mechanism of relaxation oscillation in Ashwin and Ditlevsen (2015). A number of publications focus specifically on the identification and analysis of threshold functions (Parrenin and Paillard, 2003; Parrenin and Paillard, 2012; Feng and Bailer-Jones, 2015)

The above examples do not exhaust the possible interpretations of ice age dynamics in terms of dynamical systems concepts. Le Treut and Ghil (1983), and Daruka and Ditlevsen (2015) emphasised the notion of non-linear resonance, Pelletier (2003), the coherence resonance or noise-induced excitation, Roberts et al. (2015) developed an ice age theory featuring mixed-mode oscillations associated with Canard trajectories, and Rial (1999, 2004); Rial and Saha (2011) interpreted ice ages in terms of frequency modulation theory, emphasising the role of eccentricity. These models may not all be equally relevant, but each of them unveils possible counter-intuitive effects of the astronomical forcing on non-linear climate dynamics. They are paradigmatic, in the sense that they provide research questions *and* conceptual frameworks, which can be referred to when experimenting with models of higher complexity.