The Expanding Universe

A Primer on Relativistic Cosmology

Cosmology – the science of the Universe at large – has experienced a renaissance in the decades bracketing the turn of the twenty-first century. Exploring our emerging understanding of cosmology, this text takes two complementary points of view: the physical principles underlying theories of cosmology, and the observable consequences of models of universal expansion.

- Includes a structured discussion of General Relativity, firmly based on conceptual foundations, with mathematics limited to the minimum necessary, enabling students to grasp the underlying physical principles.
- Relates modern observations to theories of cosmology, deriving and explaining the relationship between basic physical quantities and observations, to show how modern observational astronomy supports and informs cosmological theory.
- Discusses non-intuitive concepts based on the foundations of General Relativity and cosmology, supporting readers as they confront apparent paradoxes in modern cosmology.
- Carefully explains limitations on our current understanding of the Universe's structure and evolution, arising from incomplete physical theory and imperfect observations; with caveats as to possible future developments.
- Worked solutions to end-of-chapter problems are available online for instructors, via www.cambridge.org/heacox.

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Dedicated to my mentors in astronomy:
the late Phyllis Hutchings (Whitman College);
the late Tom Lutz and Sid Hacker (Washington State University);
Ann Boesgaard and Sidney Wolff (University of Hawaii).
Mahalo nui loa.

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Preface

The pace of change in cosmology has accelerated remarkably in the years bracketing the turn of the twenty-first century, so that many of the classical texts are becoming dated. The purpose of this text is to provide a coherent description of current theory underlying modern cosmology, at a level appropriate for advanced undergraduate students. To do so, the book is loosely organized around two pedagogical principles.

First, while the development of physical cosmology is heavily mathematical, the book emphasizes physical concepts over mathematical results wherever possible. The mathematics of General Relativity and of relativistic cosmology are beautiful, elegant, and seductive. It is a real temptation to develop theoretical cosmology as a purely mathematical structure, much as can be done with classical thermodynamics. But to do so is to lose sight of the deeper meaning of cosmology and to leave the student unprepared for the changes in the field that are almost certainly coming. In Einstein's inimitable phrasing,[1] "*Mathematics is all very well, but Nature leads us by the nose.*" The book endeavors to lead the student gently, if not always easily, toward a useful understanding of the physical underpinnings of modern cosmology.

Cosmology is an inherently uncertain science, because of both the remoteness (spatial and temporal) of its subjects and the incompleteness of its observational foundations. It is thus not surprising that recent technological advances in observational astronomy have produced something of a revolution in cosmological theory, from inflation to dark energy to new theories of galaxy origins. But interpretations of cosmological observations are typically based on conceptual models and (in some cases) underlying physics of uncertain validity, so wherever possible the book derives and interprets its results in a manner conducive to

[1] "Die Mathematik ist shōn und gut, aber die Natur fūhrt uns an der Nase herum"; in a postcard to Hermann Weyl dated 26 May 1923, and reproduced on page 83 of Nussbaumer and Bieri (2009).

re-interpretation when new observations and/or physics so permit. The book is also at some pains to point out the uncertainties of cosmology theory arising from incomplete observational evidence and adoption of specific physical models. Modern cosmology is truly an intellectual wonder, but is likely to experience considerable revision as new observations and physics come to bear upon it. This book will, hopefully, prepare students for such changes.

The choice of subjects to include in a text of reasonable length is largely a personal one, guided by perceived needs and the ready availability of other books and reference material. This book's main emphasis is on development of cosmological models describing the Universe's expansion, in a form that can be applied to current observations and that should be useful when and if new observations compel changes to current models. Missing from the book are extensive discussions of particles and quantum fields in the very early Universe; of the finer details of observational cosmology; and of related issues such as black holes and gravitational radiation. There is no shortage of other books covering these areas.

Conspicuously included here is the General Theory of Relativity (GR) up to the development of the Einstein Field Equations. This requires a substantial detour from cosmology *per se*, but in the author's opinion the effort required is justified in terms of deeper understandings of the physics of cosmology. General Relativity is arguably the most elegant and beautiful physical theory accessible to undergraduate students, and one to which all prospective physicists and astronomers should be introduced. The text endeavors to develop GR in a manner that avoids needless abstraction and that emphasizes the physical principles involved. Some of the more abstract and mathematical aspects of GR are relegated to Appendix A, for the edification of mathematically ambitious students.

The book's intended audience is advanced undergraduate physics and astronomy students, but much of the material can be understood by those with only modest preparation in these fields. The necessary mathematics background is limited to elementary calculus and differential equations, with a smattering of linear algebra in the form of elementary matrix manipulations. For most newcomers to the subject of GR, the most intimidating chapter will probably be Chapter 4, which describes tensor analysis at an elementary level. Tensor analysis has gotten a bad reputation because of its sophisticated applications, but at the level needed here the subject is surprisingly accessible. Very little actual tensor manipulation is required to understand GR at this level; mostly, students need only to become familiar with tensor notation. Working the problems and examples should help.

The text assumes that readers are familiar with classical and modern physics at the level usually presented in lower-division physics courses for physics and astronomy students in American universities. It would probably also help to have some knowledge of modern astronomy at an elementary level, but little of the subject will be lost to those coming to astronomy for the first time.

The values of constants and of conversion factors, and lists of symbols used in the text, appear as appendices. The natural cosmological units are giga-parsec for distance (1 Gpc = 10^9 parsecs $\approx 3 \times 10^{25}$ meters), giga-year for time (1 Gyr = 10^9 years $\approx 3 \times 10^{16}$ seconds), and solar mass (1 $M_\odot \approx 2 \times 10^{30}$ kg) for mass; these units are used throughout the text. Unless otherwise noted, section number references in the text include the chapter number: e.g., Section 9.3.1 is Subsection 1 of Section 3 of Chapter 9.

Problems for students to solve are included at the ends of most of the chapters: these range from fairly simple to rather involved, and are chosen for their pedagogical value. Students, and those reading the book for independent study, are encouraged to do them all. The book is at some pains to include references in the current literature to more extensive discussions of difficult or complex subjects, as guides for further study. Brief historical notes appear where appropriate, but the text makes no pretence of being a guide to the history of cosmology. For extensive discussions of that fascinating subject see, e.g., Nussbaumer and Bieri (2009), Ostriker and Mitton (2013), and Ferreira (2014).

Introducing the Universe

Cosmology is the study of the Universe on the largest scales. As such it deals with structures and dynamics that are quite different from those of terrestrial physics or, indeed, of most other branches of astronomy. It is a subject in which the finite speed of light plays a major role, conflating distance with time: in observing distant galaxies we see them not as they are now, but as they were when the light left them; and that is typically long enough ago to encompass significant evolution in the Universe's contents and dynamics. On cosmological scales the dynamics are dominated by gravitation and, possibly, dark energy. The current theory of gravitation is Einstein's General Theory of Relativity (GR), a non-linear and hugely complex theory entailing subtle and largely unfamiliar mathematical and physical concepts; the nature of dark energy remains speculative as of this writing. It is thus a non-trivial matter to assemble a coherent physical model of the Universe at large, requiring careful definitions of such seemingly mundane things as *distance* and *time*. We begin the effort in this introduction with a brief description of the Universe and its contents, and an assessment of our ability to understand the Universe on large scales in both space and time.

Galaxies and friends

Figure 1 illustrates the Universe as we naively think of it: a vast, crowded collection of galaxies. But this is deceptive, for such a picture collapses three dimensions into two and amplifies brightnesses, and thus under-represents the distances between galaxies and overstates both the density of matter and the degree of illumination of the Universe at large. The Universe in actuality is quite thinly populated and only faintly illuminated.

The visible contents of the Universe at large are almost entirely in the form of galaxies, mostly as large, luminous galaxies such as ours. Large galaxies, such as our Milky Way Galaxy, contain billions of stars and much diffuse matter. A typical example is shown in Figure 2: the spiral disk is $\sim$25 kilo-parsecs

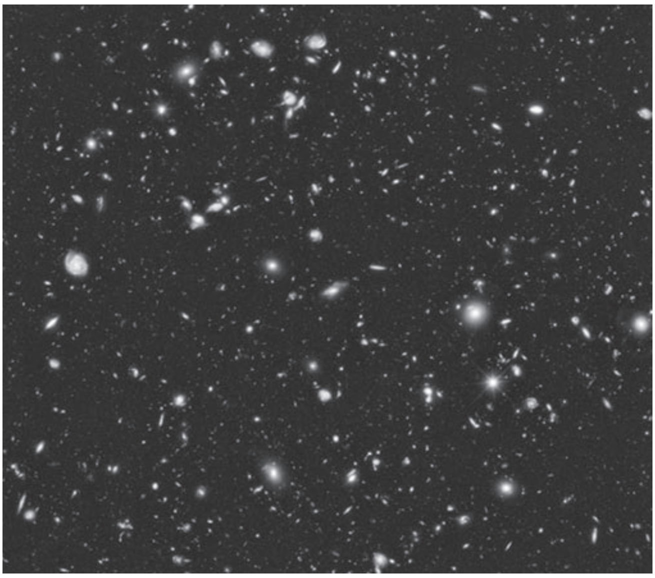

Figure I Hubble Extreme Deep Field (XDF) image, covering ~ 4 square minutes of arc and containing images of ~ 5500 galaxies. As with most digital images, the dynamical range is much higher than can be reproduced in a printed image, so only a portion of the field's contents are evident here. (NASA/ESA/G.Illingworth *et al.*/HUDF Team)

(kpc) in diameter; stars within it are, on average, ~ 1 parsec (pc) apart. On this scale, only the very brightest stars are individually visible; stars similar to, or less massive than, the Sun are too faint to resolve in all but the very nearest galaxies. Galaxies are (nearly) self-contained dynamical and chemical systems and are, in effect, the proper inhabitants of the Universe. Large galaxies such as ours are probably outnumbered by much smaller galaxies, but still account for most of the Universe's stellar content and mass. Relatively very large galaxies, such as the giant ellipticals found at the centers of many galaxy clusters, are so rare as to be relatively minor contributors to the overall galaxy population.

The total mass density of galaxies, averaged throughout the Universe, is the equivalent of less than 1 baryon (proton, neutron) per cubic meter. This is incredibly thin: 'empty' space in our Solar System, and in interstellar space in our part of the Galaxy, is denser by at least four orders of magnitude. By human standards the Universe is very nearly empty. One consequence is that luminous matter is thinly spread throughout the Universe, which is thus only dimly illuminated by the stars that account for nearly all the Universe's visible radiation. The average luminous flux in the Universe is the rough equivalent of that of a 100-watt light bulb at a distance of 10 km, effectively undetectable by the unaided human eye if at all diffuse. By human standards the Universe is, overall, very dark.

Figure 2 Spiral galaxy M101 (NASA/HST).

You can verify this simply by looking up on a dark night: visible stars are so numerous and bright that, at a good site in the right season, the stars in the Milky Way cast a visible shadow. But galaxies external to ours are so faint that, unless you know just where to look and have a very dark sky, you will not see any. The only external galaxy (not a satellite of our Galaxy) that is at all likely to be seen with unaided human vision is M31, the Andromeda Galaxy; barely visible in the northern winter skies as a faint, fuzzy patch. Get out between galaxies in an average location in the Universe and your sky would contain few, if any, visible objects. The bright glory of our night skies is an artifact arising from our quite atypical location in the Universe: relatively *very* dense and bright.

Most of the Universe's galaxies are independent or nearly so, but about 10% are organized into a hierarchy of gravitationally mediated structures ranging in size from small ($\lesssim 10$ galaxies) groups such as the Local Group in which our Galaxy is located; to larger clusters containing hundreds or thousands of galaxies, as illustrated in Figure 3. On even larger scales are clusters of clusters – 'superclusters' – containing up to 10^5 galaxies and spanning 100–150 million parsecs (Mpc). Beyond this there are no larger structures (that we can perceive): the Universe appears to be very smooth on scales greater than ~ 150 Mpc, with typical density variations limited by $|\delta\rho/\rho| \lesssim 10^{-5}$.

In our part of the Universe – the Local Group – large galaxies are typically separated by ~ 1 Mpc, or several tens of their diameters; in larger groupings the galaxy densities can be much greater and galaxy–galaxy interactions are probably common. The total number of large galaxies in the visible Universe is on the order of $\sim 10^{12}$, each of them containing $\gtrsim 10^{10}$ stars. A very large Universe,

Figure 3 Abell 2744, a large cluster of galaxies (NASA/HST.)

indeed; but probably only a tiny portion of the total number, limited as it is by the finite speed of light: since the Universe is of a finite age, only those galaxies close enough for light to have reached us are visible.

Universal expansion

Modern cosmology can fairly be said to have started with the discovery that galaxies appear to be receding from us at speeds proportional to their distances, a large-scale feature that is commonly interpreted as expansion of the entire Universe. The conceptual picture here is one of a uniformly expanding substrate that carries galaxies along with it, so that all galaxies are receding from all other galaxies at speeds proportional to distance: think of ink spots on an expanding balloon, or raisins in expanding bread dough.

The dynamics of such an expansion are best studied as those of a continuum characterized by a universal and non-dimensional expansion function $a(t)$, in terms of which the distance d between any two galaxies evolves as

$$d(t) = a(t)\, d_0 \,, \tag{I.1}$$

where $d_0 = d(t_0)$ is the current distance and a is normalized to $a(t_0) = 1$ (t_0 is the current time). The relative radial velocity between the two galaxies is then $V = \dot{d} = \dot{a}\, d_0 = (\dot{a}/a)\, d$, or velocity proportional to distance. The relative expansion rate $H \equiv \dot{a}/a$ is the **Hubble Parameter**, named for the eponymous astronomer

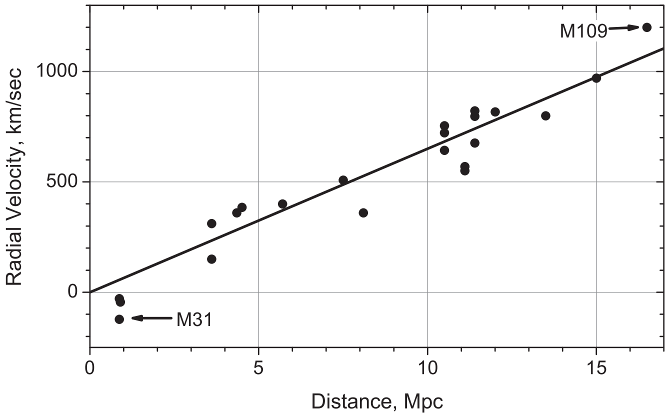

Figure 4 Velocity–distance relation for nearby bright galaxies. The straight line is a least-squares fit with slope 65 km/sec/Mpc (the Hubble Constant for nearby galaxies; see text). Two Messier catalog galaxies are identified with their Messier numbers.

who first proposed the relation

$$V = \frac{\dot{a}}{a} d = Hd \,, \tag{I.2}$$

for galaxies at cosmological distances. The units of H are properly those of inverse time but, following Equation (I.2), are usually given in the mixed units 'km/sec/Mpc'. Since both d and V are observable properties of galaxies, this **Hubble Relation** may be used to estimate the Hubble Parameter and thus the expansion rate of the Universe.

The reality of this Hubble relation is demonstrated in Figure 4, which is similar to that originally published by Hubble in the 1920s (but with more modern data). There is a fair amount of scatter here, due mostly to local effects such as mutual gravitational attractions amongst galaxies and at least partly to uncertain distance estimates; but the obvious trend is compelling evidence of an overall expansion. From such observational evidence for large numbers of galaxies, we estimate the current value of the Hubble Constant $H_0 \equiv H(t_0)$ to be $\approx 72 \pm 5$ km/sec/Mpc, or ≈ 0.074 Gyr^{-1}.

Universal evolution

The Universe shows evident signs of evolution of its content (principally in its galaxy population) and contains radiation interpreted as the remnant of an early, hot phase of the entire Universe. The current theory encompassing these matters is the 'Big Bang' theory, in which the Universe starts in a very hot, dense state from which it expands and cools. With addenda from particle and quantum physics, the Big Bang theory constitutes the current cosmological paradigm.

If the Universe were expanding at a constant rate $\dot{a}$ beginning with $a(0) = 0$, the expansion function would evolve as $a(t) = \dot{a}t$ and thus $H = \dot{a}/a = 1/t$; and the current age would be the **Hubble Time**, $t_H = 1/H_0$, a characteristic time scale in an expanding universe. With our observational estimate of $H_0 = 0.074\ \text{Gyr}^{-1}$ the Universe's current age would then be $t_0 = t_H \approx 13.5$ Gyr. This is gratifyingly similar to the estimated age of the oldest stars in our Galaxy, and suggests that our Galaxy formed early in the Universe's expansion. But gravitation and dark energy will modify the expansion rate, so such age estimates can only be approximations.

After its initial startup phases, and prior to the age of $\sim$ 400 million years, the Universe was filled with a diffuse, nearly uniform hydrogen/helium gas lit only by its own thermal radiation. As stars and galaxies formed from this gas, and the Universe expanded, it became less homogeneous but diffuse overall, and illuminated by starlight: at the time of maximum star formation rate ($t \sim 3$ Gyr) it was more than an order of magnitude denser than it currently is, and brighter by several orders of magnitude. Since that distant time the Universe has become thinner and darker, and will probably continue to do so into the foreseeable future.

Universal dynamics

The Universe's evolution is a dynamical consequence of its initial expansion – inherited from the Big Bang – and of its matter and energy contents. Since Special Relativity has it that energy and matter are equivalent, both are sources of gravitation and contribute to the Universe's dynamical history. That history is dominated by exotic forms of matter and energy – generically labelled 'dark' – of currently unknown origin and nature, that constitute $\sim$ 95% of the Universe's gravitating contents. What we observe as the visible Universe (galaxies, etc.) constitutes a sort of thin veneer over the 'real' – but unobserved, and mysterious – Universe of dark matter and energy. Observations of the effects of dark energy and matter on the visible Universe, and reasonable extrapolations from known physics, hold out hope that the dynamics of these forms of matter/energy can be understood and modelled, even if the dark stuff itself remains of a mysterious nature. Models of the Universe's expansion, such as are developed in this text, are based on such expectations which may be proven wrong in the fullness of time, and require correction. The search for the nature of these dark components is a major effort of modern cosmological research.

Universal models

Cosmologists attempt to explain large-scale features of the Universe, such as the Hubble Relation or its hierarchical structure, in terms of physical models that are based largely on explicit forms for the expansion function $a(t)$ as defined in Equation (I.1). On cosmological scales the dominant forces are those of gravitation, for which the currently accepted theory is Einstein's General Theory of Relativity (GR); and dark energy, for which there is currently no good theory but whose dynamical effects appear to be understood from observations of the Universe's expansion. Applied to a Universe of given distributions of mass and energy, GR should produce a model for $a(t)$ whose observational consequences would be testable, much as Newtonian gravitation predicts the paths of objects subject to gravitational fields produced by distributions of masses.

The cosmologists' game then becomes one of estimating the proper form of the expansion function by working the problem from both ends. On the one hand we estimate the mass/energy densities as closely as possible from direct observations of, e.g., galaxy masses and spatial densities, and electromagnetic radiation fields; and compute $a(t)$ therefrom, using GR to relate expansion to gravitation. On the other, the observational consequences of the models, such as the Hubble Relation between distance and recession velocity, are tested against actual observations in order to assess their validity. Neither of these approaches gives unambiguous results since they both rely on difficult observations of uncertain accuracy, but iteration on the pair typically converges to a self-consistent model for the expansion function that is of useful validity, to the extent that the underlying observations are correct.

The remainder of this text sets up the physics needed to model the expanding Universe (Part II) and then uses the results to construct physically meaningful models of the Universe's structure and dynamics (Parts III–V). Of necessity, the discussion is rather abstract and heavily mathematical in places, so we devote Part I to easing the student into the subject with an introduction to the underlying concepts.