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Javier Bonet, Antonio J. Gil, Richard D. Wood
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NONLINEAR SOLID MECHANICS FOR FINITE ELEMENT ANALYSIS: STATICS

Designing engineering components that make optimal use of materials requires consideration of the nonlinear static and dynamic characteristics associated with both manufacturing and working environments. The modeling of these characteristics can only be done through numerical formulation and simulation, which requires an understanding of both the theoretical background and associated computer solution techniques. By presenting both the nonlinear solid mechanics and the associated finite element techniques together, the authors provide, in the first of two books in this series, a complete, clear, and unified treatment of the static aspects of nonlinear solid mechanics.

Alongside a range of worked examples and exercises are user instructions, program descriptions, and examples for the FLagSHyP MATLAB computer implementation, for which the source code is available online.

While this book is designed to complement postgraduate courses, it is also relevant to those in industry requiring an appreciation of the way their computer simulation programs work.

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RICHARD D. WOOD is an Honorary Research Fellow in the Zienkiewicz Centre for Computational Engineering at Swansea University. He has over 20 years' experience of teaching the course "Nonlinear Continuum Mechanics for Finite Element Analysis" at Swansea University, which he originally developed at the University of Arizona. Wood's academic career has focused on finite element analysis. He has written numerous papers in international journals, and many chapter contributions, on topics related to nonlinear finite element analysis.

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Frontmatter
More Information

To Catherine, Clare, Doreen and our children

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Frontmatter
More Information

A fragment from the poem
"An Essay on Criticism"
by Alexander Pope (1688–1744)

A little Learning is a dang'rous Thing;
Drink deep, or taste not the Pierian Spring:
There shallow Draughts intoxicate the Brain,
And drinking largely sobers us again.
Fir'd at first Sight with what the Muse imparts,
In fearless Youth we tempt the Heights of Arts,
While from the bounded Level of our Mind,
Short Views we take, nor see the lengths behind,
But more advanc'd, behold with strange Surprize
New, distant Scenes of endless Science rise!

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Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

CONTENTS

Preface *page* xiii

1 INTRODUCTION **1**
1.1 NONLINEAR COMPUTATIONAL MECHANICS 1
1.2 SIMPLE EXAMPLES OF NONLINEAR STRUCTURAL BEHAVIOR 2
1.2.1 Cantilever 2
1.2.2 Column 3
1.3 NONLINEAR STRAIN MEASURES 4
1.3.1 One-Dimensional Strain Measures 4
1.3.2 Nonlinear Truss Example 6
1.3.3 Continuum Strain Measures 10
1.4 DIRECTIONAL DERIVATIVE, LINEARIZATION AND EQUATION SOLUTION 12
1.4.1 Directional Derivative 13
1.4.2 Linearization and Solution of Nonlinear Algebraic Equations 15
EXERCISES 19

2 MATHEMATICAL PRELIMINARIES **21**
2.1 INTRODUCTION 21
2.2 VECTOR AND TENSOR ALGEBRA 21
2.2.1 Vectors 22
2.2.2 Second-Order Tensors 26
2.2.3 Vector and Tensor Invariants 35
2.2.4 Higher-Order Tensors 39
2.3 LINEARIZATION AND THE DIRECTIONAL DERIVATIVE 45
2.3.1 One Degree of Freedom 45
2.3.2 General Solution to a Nonlinear Problem 46

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CAMBRIDGE

Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

2.3.3 Properties of the Directional Derivative 48
2.3.4 Examples of Linearization 49
2.4 TENSOR ANALYSIS 54
2.4.1 The Gradient and Divergence Operators 54
2.4.2 Integration Theorems 56
EXERCISES 57

3 ANALYSIS OF THREE-DIMENSIONAL TRUSS STRUCTURES 59
3.1 INTRODUCTION 59
3.2 KINEMATICS 61
3.2.1 Linearization of Geometrical Descriptors 63
3.3 INTERNAL FORCES AND HYPERELASTIC CONSTITUTIVE EQUATIONS 64
3.4 NONLINEAR EQUILIBRIUM EQUATIONS AND THE NEWTON–RAPHSON SOLUTION 66
3.4.1 Equilibrium Equations 66
3.4.2 Newton–Raphson Procedure 67
3.4.3 Tangent Elastic Stiffness Matrix 68
3.5 TOTAL POTENTIAL ENERGY 71
3.5.1 Principle of Virtual Work 72
3.6 ELASTO-PLASTIC BEHAVIOR 75
3.6.1 Multiplicative Decomposition of the Stretch 76
3.6.2 Rate-Independent Plasticity 77
3.6.3 Incremental Kinematics 81
3.6.4 Time Integration 83
3.6.5 Stress Update and Return Mapping 84
3.6.6 Algorithmic Tangent Modulus 87
3.6.7 Revised Newton–Raphson Procedure 88
3.7 EXAMPLES 89
3.7.1 Inclined Axial Rod 89
3.7.2 Trussed Frame 90
EXERCISES 91

4 KINEMATICS 96
4.1 INTRODUCTION 96
4.2 THE MOTION 96
4.3 MATERIAL AND SPATIAL DESCRIPTIONS 97
4.4 DEFORMATION GRADIENT 99
4.5 STRAIN 102
4.6 POLAR DECOMPOSITION 106
4.7 VOLUME CHANGE 112

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

4.8 DISTORTIONAL COMPONENT OF THE DEFORMATION GRADIENT 113
4.9 AREA CHANGE 116
4.10 LINEARIZED KINEMATICS 117
4.10.1 Linearized Deformation Gradient 117
4.10.2 Linearized Strain 118
4.10.3 Linearized Volume Change 119
4.11 VELOCITY AND MATERIAL TIME DERIVATIVES 119
4.11.1 Velocity 119
4.11.2 Material Time Derivative 120
4.11.3 Directional Derivative and Time Rates 121
4.11.4 Velocity Gradient 122
4.12 RATE OF DEFORMATION 123
4.13 SPIN TENSOR 126
4.14 RATE OF CHANGE OF VOLUME 129
4.15 SUPERIMPOSED RIGID BODY MOTIONS AND OBJECTIVITY 130
EXERCISES 132

5 STRESS AND EQUILIBRIUM 137
5.1 INTRODUCTION 137
5.2 CAUCHY STRESS TENSOR 137
5.2.1 Definition 137
5.2.2 Stress Objectivity 141
5.3 EQUILIBRIUM 142
5.3.1 Translational Equilibrium 142
5.3.2 Rotational Equilibrium 144
5.4 PRINCIPLE OF VIRTUAL WORK 145
5.5 WORK CONJUGACY AND ALTERNATIVE STRESS REPRESENTATIONS 146
5.5.1 The Kirchhoff Stress Tensor 146
5.5.2 The First Piola–Kirchhoff Stress Tensor 147
5.5.3 The Second Piola–Kirchhoff Stress Tensor 150
5.5.4 Deviatoric and Pressure Components 153
5.6 STRESS RATES 154
EXERCISES 156

6 HYPERELASTICITY 158
6.1 INTRODUCTION 158
6.2 HYPERELASTICITY 158
6.3 ELASTICITY TENSOR 160
6.3.1 The Material or Lagrangian Elasticity Tensor 160
6.3.2 The Spatial or Eulerian Elasticity Tensor 161

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

6.4 ISOTROPIC HYPERELASTICITY 162
6.4.1 Material Description 162
6.4.2 Spatial Description 163
6.4.3 Compressible Neo-Hookean Material 165
6.5 INCOMPRESSIBLE AND NEARLY INCOMPRESSIBLE MATERIALS 168
6.5.1 Incompressible Elasticity 168
6.5.2 Incompressible Neo-Hookean Material 170
6.5.3 Nearly Incompressible Hyperelastic Materials 172
6.6 ISOTROPIC ELASTICITY IN PRINCIPAL DIRECTIONS 175
6.6.1 Material Description 175
6.6.2 Spatial Description 176
6.6.3 Material Elasticity Tensor 177
6.6.4 Spatial Elasticity Tensor 179
6.6.5 A Simple Stretch-Based Hyperelastic Material 180
6.6.6 Nearly Incompressible Material in Principal Directions 181
6.6.7 Plane Strain and Plane Stress Cases 184
6.6.8 Uniaxial Rod Case 185
EXERCISES 186

7 LARGE ELASTO-PLASTIC DEFORMATIONS 188
7.1 INTRODUCTION 188
7.2 THE MULTIPLICATIVE DECOMPOSITION 188
7.3 RATE KINEMATICS 193
7.4 RATE-INDEPENDENT PLASTICITY 197
7.5 PRINCIPAL DIRECTIONS 199
7.6 INCREMENTAL KINEMATICS 203
7.6.1 The Radial Return Mapping 206
7.6.2 Algorithmic Tangent Modulus 208
7.7 TWO-DIMENSIONAL CASES 209
EXERCISES 212

8 LINEARIZED EQUILIBRIUM EQUATIONS 214
8.1 INTRODUCTION 214
8.2 LINEARIZATION AND THE NEWTON–RAPHSON PROCESS 214
8.3 LAGRANGIAN LINEARIZED INTERNAL VIRTUAL WORK 216
8.4 EULERIAN LINEARIZED INTERNAL VIRTUAL WORK 217
8.5 LINEARIZED EXTERNAL VIRTUAL WORK 219
8.5.1 Body Forces 219
8.5.2 Surface Forces 219
8.6 VARIATIONAL METHODS AND INCOMPRESSIBILITY 221
8.6.1 Total Potential Energy and Equilibrium 222

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Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

8.6.2 Lagrange Multiplier Approach to Incompressibility 223
8.6.3 Penalty Methods for Incompressibility 226
8.6.4 Hu–Washizu Variational Principle for Incompressibility 227
8.6.5 Mean Dilatation Procedure 229
EXERCISES 232

9 DISCRETIZATION AND SOLUTION 234
9.1 INTRODUCTION 234
9.2 DISCRETIZED KINEMATICS 234
9.3 DISCRETIZED EQUILIBRIUM EQUATIONS 239
9.3.1 General Derivation 239
9.3.2 Derivation in Matrix Notation 241
9.4 DISCRETIZATION OF THE LINEARIZED EQUILIBRIUM EQUATIONS 242
9.4.1 Constitutive Component: Indicial Form 244
9.4.2 Constitutive Component: Matrix Form 245
9.4.3 Initial Stress Component 246
9.4.4 External Force Component 247
9.4.5 Tangent Matrix 249
9.5 MEAN DILATATION METHOD FOR INCOMPRESSIBILITY 251
9.5.1 Implementation of the Mean Dilatation Method 251
9.6 NEWTON–RAPHSON ITERATION AND SOLUTION PROCEDURE 253
9.6.1 Newton–Raphson Solution Algorithm 253
9.6.2 Line Search Method 254
9.6.3 Arc Length Method 256
EXERCISES 258

10 COMPUTER IMPLEMENTATION 260
10.1 INTRODUCTION 260
10.2 USER INSTRUCTIONS 263
10.3 OUTPUT FILE DESCRIPTION 269
10.4 ELEMENT TYPES 272
10.5 SOLVER DETAILS 274
10.6 PROGRAM STRUCTURE 276
10.7 MASTER m-FILE `FLagSHyP` 278
10.8 FUNCTION `residual_and_stiffness_assembly` 285
10.9 FUNCTION `constitutive_matrix` 294
10.10 FUNCTION `geometric_matrix` 295
10.11 FUNCTION `pressure_load_and_stiffness_assembly` 296
10.12 EXAMPLES 298
10.12.1 Simple Patch Test 298
10.12.2 Nonlinear Truss 299
10.12.3 Strip with a Hole 300

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

10.12.4 Plane Strain Nearly Incompressible Strip 300
10.12.5 Twisting Column 302
10.12.6 Elasto-Plastic Cantilever 303
10.13 APPENDIX: DICTIONARY OF MAIN VARIABLES 306
10.14 APPENDIX: CONSTITUTIVE EQUATION SUMMARY 309

Bibliography 316

Index 318

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CAMBRIDGE
Cambridge University Press & Assessment
978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

PREFACE

A fundamental aspect of engineering is the desire to design artifacts that exploit materials to a maximum in terms of performance under working conditions and efficiency of manufacture. Such an activity demands an increasing understanding of the behavior of the artifact in its working environment together with an understanding of the mechanical processes occurring during manufacture.

To be able to achieve these goals it is likely that the engineer will need to consider the nonlinear characteristics associated possibly with the manufacturing process but certainly with the response to working load. Currently, analysis is most likely to involve a computer simulation of the behavior. Because of the availability of commercial finite element computer software, the opportunity for such nonlinear analysis is becoming increasingly realized.

Such a situation has an immediate educational implication because, for computer programs to be used sensibly and for the results to be interpreted wisely, it is essential that the users have some familiarity with the fundamentals of nonlinear continuum mechanics, nonlinear finite element formulations, and the solution techniques employed by the software. This book seeks to address this problem by providing a unified introduction to these three topics.

The style and content of the book obviously reflect the attributes and abilities of the authors. The authors have lectured on this material for a number of years to postgraduate classes, and the book has emerged from these courses. We hope that our complementary approaches to the topic will be in tune with the variety of backgrounds expected of our readers and, ultimately, that the book will provide a measure of enjoyment brought about by a greater understanding of what we regard as a fascinating subject.

The original edition of this book, titled *Nonlinear Continuum Mechanics for Finite Element Analysis*, published in 1997, contained a chapter on a FORTRAN program implementation of the material in the text, this being freely available at `www.flagshyp.com`. In 2008 a second edition included new chapters on

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Frontmatter
More Information

elasto-plastic behavior of trusses and solids and retained the FORTRAN implementation. It was envisioned that an expanded third edition could include dynamics, although this would involve substantial additional material not suitable to the needs of all readers. Consequently the subject has been divided into two complementary volumes, these being the present text, *Nonlinear Solid Mechanics for Finite Element Analysis: Statics*, and a companion volume, *Nonlinear Solid Mechanics for Finite Element Analysis: Dynamics*. These texts are both aimed at the same readership. Recognising its widespread adoption, particularly as a graduate training platform, this present statics text employs MATLAB®* for the implementation of the finite element analysis, the software being freely available at `www.flagshyp.com`.

This present text contains additional examples, and solutions to all exercises are given in the companion book, *Worked Examples in Nonlinear Continuum Mechanics for Finite Element Analysis*, published by Cambridge University Press (ISBN 9781107603615).

READERSHIP

This book is most suited to a postgraduate level of study by those in either higher education or industry who have graduated with an engineering or applied mathematics degree. However, the material is equally applicable to first-degree students in the final year of an applied mathematics course or an engineering course containing some additional emphasis on maths and numerical analysis. A familiarity with statics and elementary stress analysis is assumed, as is some exposure to the principles of the finite element method. However, a primary objective of the book is that it be reasonably self-contained, particularly with respect to the nonlinear continuum mechanics chapters, which comprise a large portion of the content.

When dealing with such a complex set of topics it is unreasonable to expect all readers to become familiar with all aspects of the book. If the reader is prepared not to get too hung up on details, it is possible to use the book to obtain a reasonable overview of the subject. Such an approach may be suitable for someone starting to use a nonlinear computer program. Alternatively, the requirements of a research project may necessitate a deeper understanding of the concepts discussed. To assist in this latter endeavor the book provides access to a computer program for the nonlinear finite deformation finite element analysis of two- and three-dimensional solids. Such a program provides the basis for a contemporary approach to finite deformation elasto-plastic analysis.

* Mathworks, Inc.

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Frontmatter
More Information

LAYOUT

Chapter 1: Introduction

Here, the nature of nonlinear computational mechanics is discussed, and followed by a series of very simple examples that demonstrate various aspects of nonlinear structural behavior. These examples are intended, to an extent, to upset the reader's preconceived ideas inherited from an overexposure to linear analysis and, we hope, provide a motivation for reading the rest of the book! Nonlinear strain measures are introduced and illustrated using a simple one-degree-of-freedom truss analysis. The concepts of linearization and the directional derivative are of sufficient importance to merit a gentle introduction in this chapter. Linearization naturally leads on to the Newton–Raphson iterative solution, which is the fundamental way of solving the nonlinear equilibrium equations occurring in finite element analysis. Consequently, by way of an example, the simple truss is solved and a short MATLAB program is presented that, in essence, is the prototype for the main finite element program discussed later in the book.

Chapter 2: Mathematical Preliminaries

Vector and tensor manipulations occur throughout the book, and these are introduced in this chapter. Although vector algebra is a well-known topic, tensor algebra is less familiar, certainly, to many approaching the subject with an engineering educational background. Consequently, tensor algebra is considered in enough detail to cover the needs of the subsequent chapters; in particular, it is hoped that readers will understand the physical interpretation of a second-order tensor. Crucial to the development of the finite element solution scheme are the concepts of linearization and the directional derivative. The introduction provided in Chapter 1 is now thoroughly developed. Finally, for completeness, some standard analysis topics are briefly presented.

Chapter 3: Analysis of Three-dimensional Truss Structures

This chapter is largely independent of the remainder of the book and deals with the large strain elasto-plastic behavior of trusses. The chapter begins with a discussion of the nonlinear kinematics of a simple two-noded truss member, which leads to a definition of logarithmic strain. A hyperelastic stress–strain relationship is then derived and used to obtain the equilibrium equations at a node. In preparation for the variational formulation in Chapter 8 the equilibrium equations are re-derived

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Frontmatter
More Information

using an energy approach. These equations are then linearized with respect to small incremental displacements to provide a Newton–Raphson solution process. The chapter then moves on to discuss a simple hyperelastic plastic model for the truss member based on the multiplicative decomposition of the total stretch into elastic and plastic components. The constitutive model is also linearized to provide a tangent modulus. The chapter concludes with some examples of the use of the formulation obtained using the FLagSHyP program.

Chapter 4: Kinematics

This chapter deals with the kinematics of finite deformation, that is, the study of motion without reference to the cause. Central to this concept is the deformation gradient tensor, which describes the relationship between elemental vectors defining neighboring particles in the undeformed and deformed configurations of the body whose motion is under consideration. The deformation gradient permeates most of the development of finite deformation kinematics because, among other things, it enables a variety of definitions of strain to be established. Material (initial) and spatial (current) descriptions of various items are discussed, as is the linearization of kinematic quantities. Although dynamics is not the subject of this book, it is nevertheless necessary to consider velocity and the rate of deformation. The chapter concludes with a brief discussion of rigid body motion and objectivity.

Chapter 5: Stress and Equilibrium

The definition of the true or Cauchy stress is followed by the development of standard differential equilibrium equations. As a prelude to the finite element development the equilibrium equations are recast in the weak integral virtual work form. Although initially in the spatial or current deformed configuration, these equations are reformulated in terms of the material or undeformed configuration, and as a consequence alternative stress measures emerge. Finally, stress rates are discussed in preparation for the following chapter on hyperelasticity.

Chapter 6: Hyperelasticity

Hyperelasticity, whereby the stress is found as a derivative of some potential energy function, encompasses many types of nonlinear material behavior and provides the basis for the finite element treatment of elasto-plastic behavior. Isotropic

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Frontmatter
More Information

hyperelasticity is considered both in a material and in a spatial description for compressible and incompressible behavior. The topic is extended to a general description in principal directions that is specialized for the cases of plane strain, plane stress, and uniaxial behavior.

Chapter 7: Large Elasto-Plastic Deformations

This chapter provides an introduction to the formulation of inelastic deformation processes based on the multiplicative decomposition of the deformation gradient into recoverable and permanent components. Although only a basic Von Mises model with a radial return-mapping procedure is presented, the use of principal directions and logarithmic stretches provides a simple mechanism whereby small strain concepts can be extended to large strains. From the outset, the approach followed, to derive the kinematic rate equations necessary for the flow rule, anticipates the standard trial stress and return-map procedure required to satisfy the plasticity constraints. Such a development clarifies the kinematic rate equations in the context of the eventual incremental algorithmic procedure.

Chapter 8: Linearized Equilibrium Equations

To establish the Newton–Raphson solution procedure the virtual work expression of equilibrium may be linearized either before or after discretization. Here the former approach is adopted. Linearization of the equilibrium equations includes consideration of deformation-dependent surface pressure loading. A large proportion of this chapter is devoted to incompressibility and to the development, via the Hu–Washizu principle, of the mean dilatation technique.

Chapter 9: Discretization and Solution

All previous chapters have provided the foundation for the development of the discretized equilibrium and linearized equilibrium equations considered in this chapter. Linearization of the virtual work equation leads to the familiar finite element expression of equilibrium involving $\int \boldsymbol{B}^T \boldsymbol{\sigma} dv$, whereas discretization of the linearized equilibrium equations leads to the tangent matrix, which comprises constitutive and initial stress components. Discretization of the mean dilatation technique is presented in detail. The tangent matrix forms the basis of the Newton–Raphson solution procedure, which is presented as the fundamental solution technique enshrined in the computer program discussed in the following chapter. The

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Javier Bonet, Antonio J. Gil, Richard D. Wood
Frontmatter
More Information

chapter concludes with a discussion of line search and arc length enhancements to the Newton–Raphson procedure.

Chapter 10: Computer Implementation

Here, information is presented on a nonlinear finite element computer program called FLagSHyP,[†] for the solution of finite deformation elasto-plastic finite element problems employing the neo-Hookean hyperelastic compressible and incompressible constitutive equations developed in Chapters 6 and 7. The usage and layout of the MATLAB program is discussed together with the description of the various key functions. The program is available free on the Internet from the website `www.flagshyp.com`. The website also contains the original FORTRAN version of the program which is detailed in previous editions of this text. Note that the user instructions and output layout are the same for both the FORTRAN and MATLAB versions of the program. The software can also be obtained by `email` request to any of the authors: `a.j.gil@swansea.ac.uk`, `r.d.wood@swansea.ac.uk` or `j.bonet@swansea.ac.uk`. The authors would like to acknowledge the assistance given by Dr. Rogelio Ortigosa in the development of this computer program.

A bibliography is provided that enables the reader to access the background to the more standard aspects of finite element analysis. Also listed are books and papers that have been of use in the preparation of this book or that cover similar material in greater depth.

Note on equation numbering: Typically, Equation (x.yz a, b, c, d)$_b$ refers to Equation (x.yz, b).

[†] **F**inite element **Larg**e **S**train **Hy**perelasto-plastic **P**rogram.

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