NONLINEAR SOLID MECHANICS FOR FINITE ELEMENT ANALYSIS: STATICS

Designing engineering components that make optimal use of materials requires consideration of the nonlinear static and dynamic characteristics associated with both manufacturing and working environments. The modeling of these characteristics can only be done through numerical formulation and simulation, which requires an understanding of both the theoretical background and associated computer solution techniques. By presenting both the nonlinear solid mechanics and the associated finite element techniques together, the authors provide, in the first of two books in this series, a complete, clear, and unified treatment of the static aspects of nonlinear solid mechanics.

Alongside a range of worked examples and exercises are user instructions, program descriptions, and examples for the FLagSHyP MATLAB computer implementation, for which the source code is available online.

While this book is designed to complement postgraduate courses, it is also relevant to those in industry requiring an appreciation of the way their computer simulation programs work.

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To Catherine, Clare, Doreen and our children

A fragment from the poem
"An Essay on Criticism"
by Alexander Pope (1688–1744)

A little Learning is a dang'rous Thing;
Drink deep, or taste not the Pierian Spring:
There shallow Draughts intoxicate the Brain,
And drinking largely sobers us again.
Fir'd at first Sight with what the Muse imparts,
In fearless Youth we tempt the Heights of Arts,
While from the bounded Level of our Mind,
Short Views we take, nor see the lengths behind,
But more advanc'd, behold with strange Surprize
New, distant Scenes of endless Science rise!

CONTENTS

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978-1-107-11579-8 — Nonlinear Solid Mechanics for Finite Element Analysis: Statics
Javier Bonet , Antonio J. Gil , Richard D. Wood
Frontmatter
More Information

PREFACE

A fundamental aspect of engineering is the desire to design artifacts that exploit materials to a maximum in terms of performance under working conditions and efficiency of manufacture. Such an activity demands an increasing understanding of the behavior of the artifact in its working environment together with an understanding of the mechanical processes occurring during manufacture.

To be able to achieve these goals it is likely that the engineer will need to consider the nonlinear characteristics associated possibly with the manufacturing process but certainly with the response to working load. Currently, analysis is most likely to involve a computer simulation of the behavior. Because of the availability of commercial finite element computer software, the opportunity for such nonlinear analysis is becoming increasingly realized.

Such a situation has an immediate educational implication because, for computer programs to be used sensibly and for the results to be interpreted wisely, it is essential that the users have some familiarity with the fundamentals of nonlinear continuum mechanics, nonlinear finite element formulations, and the solution techniques employed by the software. This book seeks to address this problem by providing a unified introduction to these three topics.

The style and content of the book obviously reflect the attributes and abilities of the authors. The authors have lectured on this material for a number of years to postgraduate classes, and the book has emerged from these courses. We hope that our complementary approaches to the topic will be in tune with the variety of backgrounds expected of our readers and, ultimately, that the book will provide a measure of enjoyment brought about by a greater understanding of what we regard as a fascinating subject.

The original edition of this book, titled *Nonlinear Continuum Mechanics for Finite Element Analysis*, published in 1997, contained a chapter on a FORTRAN program implementation of the material in the text, this being freely available at `www.flagshyp.com`. In 2008 a second edition included new chapters on elasto-plastic behavior of trusses and solids and retained the FORTRAN implementation. It was envisioned that an expanded third edition could include dynamics, although this would involve substantial additional material not suitable to the needs of all readers. Consequently the subject has been divided into two complementary volumes, these being the present text, *Nonlinear Solid Mechanics for Finite Element Analysis: Statics*, and a companion volume, *Nonlinear Solid Mechanics for Finite Element Analysis: Dynamics*. These texts are both aimed at the same readership. Recognising its widespread adoption, particularly as a graduate training platform, this present statics text employs MATLAB®* for the implementation of the finite element analysis, the software being freely available at `www.flagshyp.com`.

This present text contains additional examples, and solutions to all exercises are given in the companion book, *Worked Examples in Nonlinear Continuum Mechanics for Finite Element Analysis*, published by Cambridge University Press (ISBN 9781107603615).

READERSHIP

This book is most suited to a postgraduate level of study by those in either higher education or industry who have graduated with an engineering or applied mathematics degree. However, the material is equally applicable to first-degree students in the final year of an applied mathematics course or an engineering course containing some additional emphasis on maths and numerical analysis. A familiarity with statics and elementary stress analysis is assumed, as is some exposure to the principles of the finite element method. However, a primary objective of the book is that it be reasonably self-contained, particularly with respect to the nonlinear continuum mechanics chapters, which comprise a large portion of the content.

When dealing with such a complex set of topics it is unreasonable to expect all readers to become familiar with all aspects of the book. If the reader is prepared not to get too hung up on details, it is possible to use the book to obtain a reasonable overview of the subject. Such an approach may be suitable for someone starting to use a nonlinear computer program. Alternatively, the requirements of a research project may necessitate a deeper understanding of the concepts discussed. To assist in this latter endeavor the book provides access to a computer program for the nonlinear finite deformation finite element analysis of two- and three-dimensional solids. Such a program provides the basis for a contemporary approach to finite deformation elasto-plastic analysis.

* Mathworks, Inc.

LAYOUT

Chapter 1: Introduction

Here, the nature of nonlinear computational mechanics is discussed, and followed by a series of very simple examples that demonstrate various aspects of nonlinear structural behavior. These examples are intended, to an extent, to upset the reader's preconceived ideas inherited from an overexposure to linear analysis and, we hope, provide a motivation for reading the rest of the book! Nonlinear strain measures are introduced and illustrated using a simple one-degree-of-freedom truss analysis. The concepts of linearization and the directional derivative are of sufficient importance to merit a gentle introduction in this chapter. Linearization naturally leads on to the Newton–Raphson iterative solution, which is the fundamental way of solving the nonlinear equilibrium equations occurring in finite element analysis. Consequently, by way of an example, the simple truss is solved and a short MATLAB program is presented that, in essence, is the prototype for the main finite element program discussed later in the book.

Chapter 2: Mathematical Preliminaries

Vector and tensor manipulations occur throughout the book, and these are introduced in this chapter. Although vector algebra is a well-known topic, tensor algebra is less familiar, certainly, to many approaching the subject with an engineering educational background. Consequently, tensor algebra is considered in enough detail to cover the needs of the subsequent chapters; in particular, it is hoped that readers will understand the physical interpretation of a second-order tensor. Crucial to the development of the finite element solution scheme are the concepts of linearization and the directional derivative. The introduction provided in Chapter 1 is now thoroughly developed. Finally, for completeness, some standard analysis topics are briefly presented.

Chapter 3: Analysis of Three-dimensional Truss Structures

This chapter is largely independent of the remainder of the book and deals with the large strain elasto-plastic behavior of trusses. The chapter begins with a discussion of the nonlinear kinematics of a simple two-noded truss member, which leads to a definition of logarithmic strain. A hyperelastic stress–strain relationship is then derived and used to obtain the equilibrium equations at a node. In preparation for the variational formulation in Chapter 8 the equilibrium equations are re-derived using an energy approach. These equations are then linearized with respect to small incremental displacements to provide a Newton–Raphson solution process. The chapter then moves on to discuss a simple hyperelastic plastic model for the truss member based on the multiplicative decomposition of the total stretch into elastic and plastic components. The constitutive model is also linearized to provide a tangent modulus. The chapter concludes with some examples of the use of the formulation obtained using the FLagSHyP program.

Chapter 4: Kinematics

This chapter deals with the kinematics of finite deformation, that is, the study of motion without reference to the cause. Central to this concept is the deformation gradient tensor, which describes the relationship between elemental vectors defining neighboring particles in the undeformed and deformed configurations of the body whose motion is under consideration. The deformation gradient permeates most of the development of finite deformation kinematics because, among other things, it enables a variety of definitions of strain to be established. Material (initial) and spatial (current) descriptions of various items are discussed, as is the linearization of kinematic quantities. Although dynamics is not the subject of this book, it is nevertheless necessary to consider velocity and the rate of deformation. The chapter concludes with a brief discussion of rigid body motion and objectivity.

Chapter 5: Stress and Equilibrium

The definition of the true or Cauchy stress is followed by the development of standard differential equilibrium equations. As a prelude to the finite element development the equilibrium equations are recast in the weak integral virtual work form. Although initially in the spatial or current deformed configuration, these equations are reformulated in terms of the material or undeformed configuration, and as a consequence alternative stress measures emerge. Finally, stress rates are discussed in preparation for the following chapter on hyperelasticity.

Chapter 6: Hyperelasticity

Hyperelasticity, whereby the stress is found as a derivative of some potential energy function, encompasses many types of nonlinear material behavior and provides the basis for the finite element treatment of elasto-plastic behavior. Isotropic hyperelasticity is considered both in a material and in a spatial description for compressible and incompressible behavior. The topic is extended to a general description in principal directions that is specialized for the cases of plane strain, plane stress, and uniaxial behavior.

Chapter 7: Large Elasto-Plastic Deformations

This chapter provides an introduction to the formulation of inelastic deformation processes based on the multiplicative decomposition of the deformation gradient into recoverable and permanent components. Although only a basic Von Mises model with a radial return-mapping procedure is presented, the use of principal directions and logarithmic stretches provides a simple mechanism whereby small strain concepts can be extended to large strains. From the outset, the approach followed, to derive the kinematic rate equations necessary for the flow rule, anticipates the standard trial stress and return-map procedure required to satisfy the plasticity constraints. Such a development clarifies the kinematic rate equations in the context of the eventual incremental algorithmic procedure.

Chapter 8: Linearized Equilibrium Equations

To establish the Newton–Raphson solution procedure the virtual work expression of equilibrium may be linearized either before or after discretization. Here the former approach is adopted. Linearization of the equilibrium equations includes consideration of deformation-dependent surface pressure loading. A large proportion of this chapter is devoted to incompressibility and to the development, via the Hu–Washizu principle, of the mean dilatation technique.

Chapter 9: Discretization and Solution

All previous chapters have provided the foundation for the development of the discretized equilibrium and linearized equilibrium equations considered in this chapter. Linearization of the virtual work equation leads to the familiar finite element expression of equilibrium involving $\int \boldsymbol{B}^T \boldsymbol{\sigma} dv$, whereas discretization of the linearized equilibrium equations leads to the tangent matrix, which comprises constitutive and initial stress components. Discretization of the mean dilatation technique is presented in detail. The tangent matrix forms the basis of the Newton–Raphson solution procedure, which is presented as the fundamental solution technique enshrined in the computer program discussed in the following chapter. The chapter concludes with a discussion of line search and arc length enhancements to the Newton–Raphson procedure.

Chapter 10: Computer Implementation

Here, information is presented on a nonlinear finite element computer program called FLagSHyP,† for the solution of finite deformation elasto-plastic finite element problems employing the neo-Hookean hyperelastic compressible and incompressible constitutive equations developed in Chapters 6 and 7. The usage and layout of the MATLAB program is discussed together with the description of the various key functions. The program is available free on the Internet from the website `www.flagshyp.com`. The website also contains the original FORTRAN version of the program which is detailed in previous editions of this text. Note that the user instructions and output layout are the same for both the FORTRAN and MATLAB versions of the program. The software can also be obtained by `email` request to any of the authors: `a.j.gil@swansea.ac.uk`, `r.d.wood@swansea.ac.uk` or `j.bonet@swansea.ac.uk`. The authors would like to acknowledge the assistance given by Dr. Rogelio Ortigosa in the development of this computer program.

A bibliography is provided that enables the reader to access the background to the more standard aspects of finite element analysis. Also listed are books and papers that have been of use in the preparation of this book or that cover similar material in greater depth.

Note on equation numbering: Typically, Equation (x.yz a, b, c, d)$_b$ refers to Equation (x.yz, b).

† **F**inite element **La**r**g**e **S**train **Hy**perelasto-plastic **P**rogram.