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978-1-107-11579-8 - Nonlinear Solid Mechanics for Finite Element Analysis: Statics

Javier Bonet, Antonio J. Gil and Richard D. Wood

Frontmatter

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NONLINEAR SOLID MECHANICS FOR FINITE ELEMENT ANALYSIS: STATICS

Designing engineering components that make optimal use of materials requires consideration of the nonlinear static and dynamic characteristics associated with both manufacturing and working environments. The modeling of these characteristics can only be done through numerical formulation and simulation, which requires an understanding of both the theoretical background and associated computer solution techniques. By presenting both the nonlinear solid mechanics and the associated finite element techniques together, the authors provide, in the first of two books in this series, a complete, clear, and unified treatment of the static aspects of nonlinear solid mechanics.

Alongside a range of worked examples and exercises are user instructions, program descriptions, and examples for the FLaGSHyP MATLAB computer implementation, for which the source code is available online.

While this book is designed to complement postgraduate courses, it is also relevant to those in industry requiring an appreciation of the way their computer simulation programs work.

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University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107115798

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First published 2016

Printed in the United Kingdom by TJ International Ltd. Padstow Cornwall

A catalogue record for this publication is available from the British Library.

Library of Congress Cataloging-in-Publication Data

Names: Bonet, Javier, 1961– author. | Gil, Antonio J. author. | Wood, Richard D., 1943– author.
Title: Nonlinear solid mechanics for finite element analysis : statics / Javier Bonet, University of Wales, Swansea, Antonio J. Gil, University of Wales, Swansea, Richard D. Wood, University of Wales, Swansea.
Description: Cambridge : Cambridge University Press is part of the University of Cambridge, [2016]
Identifiers: LCCN 2016012972 | ISBN 9781107115798
Subjects: LCSH: Continuum mechanics. | Statics. | Finite element method.
Classification: LCC QA808.2 .B658 2016 | DDC 531/.2–dc23
LC record available at <https://lccn.loc.gov/2016012972>

ISBN 978-1-107-11579-8 Hardback

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To Catherine, Clare, Doreen and our children

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A fragment from the poem
“An Essay on Criticism”
by Alexander Pope (1688–1744)

A little Learning is a dang'rous Thing;
Drink deep, or taste not the Pierian Spring:
There shallow Draughts intoxicate the Brain,
And drinking largely sobers us again.
Fir'd at first Sight with what the Muse imparts,
In fearless Youth we tempt the Heights of Arts,
While from the bounded Level of our Mind,
Short Views we take, nor see the lengths behind,
But more advanc'd, behold with strange Surprise
New, distant Scenes of endless Science rise!

CONTENTS

	<i>Preface</i>	<i>page xiii</i>
1	INTRODUCTION	1
1.1	NONLINEAR COMPUTATIONAL MECHANICS	1
1.2	SIMPLE EXAMPLES OF NONLINEAR STRUCTURAL BEHAVIOR	2
1.2.1	Cantilever	2
1.2.2	Column	3
1.3	NONLINEAR STRAIN MEASURES	4
1.3.1	One-Dimensional Strain Measures	4
1.3.2	Nonlinear Truss Example	6
1.3.3	Continuum Strain Measures	10
1.4	DIRECTIONAL DERIVATIVE, LINEARIZATION AND EQUATION SOLUTION	12
1.4.1	Directional Derivative	13
1.4.2	Linearization and Solution of Nonlinear Algebraic Equations	15
	EXERCISES	19
2	MATHEMATICAL PRELIMINARIES	21
2.1	INTRODUCTION	21
2.2	VECTOR AND TENSOR ALGEBRA	21
2.2.1	Vectors	22
2.2.2	Second-Order Tensors	26
2.2.3	Vector and Tensor Invariants	35
2.2.4	Higher-Order Tensors	39
2.3	LINEARIZATION AND THE DIRECTIONAL DERIVATIVE	45
2.3.1	One Degree of Freedom	45
2.3.2	General Solution to a Nonlinear Problem	46

2.3.3	Properties of the Directional Derivative	48
2.3.4	Examples of Linearization	49
2.4	TENSOR ANALYSIS	54
2.4.1	The Gradient and Divergence Operators	54
2.4.2	Integration Theorems	56
	EXERCISES	57
3	ANALYSIS OF THREE-DIMENSIONAL TRUSS STRUCTURES	59
3.1	INTRODUCTION	59
3.2	KINEMATICS	61
3.2.1	Linearization of Geometrical Descriptors	63
3.3	INTERNAL FORCES AND HYPERELASTIC CONSTITUTIVE EQUATIONS	64
3.4	NONLINEAR EQUILIBRIUM EQUATIONS AND THE NEWTON–RAPHSON SOLUTION	66
3.4.1	Equilibrium Equations	66
3.4.2	Newton–Raphson Procedure	67
3.4.3	Tangent Elastic Stiffness Matrix	68
3.5	TOTAL POTENTIAL ENERGY	71
3.5.1	Principle of Virtual Work	72
3.6	ELASTO-PLASTIC BEHAVIOR	75
3.6.1	Multiplicative Decomposition of the Stretch	76
3.6.2	Rate-Independent Plasticity	77
3.6.3	Incremental Kinematics	81
3.6.4	Time Integration	83
3.6.5	Stress Update and Return Mapping	84
3.6.6	Algorithmic Tangent Modulus	87
3.6.7	Revised Newton–Raphson Procedure	88
3.7	EXAMPLES	89
3.7.1	Inclined Axial Rod	89
3.7.2	Trussed Frame	90
	EXERCISES	91
4	KINEMATICS	96
4.1	INTRODUCTION	96
4.2	THE MOTION	96
4.3	MATERIAL AND SPATIAL DESCRIPTIONS	97
4.4	DEFORMATION GRADIENT	99
4.5	STRAIN	102
4.6	POLAR DECOMPOSITION	106
4.7	VOLUME CHANGE	112

CONTENTS

ix

4.8	DISTORTIONAL COMPONENT OF THE DEFORMATION GRADIENT	113
4.9	AREA CHANGE	116
4.10	LINEARIZED KINEMATICS	117
4.10.1	Linearized Deformation Gradient	117
4.10.2	Linearized Strain	118
4.10.3	Linearized Volume Change	119
4.11	VELOCITY AND MATERIAL TIME DERIVATIVES	119
4.11.1	Velocity	119
4.11.2	Material Time Derivative	120
4.11.3	Directional Derivative and Time Rates	121
4.11.4	Velocity Gradient	122
4.12	RATE OF DEFORMATION	123
4.13	SPIN TENSOR	126
4.14	RATE OF CHANGE OF VOLUME	129
4.15	SUPERIMPOSED RIGID BODY MOTIONS AND OBJECTIVITY	130
	EXERCISES	132
5	STRESS AND EQUILIBRIUM	137
5.1	INTRODUCTION	137
5.2	CAUCHY STRESS TENSOR	137
5.2.1	Definition	137
5.2.2	Stress Objectivity	141
5.3	EQUILIBRIUM	142
5.3.1	Translational Equilibrium	142
5.3.2	Rotational Equilibrium	144
5.4	PRINCIPLE OF VIRTUAL WORK	145
5.5	WORK CONJUGACY AND ALTERNATIVE STRESS REPRESENTATIONS	146
5.5.1	The Kirchhoff Stress Tensor	146
5.5.2	The First Piola–Kirchhoff Stress Tensor	147
5.5.3	The Second Piola–Kirchhoff Stress Tensor	150
5.5.4	Deviatoric and Pressure Components	153
5.6	STRESS RATES	154
	EXERCISES	156
6	HYPERELASTICITY	158
6.1	INTRODUCTION	158
6.2	HYPERELASTICITY	158
6.3	ELASTICITY TENSOR	160
6.3.1	The Material or Lagrangian Elasticity Tensor	160
6.3.2	The Spatial or Eulerian Elasticity Tensor	161

6.4	ISOTROPIC HYPERELASTICITY	162
6.4.1	Material Description	162
6.4.2	Spatial Description	163
6.4.3	Compressible Neo-Hookean Material	165
6.5	INCOMPRESSIBLE AND NEARLY INCOMPRESSIBLE MATERIALS	168
6.5.1	Incompressible Elasticity	168
6.5.2	Incompressible Neo-Hookean Material	170
6.5.3	Nearly Incompressible Hyperelastic Materials	172
6.6	ISOTROPIC ELASTICITY IN PRINCIPAL DIRECTIONS	175
6.6.1	Material Description	175
6.6.2	Spatial Description	176
6.6.3	Material Elasticity Tensor	177
6.6.4	Spatial Elasticity Tensor	179
6.6.5	A Simple Stretch-Based Hyperelastic Material	180
6.6.6	Nearly Incompressible Material in Principal Directions	181
6.6.7	Plane Strain and Plane Stress Cases	184
6.6.8	Uniaxial Rod Case	185
	EXERCISES	186
7	LARGE ELASTO-PLASTIC DEFORMATIONS	188
7.1	INTRODUCTION	188
7.2	THE MULTIPLICATIVE DECOMPOSITION	188
7.3	RATE KINEMATICS	193
7.4	RATE-INDEPENDENT PLASTICITY	197
7.5	PRINCIPAL DIRECTIONS	199
7.6	INCREMENTAL KINEMATICS	203
7.6.1	The Radial Return Mapping	206
7.6.2	Algorithmic Tangent Modulus	208
7.7	TWO-DIMENSIONAL CASES	209
	EXERCISES	212
8	LINEARIZED EQUILIBRIUM EQUATIONS	214
8.1	INTRODUCTION	214
8.2	LINEARIZATION AND THE NEWTON–RAPHSON PROCESS	214
8.3	LAGRANGIAN LINEARIZED INTERNAL VIRTUAL WORK	216
8.4	EULERIAN LINEARIZED INTERNAL VIRTUAL WORK	217
8.5	LINEARIZED EXTERNAL VIRTUAL WORK	219
8.5.1	Body Forces	219
8.5.2	Surface Forces	219
8.6	VARIATIONAL METHODS AND INCOMPRESSIBILITY	221
8.6.1	Total Potential Energy and Equilibrium	222

CONTENTS		xi
	8.6.2 Lagrange Multiplier Approach to Incompressibility	223
	8.6.3 Penalty Methods for Incompressibility	226
	8.6.4 Hu–Washizu Variational Principle for Incompressibility	227
	8.6.5 Mean Dilatation Procedure	229
	EXERCISES	232
9	DISCRETIZATION AND SOLUTION	234
9.1	INTRODUCTION	234
9.2	DISCRETIZED KINEMATICS	234
9.3	DISCRETIZED EQUILIBRIUM EQUATIONS	239
	9.3.1 General Derivation	239
	9.3.2 Derivation in Matrix Notation	241
9.4	DISCRETIZATION OF THE LINEARIZED EQUILIBRIUM EQUATIONS	242
	9.4.1 Constitutive Component: Indicial Form	244
	9.4.2 Constitutive Component: Matrix Form	245
	9.4.3 Initial Stress Component	246
	9.4.4 External Force Component	247
	9.4.5 Tangent Matrix	249
9.5	MEAN DILATATION METHOD FOR INCOMPRESSIBILITY	251
	9.5.1 Implementation of the Mean Dilatation Method	251
9.6	NEWTON–RAPHSON ITERATION AND SOLUTION PROCEDURE	253
	9.6.1 Newton–Raphson Solution Algorithm	253
	9.6.2 Line Search Method	254
	9.6.3 Arc Length Method	256
	EXERCISES	258
10	COMPUTER IMPLEMENTATION	260
10.1	INTRODUCTION	260
10.2	USER INSTRUCTIONS	263
10.3	OUTPUT FILE DESCRIPTION	269
10.4	ELEMENT TYPES	272
10.5	SOLVER DETAILS	274
10.6	PROGRAM STRUCTURE	276
10.7	MASTER m-FILE FLagSHyP	278
10.8	FUNCTION residual_and_stiffness_assembly	285
10.9	FUNCTION constitutive_matrix	294
10.10	FUNCTION geometric_matrix	295
10.11	FUNCTION pressure_load_and_stiffness_assembly	296
10.12	EXAMPLES	298
	10.12.1 Simple Patch Test	298
	10.12.2 Nonlinear Truss	299
	10.12.3 Strip with a Hole	300

10.12.4	Plane Strain Nearly Incompressible Strip	300
10.12.5	Twisting Column	302
10.12.6	Elasto-Plastic Cantilever	303
10.13	APPENDIX: DICTIONARY OF MAIN VARIABLES	306
10.14	APPENDIX: CONSTITUTIVE EQUATION SUMMARY	309
	<i>Bibliography</i>	316
	<i>Index</i>	318

PREFACE

A fundamental aspect of engineering is the desire to design artifacts that exploit materials to a maximum in terms of performance under working conditions and efficiency of manufacture. Such an activity demands an increasing understanding of the behavior of the artifact in its working environment together with an understanding of the mechanical processes occurring during manufacture.

To be able to achieve these goals it is likely that the engineer will need to consider the nonlinear characteristics associated possibly with the manufacturing process but certainly with the response to working load. Currently, analysis is most likely to involve a computer simulation of the behavior. Because of the availability of commercial finite element computer software, the opportunity for such nonlinear analysis is becoming increasingly realized.

Such a situation has an immediate educational implication because, for computer programs to be used sensibly and for the results to be interpreted wisely, it is essential that the users have some familiarity with the fundamentals of nonlinear continuum mechanics, nonlinear finite element formulations, and the solution techniques employed by the software. This book seeks to address this problem by providing a unified introduction to these three topics.

The style and content of the book obviously reflect the attributes and abilities of the authors. The authors have lectured on this material for a number of years to postgraduate classes, and the book has emerged from these courses. We hope that our complementary approaches to the topic will be in tune with the variety of backgrounds expected of our readers and, ultimately, that the book will provide a measure of enjoyment brought about by a greater understanding of what we regard as a fascinating subject.

The original edition of this book, titled *Nonlinear Continuum Mechanics for Finite Element Analysis*, published in 1997, contained a chapter on a FORTRAN program implementation of the material in the text, this being freely available at www.flagshyp.com. In 2008 a second edition included new chapters on

elasto-plastic behavior of trusses and solids and retained the FORTRAN implementation. It was envisioned that an expanded third edition could include dynamics, although this would involve substantial additional material not suitable to the needs of all readers. Consequently the subject has been divided into two complementary volumes, these being the present text, *Nonlinear Solid Mechanics for Finite Element Analysis: Statics*, and a companion volume, *Nonlinear Solid Mechanics for Finite Element Analysis: Dynamics*. These texts are both aimed at the same readership. Recognising its widespread adoption, particularly as a graduate training platform, this present statics text employs MATLAB[®]* for the implementation of the finite element analysis, the software being freely available at www.flagshyp.com.

This present text contains additional examples, and solutions to all exercises are given in the companion book, *Worked Examples in Nonlinear Continuum Mechanics for Finite Element Analysis*, published by Cambridge University Press (ISBN 9781107603615).

READERSHIP

This book is most suited to a postgraduate level of study by those in either higher education or industry who have graduated with an engineering or applied mathematics degree. However, the material is equally applicable to first-degree students in the final year of an applied mathematics course or an engineering course containing some additional emphasis on maths and numerical analysis. A familiarity with statics and elementary stress analysis is assumed, as is some exposure to the principles of the finite element method. However, a primary objective of the book is that it be reasonably self-contained, particularly with respect to the nonlinear continuum mechanics chapters, which comprise a large portion of the content.

When dealing with such a complex set of topics it is unreasonable to expect all readers to become familiar with all aspects of the book. If the reader is prepared not to get too hung up on details, it is possible to use the book to obtain a reasonable overview of the subject. Such an approach may be suitable for someone starting to use a nonlinear computer program. Alternatively, the requirements of a research project may necessitate a deeper understanding of the concepts discussed. To assist in this latter endeavor the book provides access to a computer program for the nonlinear finite deformation finite element analysis of two- and three-dimensional solids. Such a program provides the basis for a contemporary approach to finite deformation elasto-plastic analysis.

* Mathworks, Inc.

LAYOUT

Chapter 1: Introduction

Here, the nature of nonlinear computational mechanics is discussed, and followed by a series of very simple examples that demonstrate various aspects of nonlinear structural behavior. These examples are intended, to an extent, to upset the reader's preconceived ideas inherited from an overexposure to linear analysis and, we hope, provide a motivation for reading the rest of the book! Nonlinear strain measures are introduced and illustrated using a simple one-degree-of-freedom truss analysis. The concepts of linearization and the directional derivative are of sufficient importance to merit a gentle introduction in this chapter. Linearization naturally leads on to the Newton–Raphson iterative solution, which is the fundamental way of solving the nonlinear equilibrium equations occurring in finite element analysis. Consequently, by way of an example, the simple truss is solved and a short MATLAB program is presented that, in essence, is the prototype for the main finite element program discussed later in the book.

Chapter 2: Mathematical Preliminaries

Vector and tensor manipulations occur throughout the book, and these are introduced in this chapter. Although vector algebra is a well-known topic, tensor algebra is less familiar, certainly, to many approaching the subject with an engineering educational background. Consequently, tensor algebra is considered in enough detail to cover the needs of the subsequent chapters; in particular, it is hoped that readers will understand the physical interpretation of a second-order tensor. Crucial to the development of the finite element solution scheme are the concepts of linearization and the directional derivative. The introduction provided in Chapter 1 is now thoroughly developed. Finally, for completeness, some standard analysis topics are briefly presented.

Chapter 3: Analysis of Three-dimensional Truss Structures

This chapter is largely independent of the remainder of the book and deals with the large strain elasto-plastic behavior of trusses. The chapter begins with a discussion of the nonlinear kinematics of a simple two-noded truss member, which leads to a definition of logarithmic strain. A hyperelastic stress–strain relationship is then derived and used to obtain the equilibrium equations at a node. In preparation for the variational formulation in Chapter 8 the equilibrium equations are re-derived

using an energy approach. These equations are then linearized with respect to small incremental displacements to provide a Newton–Raphson solution process. The chapter then moves on to discuss a simple hyperelastic plastic model for the truss member based on the multiplicative decomposition of the total stretch into elastic and plastic components. The constitutive model is also linearized to provide a tangent modulus. The chapter concludes with some examples of the use of the formulation obtained using the FLaGSHyP program.

Chapter 4: Kinematics

This chapter deals with the kinematics of finite deformation, that is, the study of motion without reference to the cause. Central to this concept is the deformation gradient tensor, which describes the relationship between elemental vectors defining neighboring particles in the undeformed and deformed configurations of the body whose motion is under consideration. The deformation gradient permeates most of the development of finite deformation kinematics because, among other things, it enables a variety of definitions of strain to be established. Material (initial) and spatial (current) descriptions of various items are discussed, as is the linearization of kinematic quantities. Although dynamics is not the subject of this book, it is nevertheless necessary to consider velocity and the rate of deformation. The chapter concludes with a brief discussion of rigid body motion and objectivity.

Chapter 5: Stress and Equilibrium

The definition of the true or Cauchy stress is followed by the development of standard differential equilibrium equations. As a prelude to the finite element development the equilibrium equations are recast in the weak integral virtual work form. Although initially in the spatial or current deformed configuration, these equations are reformulated in terms of the material or undeformed configuration, and as a consequence alternative stress measures emerge. Finally, stress rates are discussed in preparation for the following chapter on hyperelasticity.

Chapter 6: Hyperelasticity

Hyperelasticity, whereby the stress is found as a derivative of some potential energy function, encompasses many types of nonlinear material behavior and provides the basis for the finite element treatment of elasto-plastic behavior. Isotropic

hyperelasticity is considered both in a material and in a spatial description for compressible and incompressible behavior. The topic is extended to a general description in principal directions that is specialized for the cases of plane strain, plane stress, and uniaxial behavior.

Chapter 7: Large Elasto-Plastic Deformations

This chapter provides an introduction to the formulation of inelastic deformation processes based on the multiplicative decomposition of the deformation gradient into recoverable and permanent components. Although only a basic Von Mises model with a radial return-mapping procedure is presented, the use of principal directions and logarithmic stretches provides a simple mechanism whereby small strain concepts can be extended to large strains. From the outset, the approach followed, to derive the kinematic rate equations necessary for the flow rule, anticipates the standard trial stress and return-map procedure required to satisfy the plasticity constraints. Such a development clarifies the kinematic rate equations in the context of the eventual incremental algorithmic procedure.

Chapter 8: Linearized Equilibrium Equations

To establish the Newton–Raphson solution procedure the virtual work expression of equilibrium may be linearized either before or after discretization. Here the former approach is adopted. Linearization of the equilibrium equations includes consideration of deformation-dependent surface pressure loading. A large proportion of this chapter is devoted to incompressibility and to the development, via the Hu–Washizu principle, of the mean dilatation technique.

Chapter 9: Discretization and Solution

All previous chapters have provided the foundation for the development of the discretized equilibrium and linearized equilibrium equations considered in this chapter. Linearization of the virtual work equation leads to the familiar finite element expression of equilibrium involving $\int \mathbf{B}^T \boldsymbol{\sigma} dv$, whereas discretization of the linearized equilibrium equations leads to the tangent matrix, which comprises constitutive and initial stress components. Discretization of the mean dilatation technique is presented in detail. The tangent matrix forms the basis of the Newton–Raphson solution procedure, which is presented as the fundamental solution technique enshrined in the computer program discussed in the following chapter. The

chapter concludes with a discussion of line search and arc length enhancements to the Newton–Raphson procedure.

Chapter 10: Computer Implementation

Here, information is presented on a nonlinear finite element computer program called FFlagSHyP,[†] for the solution of finite deformation elasto-plastic finite element problems employing the neo-Hookean hyperelastic compressible and incompressible constitutive equations developed in Chapters 6 and 7. The usage and layout of the MATLAB program is discussed together with the description of the various key functions. The program is available free on the Internet from the website www.flagshyp.com. The website also contains the original FORTRAN version of the program which is detailed in previous editions of this text. Note that the user instructions and output layout are the same for both the FORTRAN and MATLAB versions of the program. The software can also be obtained by email request to any of the authors: a.j.gil@swansea.ac.uk, r.d.wood@swansea.ac.uk or j.bonet@swansea.ac.uk. The authors would like to acknowledge the assistance given by Dr. Rogelio Ortigosa in the development of this computer program.

A bibliography is provided that enables the reader to access the background to the more standard aspects of finite element analysis. Also listed are books and papers that have been of use in the preparation of this book or that cover similar material in greater depth.

Note on equation numbering: Typically, Equation (x.yz a, b, c, d)_b refers to Equation (x.yz, b).

[†] Finite element Large Strain Hyperelasto-plastic Program.