
CONTENTS

<i>Foreword by Robert M. Solow</i>	<i>page</i> xiii
<i>Preface to the second edition</i>	xv
<i>Introduction to the first edition</i>	xviii

PART I POSITIVE GROWTH THEORY

1	The welfare of society and economic growth	3
1	Income as a measure of economic activity	3
2	Is income per person a fair gauge of society's welfare?	12
3	A major caveat	14
2	The growth process	26
1	The growth process: an intuitive approach	26
2	A more precise approach: a simple model of economic growth	28
3	Introducing technical progress	49
3	Poverty traps	68
1	Introduction	68
2	The bare facts	68
3	Correcting a serious mistake	69
4	Escaping poverty traps	74
4	A production function of central importance	76
1	Motivation	76
2	The links between the elasticity of substitution and income distribution	83
3	Determining the constant elasticity of substitution production function	85
5	The CES production function as a general mean (in collaboration with Robert M. Solow)	92
1	The concept of the general mean of order p , and its fundamental properties	94
2	Applications to the CES production function	96
3	The qualitative behaviour of the CES function as σ changes	98
6	Capital–labour substitution and economic growth (in collaboration with Robert M. Solow)	114
1	Further analytics of the CES function in a growth model	115
2	The elasticity of substitution at work	127

<i>x</i>	<i>Contents</i>	
	3 Introducing technical progress	132
	4 Time-series and cross-section estimates	145
	5 The broader significance of the elasticity of substitution in the context of economic growth	151
7	Why has the elasticity of substitution most often been observed as smaller than 1? And why is it of importance?	155
	1 Introduction	155
	2 The unsustainability of competitive equilibrium with $\sigma > 1$	157
	3 A vivid contrast: the sustainability of competitive equilibrium and its associated growth paths with $\sigma < 1$	162
8	The long-term growth rate as a random variable, with an application to the US economy	170
	1 From daily to yearly growth rates	172
	2 The first moments of the long-term yearly growth rate	177
	3 Application to the long-term growth rates of the US economy	183
PART II OPTIMAL GROWTH THEORY		
9	Optimal growth theory: an introduction to the calculus of variations	191
	1 The Euler equation	192
	2 Fundamental properties of the Euler equation	197
	3 Particular cases of the Euler equation	197
	4 Functionals depending on n functions $y_1(x), \dots, y_n(x)$	199
	5 A necessary and sufficient condition for $y(x)$ to maximize the functional $\int_a^b F(x, y, y') dx$	201
	6 End-point and transversality conditions	203
	7 The case of improper integrals $\int_0^\infty F(y, y', t) dt$ and transversality conditions at infinity	207
10	Deriving the central equations of the calculus of variations with a single stroke of the pen	222
	1 A one-line derivation of the Euler equation through economic reasoning: the case of an extremum for $\int_{x_0}^{x_1} F(x, y, y') dx$	223
	2 Extending this reasoning to the derivation of the Ostrogradski equation: the case of an extremum for $\int \int_R F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}) dx dy$	225
	3 An intuitive derivation of the Beltrami equation	227
	4 End-point and transversality conditions: derivations through direct reasoning	228
	5 Conclusions	230
11	Other major tools for optimal growth theory: the Pontryagin maximum principle and the Dorfmanian	231
	1 The maximum principle in its simplest form	232
	2 The relationship between the Pontryagin maximum principle and the calculus of variations	233
	3 An economic derivation of the maximum principle	234

*Contents**xi*

4	First application: deriving the Euler equation from economic reasoning	237
5	Further applications: deriving high-order equations of the calculus of variations	238
12	First applications to optimal growth	243
1	The mainstream problem of optimal growth: a simplified presentation	244
2	The calculus of variations approach	244
3	The Pontryagin maximum principle approach	249
4	Optimal paths	251
13	Optimal growth and the optimal savings rate	266
1	The central model of optimal growth theory	267
2	The consequences of using power utility functions	269
3	The consequences of using exponential utility functions	281
PART III A UNIFIED APPROACH		
14	Preliminaries: interest rates and capital valuation	297
1	The reason for the existence of interest rates	298
2	The various types of interest rates and their fundamental properties	301
3	Applications to the model of economic growth	316
15	From arbitrage to equilibrium	327
1	The case of risk-free transactions	327
2	Introducing uncertainty and a risk premium	332
16	Why is traditional optimal growth theory mute? Restoring its rightful voice	335
1	Overview	335
2	Three papers that should have rung alarm bells: Ramsey (1928), Goodwin (1961), King and Rebelo (1993)	338
3	The ill-fated role of utility functions	346
4	How the strict concavity of utility functions makes competitive equilibrium unsustainable	349
5	A suggested solution	360
6	The robustness of the optimal savings rate: the normal impact of different scenarios	369
7	Conclusion	376
17	Problems in growth: common traits between planned economies and poor countries	383
1	The consequences of planning	384
2	Common traits of centrally planned economies and poor countries	388
18	From Ibn Khaldun to Adam Smith; a proof of their conjecture	392
1	Ibn Khaldun's message	393
2	Two illustrations of the message of Ibn Khaldun and Adam Smith	400
3	A proof of their conjecture	405

19	Capital and economic growth in the coming century	406
	1 What are the hypotheses we can agree upon?	407
	2 What can we conclude?	408
	<i>In conclusion: on the convergence of ideas and values through civilizations</i>	415
	<i>Further reading, data on growth and references</i>	417
	<i>Index</i>	423