

Economic Growth

In the second edition of this user-friendly book, Olivier de La Grandville provides a clear and original introduction to the theory of economic growth, its mechanisms and its challenges. The book has been fully updated to incorporate several important new results and proofs since the first edition. In addition to a progressive treatment of dynamic optimization, readers will find intuitive derivations of all central equations of the calculus of variations and of optimal control theory. It offers a new solution to the fundamental question: How much should a nation save and invest? La Grandville shows that the optimal savings rule he suggests not only corresponds to the maximization of future welfare flows for society, but also maximizes the value of society's activity, as well as the total remuneration of labour. The rule offers a fresh alternative to dire current predictions about an ever-increasing capital–output ratio and a decrease of the labour share in national income.

OLIVIER DE LA GRANDVILLE is Senior Professor at Frankfurt University and Visiting Professor in the Management Science and Engineering Department at Stanford University. He was Professor of Economics at the University of Geneva from 1978 to 2007 and held visiting positions at the Massachusetts Institute of Technology, at École Polytechnique Fédérale de Lausanne, at the University of Neuchâtel and at the University of Western Australia. He is the author of seven books on topics ranging from microeconomics to macroeconomics and finance, and his research work has been published in international journals such as the *American Economic Review* and *Econometrica*.

Praise for *Economic Growth*, Second Edition

‘Olivier de La Grandville has written a sparkling, wide-ranging and provocative analysis of economic growth models. The work is marked by a large number of novel specific analytic results which will be of wide use.’

Kenneth J. Arrow, Stanford University, Nobel Laureate

‘This is a very useful book. It covers in extensive detail the neoclassical perspective on optimal economic growth, going beyond what is available in the current textbook expositions. Especially noteworthy are the new results in Theorem 16.1. Researchers working on the topic will greatly benefit from the attention that the book pays to the analytical foundations of the approach and its numerical exploration of specifications of the main model that often do not get the attention they deserve. Indeed, the quantitative analysis is what makes this book especially useful for fully understanding what the standard model of capital accumulation really teaches us about economic growth.’

Pietro F. Peretto, Duke University

‘What strikes in this book is that the author, when confronted with a difficult problem in economic growth, first checks for the validity of the standard theoretical solution to such a problem, and, if he finds it wrong, offers his own solution, which turns out to be the correct one. An example is the new chapter 3 on poverty traps. I repeat what I already wrote on the first edition: this is an important book that every economist should read.’

Giancarlo Gandolfo, Accademia Nazionale dei Lincei, Rome

‘Now in a new edition, this book combines rigorous analysis with a keen attention to its practical implications. This applies – for instance – to the implausibly high level of the saving rate required by standard growth theory, for which the author offers an innovative solution, and to the diagnosis provided for poverty traps, which also suggests how they can be escaped.’

Graziella Bertocchi, University of Modena and Reggio Emilia

‘Olivier de La Grandville takes the reader on a fascinating tour through neoclassical growth theory. Idiosyncratic in scope and style, the tour stops at major intellectual sights. In addition, the author guides us to new and important places of interest that emanate from his own research. All this is accomplished in a formidable self-contained manner.’

Andreas Irmen, University of Luxembourg

‘Economists need a better understanding of the Euler and Pontryagin dynamic equations, both from an analytical and a computational point of view. They also need a new, reasonable solution to the crucial problem of optimal growth. They will find both in this remarkable book by Olivier de La Grandville.’

Bjarne S. Jensen, University of Southern Denmark

‘With exceptional clarity, de la Grandville presents the theory of economic growth, incorporating, in this second edition, new results and raising interesting research questions. Its rigorous theoretical and insightful analysis provides a foundation on which future students and academic researchers, dealing with complex issues of growth, inequality, poverty, and social welfare, are sure to build.’

Daniela Federici, University of Cassino and Southern Lazio-Italy

‘Olivier de La Grandville continues his profound research on economic growth and development in the second edition of his fascinating book. His argument that a realistic model of economic growth requires competitive equilibrium warrants widespread recognition in a graduate courses. Economic policy makers should heed his findings on the critical role of the elasticity of substitution between input factors for economic growth and the distribution of factor incomes.’

E. Juerg Weber, University of Western Australia

‘This book is much more than an excellent textbook on growth economics: it examines some fundamental questions in the neoclassical growth theory that have thus far not been fully articulated. Olivier de La Grandville’s penetrating discussion on the role of factor substitutability and the relation between positive and normative growth theories are particularly insightful. I highly recommend this book for anyone interested in the theories of growth and development.’

Kazuo Mino, Doshisha University and Kyoto University

‘A remarkable and masterfully written text. De La Grandville’s approach to growth theory is insightful and reveals how much more there is to learn from the workhorse neoclassical growth model. The new edition incorporates substantive and original new material. It is a thought provoking combination of a textbook and original essays. Essential material for researchers and graduate students interested in growth and development theory.’

Miguel León-Ledesma, University of Kent

‘Olivier de La Grandville presents a sound and stimulating introduction to modern growth theory. His analysis is at once rigorous and intuitive, opening new perspectives along the Solovian growth model tradition. His approach to the problem of optimal growth leads to a deeper understanding of main theoretical results of current growth literature, and also uncovers some of its more serious drawbacks. His solution is convincing, always leading to reasonable time paths for the economy. This book should be read by all scholars interested in growth theory.’

Davide Fiaschi, University of Pisa

‘This book is a truly delightful revisiting of the theory of economic growth starting with the very essential foundations of the theory: preferences of consumers and technology in the hands of producers. Olivier de La Grandville digs deeper into these foundations compared to other textbooks, and provides us with an original light on the non-trivial role of several key assumptions inherent in neoclassical growth. In particular, the systematic analysis of

the implications strictly concave utility functions for sustainable growth and underlying competitive equilibria is a very stimulating contribution. La Grandville's determination to take the neoclassical model to the data and his incomparable intuitive use of optimal control theory are other remarkable features of this otherwise highly pedagogical and informative textbook.'

Raouf Boucekkine, University of Louvain, Center of Center of Operations Research and Econometrics, and Aix-Marseille University

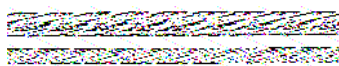
OLIVIER DE LA GRANDVILLE

Economic Growth

A Unified Approach

Second edition

With a foreword and two contributions by Robert M. Solow



CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9781107115231

© Olivier de La Grandville 2017

This publication is in copyright. Subject to statutory exception and to the provisions of relevant collective licensing agreements, no reproduction of any part may take place without the written permission of Cambridge University Press.

First published 2009

Second edition 2017

Printed in the United Kingdom by Clays, St Ives plc

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Names: La Grandville, Olivier de, author. | Solow, Robert M., contributor.

Title: Economic growth : a unified approach / Olivier de La Grandville ; with a foreword and two contributions by Robert M. Solow.

Description: Second edition. | Cambridge, United Kingdom : Cambridge University Press, 2016.

Identifiers: LCCN 2016004780 | ISBN 9781107115231 (hardback)

Subjects: LCSH: Economic development. | Economics – Mathematical models. |

BISAC: BUSINESS & ECONOMICS / Development / Economic Development.

Classification: LCC HD75 .L295 2016 | DDC 338.9001 – dc23

LC record available at <https://lccn.loc.gov/2016004780>

ISBN 978-1-107-11523-1 Hardback

ISBN 978-1-107-53560-2 Paperback

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate.

*This book is lovingly dedicated
to my wife Ann,
to our children Diane, Isabelle and Henri,
and to their own children
Ferdinand and Théodore, Eloïse, Maxime and Margaux*

Cambridge University Press
978-1-107-11523-1 — Economic Growth
2nd Edition
Frontmatter
[More Information](#)

CONTENTS

<i>Foreword by Robert M. Solow</i>	<i>page</i> xiii
<i>Preface to the second edition</i>	xv
<i>Introduction to the first edition</i>	xviii

PART I POSITIVE GROWTH THEORY

1	The welfare of society and economic growth	3
1	Income as a measure of economic activity	3
2	Is income per person a fair gauge of society’s welfare?	12
3	A major caveat	14
2	The growth process	26
1	The growth process: an intuitive approach	26
2	A more precise approach: a simple model of economic growth	28
3	Introducing technical progress	49
3	Poverty traps	68
1	Introduction	68
2	The bare facts	68
3	Correcting a serious mistake	69
4	Escaping poverty traps	74
4	A production function of central importance	76
1	Motivation	76
2	The links between the elasticity of substitution and income distribution	83
3	Determining the constant elasticity of substitution production function	85
5	The CES production function as a general mean (in collaboration with Robert M. Solow)	92
1	The concept of the general mean of order p , and its fundamental properties	94
2	Applications to the CES production function	96
3	The qualitative behaviour of the CES function as σ changes	98
6	Capital–labour substitution and economic growth (in collaboration with Robert M. Solow)	114
1	Further analytics of the CES function in a growth model	115
2	The elasticity of substitution at work	127

x

Contents

3	Introducing technical progress	132
4	Time-series and cross-section estimates	145
5	The broader significance of the elasticity of substitution in the context of economic growth	151
7	Why has the elasticity of substitution most often been observed as smaller than 1? And why is it of importance?	155
1	Introduction	155
2	The unsustainability of competitive equilibrium with $\sigma > 1$	157
3	A vivid contrast: the sustainability of competitive equilibrium and its associated growth paths with $\sigma < 1$	162
8	The long-term growth rate as a random variable, with an application to the US economy	170
1	From daily to yearly growth rates	172
2	The first moments of the long-term yearly growth rate	177
3	Application to the long-term growth rates of the US economy	183
PART II OPTIMAL GROWTH THEORY		
9	Optimal growth theory: an introduction to the calculus of variations	191
1	The Euler equation	192
2	Fundamental properties of the Euler equation	197
3	Particular cases of the Euler equation	197
4	Functionals depending on n functions $y_1(x), \dots, y_n(x)$	199
5	A necessary and sufficient condition for $y(x)$ to maximize the functional $\int_a^b F(x, y, y')dx$	201
6	End-point and transversality conditions	203
7	The case of improper integrals $\int_0^\infty F(y, y', t)dt$ and transversality conditions at infinity	207
10	Deriving the central equations of the calculus of variations with a single stroke of the pen	222
1	A one-line derivation of the Euler equation through economic reasoning: the case of an extremum for $\int_{x_0}^{x_n} F(x, y, y')dx$	223
2	Extending this reasoning to the derivation of the Ostrogradski equation: the case of an extremum for $\int \int_R F(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y})dx dy$	225
3	An intuitive derivation of the Beltrami equation	227
4	End-point and transversality conditions: derivations through direct reasoning	228
5	Conclusions	230
11	Other major tools for optimal growth theory: the Pontryagin maximum principle and the Dorfmanian	231
1	The maximum principle in its simplest form	232
2	The relationship between the Pontryagin maximum principle and the calculus of variations	233
3	An economic derivation of the maximum principle	234

	<i>Contents</i>	<i>xi</i>
	4 First application: deriving the Euler equation from economic reasoning	237
	5 Further applications: deriving high-order equations of the calculus of variations	238
12	First applications to optimal growth	243
	1 The mainstream problem of optimal growth: a simplified presentation	244
	2 The calculus of variations approach	244
	3 The Pontryagin maximum principle approach	249
	4 Optimal paths	251
13	Optimal growth and the optimal savings rate	266
	1 The central model of optimal growth theory	267
	2 The consequences of using power utility functions	269
	3 The consequences of using exponential utility functions	281
PART III A UNIFIED APPROACH		
14	Preliminaries: interest rates and capital valuation	297
	1 The reason for the existence of interest rates	298
	2 The various types of interest rates and their fundamental properties	301
	3 Applications to the model of economic growth	316
15	From arbitrage to equilibrium	327
	1 The case of risk-free transactions	327
	2 Introducing uncertainty and a risk premium	332
16	Why is traditional optimal growth theory mute? Restoring its rightful voice	335
	1 Overview	335
	2 Three papers that should have rung alarm bells: Ramsey (1928), Goodwin (1961), King and Rebelo (1993)	338
	3 The ill-fated role of utility functions	346
	4 How the strict concavity of utility functions makes competitive equilibrium unsustainable	349
	5 A suggested solution	360
	6 The robustness of the optimal savings rate: the normal impact of different scenarios	369
	7 Conclusion	376
17	Problems in growth: common traits between planned economies and poor countries	383
	1 The consequences of planning	384
	2 Common traits of centrally planned economies and poor countries	388
18	From Ibn Khaldun to Adam Smith; a proof of their conjecture	392
	1 Ibn Khaldun's message	393
	2 Two illustrations of the message of Ibn Khaldun and Adam Smith	400
	3 A proof of their conjecture	405

<i>xii</i>	<i>Contents</i>	
19	Capital and economic growth in the coming century	406
	1 What are the hypotheses we can agree upon?	407
	2 What can we conclude?	408
	<i>In conclusion: on the convergence of ideas and values through civilizations</i>	415
	<i>Further reading, data on growth and references</i>	417
	<i>Index</i>	423

FOREWORD

by Robert M. Solow

There are several different directions from which one can approach the theory of economic growth, even with a fairly strict definition of “theory”. The theory can help to account for particular historical episodes: Europe after the War, China after the reforms, for example. More particularly, the theory may help to codify, though not explain, the story of technological progress. Alternatively it can be used to try to understand the effects, on inequality, say, of social and economic institutions and their changes. Or, in still other hands, it can serve as a vehicle for an abstract picture of a whole economy. Each of these angles could give rise to a different sort of book.

Olivier de La Grandville’s remarkable text is not any of those. It is, of course, a careful and clear exposition of the theory itself, with a transparent proof of everything that needs proving. Yes, but a reader is struck – at least this reader was struck – by the number of tables and graphs that amount to numerical experiments. They depict in great detail the consequences for the message of the theory of choosing different values for the main parameters. The book is written in the spirit of a master model-builder: it tells you how to do it, and why to do it. And interestingly, the text is more self-contained than the usual: for example, some central results in the calculus of variations are derived from economic first principles. And why not? Economic logic is also logic.

This instinct of calculation (I intend the echo of Veblen’s “instinct of workmanship”) is a useful habit of thought. It is always comforting to build a model from first principles, using assumptions that one learned in school and that have become internalized. Then the model can just be left to do whatever it does. Olivier de La Grandville (and I) think that this is not such a good idea. Sometimes what seems intuitively inevitable seems that way only because it is customary. Trying out alternative parameter-values is a good way to discover whether a model actually gives sensible results. Peculiar results may be a warning that some traditional assumptions may not be so reasonable, or not any more.

The main example of this process in the book is concerned with the long-standing Ramsey problem: How much should a society save? That, of course, depends on the society’s objective (and on the presumption, famously denied by Margaret Thatcher, that there is such a thing as “society”). The traditional assumption is that at any time there is a social value of current consumption (or

consumption per person) and this social value is subject to the principle of diminishing marginal utility. In the jargon, there is a strictly concave social utility function with consumption at any time as its argument. These have to be added up or otherwise integrated over time, and then optimized. This way of thinking is by now second nature.

Alas, the text shows, in considerable detail, that proceeding in this way always – yes, always – gives answers that are simply unacceptable. In particular, the proposed initial saving rate is always ridiculously high. There is no doubt that the calculation is giving us the “correct” answer to the problem as posed. The whole point is that if the answer is ridiculous the problem must be badly posed or formulated.

The author’s solution to this impasse is to abandon the presumption of diminishing marginal social utility of consumption per person. He reformulates the Ramsey problem as simply optimizing the present value of the flows of consumption itself (or an affine function of it). Then the recommended saving rate turns out to be reasonable, something one can imagine happening. Moreover, the solution to this reformulated problem turns out to have other desirable features.

As the text (chapter 16, section 4) shows in a comprehensive theorem, this formulation of the Ramsey problem yields a path corresponding to competitive equilibrium; furthermore this path also maximizes society’s net product and its compensation of labour both in the short run and in the discounted long run. It is a fresh window on the whole problem of optimal growth. Just as interestingly it points the way to what I think is a major open issue in empirically grounded macroeconomics: the magnitude and role of monopoly (and other) rents in modern capitalist economies.

To take another example of the constructive, calculational approach, the text explores meticulously the influence on growth paths of changes in the elasticity of substitution between labour and capital, breaking some new ground in the process. Of course the effect of capital accumulation on the return to new investment has been a central topic in economics at least since the days of David Ricardo and John Stuart Mill. It has taken on renewed interest recently as the spectre of increasing inequality in the distribution of income and wealth has come to public attention. Only part of the needed analytical apparatus appears in this book, but it is an important part. The many calculations add measurably to the force of the analysis.

Any economist or other social scientist who wants a working knowledge of growth theory, and a feel for what matters more and what matters less, will find Olivier de La Grandville’s book a useful, faithful and stimulating guide. It should be read with a pad of paper, a pencil, and an eraser in hand. I know from experience.

Robert M. Solow

PREFACE TO THE SECOND EDITION

It is hard to imagine a more important and difficult economic challenge than to define a policy that would shape society’s welfare in an optimal way, both for the present day and for our future generations. This is precisely the challenge that Frank Ramsey took up nearly a century ago when he asked the question: “How much should a nation save?” In trying to answer it, he founded the theory of optimal growth.

It turns out that neither Ramsey nor subsequent theorists came up with a reasonable, convincing answer to that famous question. Ramsey’s disappointment is palpable when the answer he reached – an “optimal” savings rate equal to 60% – was, in his own words “greatly in excess of that which anyone would suggest”, adding that the utility function he used was “put forward merely as an illustration”. Subsequent essays either remained in the realm of theory or produced the strangest results: some authors gave short shrift to excessive savings rates or, when they worried about those, they had recourse to utility functions that would hardly be recognizable by anybody, and even less by a whole society; or they resorted to changing the values and the very significance of the parameters they used. Despite such bold moves, they could never prevent at least one central variable of the economy from taking a time path that was never observed historically or that was simply unacceptable.

Until now, the main problem was to define a model that would lead to reasonable trajectories for *all* central variables of the economy: not only the savings rate, but the marginal productivity of capital, the growth rate of income per person or the capital–output ratio, to name a few.

In the first edition of this book, we started unveiling the reason for the failure of the theory to yield consistent, sensible results: we showed that it hinged on the systematic use of a strictly concave utility function. This second edition will show that whenever we try to modify the utility function to obtain a more acceptable savings rate, we inevitably induce trajectories for other central variables of the economy that either do not fit with the capabilities of the economy or are inconceivable.

In the new chapter 16 we also demonstrate that the concavity of the utility functions impedes any possibility of a sustained competitive equilibrium; any economy

initially in such equilibrium will *always* veer off from that situation into unwanted trajectories if it is governed by the standard model. We then propose the following solution to the problem of optimal growth: optimal trajectories of the economy, and first and foremost the optimal savings rate, should be determined by the Euler equation resulting from competitive equilibrium.

While traditional theory aims at a single objective, the maximization of the sum of discounted utility flows, the model we offer here leads to the simultaneous, intertemporal maximization of three magnitudes; indeed, by saving and investing along lines defined by such an equilibrium, along with minimizing production costs society is able to maximize:

- the sum of discounted consumption flows;
- the total value of society's activity, defined by the sum of consumption and the rate of increase in the value of the capital stock;
- last but not least, the total remuneration of labour.

We show that for all parameters in the range of observed or predictable values, as well as for quite different hypotheses regarding the future evolution of population or technological progress, we are always led to very reasonable time paths for all central variables of the economy. The model we propose is also highly robust to very different hypotheses regarding the future of societies, in the form of *S*-shaped evolutions of population and technological progress.

Furthermore, the model brings comforting news: contrary to contemporary gloomy predictions about the inevitability of an increase in the capital–output ratio and a decrease in the share of labour in national income, we show that if economic policy can bring a society close to competitive equilibrium, then the capital–output ratio will decrease while the share of labour will increase.

We hope that you will enjoy this second edition and its five entirely new chapters, not only for its results – surprising, and at the same time reassuring and challenging for our future – but also for the methodological novelties it offers. While the basic principles of the calculus of variations were discussed in a standard, classical way in the first edition, in this one you will discover (in the new chapter 10) how the Euler equation, its generalization to multiple integrals as well as the Beltrami equation and the transversality conditions can be derived intuitively with a single stroke of the pen, rendering those beautiful but apparently difficult to comprehend equations almost evident – in fact you will not even need a pen to explain the Euler equation to your neighbour at the ballpark; and you will see: he will be interested.

It is with great pleasure that I express my gratitude to a number of individuals. First I want to acknowledge the invaluable help given to me by my colleague Ernst Hairer, whose mastery at solving numerically differential equations is at

Preface to the second edition

xvii

the origin of the spectacular diagrams in chapters 12, 13 and 16. Without his help I would not have been able to put the utility functions to the test of competitive equilibrium in chapter 16. And of course I want to heartily thank Robert Solow, the co-author of two chapters of the first edition – now chapters 5 and 6 – who was also kind enough to write a foreword for this second edition. I am also indebted to Kenneth Arrow, Michael Binder, Giuseppe De Archangelis, Giovanni Di Bartolomeo, Maria Dimakopoulou, Robert Chirinko, Daniela Federici, Robert Feicht, Giancarlo Gandolfo, Jean-Marie Grether, Erich Gundlach, Andreas Irmen, Bjarne Jensen, Mathias Jonsson, Rainer Klump, Anastasia Litina, Miguel León-Ledesma, Henri Loubergé, Peter McAdam, Bernardo Maggi, Scott Murff, Srinivasan Muthukrishnan, Elisabeth Paté-Cornell, Enrico Saltari, Wolfgang Stummer, Jim Sweeney, Richard Waswo, Juerg Weber and Milad Zarin-Nejadan, as well as to participants in seminars at Stanford, Frankfurt, Luxembourg and Rome (La Sapienza) for their highly constructive remarks. My warmest thanks go also to Mary Catherine Bongiovi, our very efficient production editor, to Glenis Starling, who copy-edited this new edition with remarkable acumen and superb skill, and to William Jack who prepared a detailed index.

Olivier de La Grandville

INTRODUCTION TO THE FIRST EDITION

Why should you read a book on economic growth? Because the subject is important: it is about the well-being of our societies today and in the future; and because it is beautiful. It carries wonderful ideas, some exposed more than 2000 years ago, spanning all civilizations. You will certainly marvel at Ibn Khaldun’s prescience, at Mo Tzu’s wisdom, at Solow’s depiction of transition phases, at Dorfman’s incredible intuition in solving variational problems.

This book is not quite the same as other books. Economic growth has attracted, particularly in the last hundred years, countless, excellent writers who have developed the field into an immense array of topics, from theoretical to empirical. Rather than trying to cover all developments – of which you can have an idea through the bibliography – I have wanted to tell you what I found fascinating in the subject. But my hope is also that you will find here a useful introduction to this wide area of research, because a lot of the book is not only on ideas but on methodology as well.

A further reason for me to write this text was to submit personal views and present new results. For too many years I have expounded growth theory by dividing the subject, as many did, into two main strands of thought: positive, or descriptive theory on the one hand, normative on the other. I am now convinced that those two strands should be unified – hence the title of the book. For clarity’s sake, I think however that both approaches to the theory should be first presented separately (parts I and II), and then unified (part III). Such unification proceeds not from any personal whim, but from logical reasons: the results of both strands of thought mutually imply each other, as will be shown.

Let me now underline the new results you will find in this book:

- A proof of one of the most important, daring conjectures ever made in economics or social sciences. We owe this conjecture, known as “the invisible hand”, to Adam Smith who wrote, in his *Inquiry into the nature and the causes of the wealth of nations* (1776):

Every individual is continually exerting himself to find out the most advantageous employment for whatever capital he can command. It is his own advantage, indeed, and not that of the society, which he has in view. But

the study of his own advantage naturally, or rather necessarily, leads him to prefer that employment which is most advantageous to the society . . .

He generally, indeed, neither intends to promote the public interest, nor knows how much he is promoting it He intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention.¹

Note how far reaching Adam Smith's conjecture is. Not only does the author hold that the search by individuals for the most advantageous employment of their capital stock is advantageous to society; he takes one step further by stating that this advantage is maximized.

What is then the exact benefit that Smith could have referred to in his conjecture? How is it to be measured? The answer is quite surprising. It will be shown that it is not only one, but *two* magnitudes that are simultaneously maximized for society:

(1) *the sum of the discounted consumption flows society can acquire from now to infinity*

(2) *the benefits of society's activity at any point of time t – including today; those benefits are the sum of the consumption flows received at time t and the rate of increase in the value of the capital stock at that time.*

The proof of this theorem, which you will find in the last chapter, uses the methodology and results expounded throughout this text, and draws both on positive and normative theory. If only for this very proof, there would be ample reason to justify the unified approach I am advocating.

- A thorough analysis of the importance of the elasticity of substitution in the growth process. Too often do we see growth models carrying the convenient, beloved hypothesis of an elasticity of substitution equal to 1, equivalent to the Cobb–Douglas function. We now have evidence, however, that in any economy we might consider the elasticity of substitution is significantly different from 1, with a tendency of growing – which is good news, as the reader will discover. A surprise is in store: an increase in the elasticity of substitution will be shown to have far more importance for society's future welfare than a similar increase in the rate of technical progress.
- A detailed examination of the consequences of using utility functions in optimal growth theory. Traditional treatment of the theory usually leads to a system of differential equations which does not possess analytical solutions. In my opinion, this issue, involving solving numerically such a system, has been taken too lightly,

¹ Adam Smith (1776), *An inquiry into the nature and causes of the wealth of nations*, 1975 edition by Dent & Sons, pp. 398–400.

and short shrift has been given to results taking the form of exceedingly high “optimal” savings rates. It can even be read in the literature that some family of utility functions can be used, although it turns out that no equilibrium point exists. Great care will be taken in analysing the optimal time-paths of the economy, both in terms of their associated initial values and their ultimate evolution.

- A formula for the optimal savings rate of an economy. Until now optimal savings rates could be calculated numerically only – no closed form was available and, as mentioned, those values often made little sense. The formula I submit is expressed in terms of the fundamental characteristics of the economy and society’s rate of preference for the present, and yields reasonable, very reachable values.
- Applications and extensions of Dorfman’s modified Hamiltonian. With remarkable insight, Robert Dorfman had introduced a new Hamiltonian to tackle the variational problems encountered in optimal growth theory. To honour Professor Dorfman’s memory, I propose to call his concept a Dorfmanian. It will play a fundamental role in the proof of Smith’s conjecture. The reader will also find here extensions of the Dorfmanian which can yield all high-order equations of the calculus of variations – including the Euler–Poisson and the Ostrogradski equations.
- The final reason why you should read this book is that Robert Solow and myself need your help: you will be invited to exercise your sagacity and try to prove a conjecture we are offering at the end of chapter 3. The conjecture, of a mathematical nature, is as formidable a challenge to prove as it is easy to express: the general mean of two numbers, considered as a function of its order, has one and only one inflection point. Why is it important? Because as a result, income per person behaves exactly like a function of production whose dependent variable is the elasticity of substitution, with a first phase of increasing returns, followed by decreasing returns, and our economies seem to be in the very neighbourhood of this point of inflection.

It is my pleasure to thank a number of persons whose role has been essential in the realization of this book. First and foremost, I would like to express my deepest gratitude to Robert Solow, who not only co-authored a large chapter (chapter 6) and the appendix of chapter 5, but also gave me invaluable advice on many other important parts of the book. Needless to say, I alone remain responsible for any remaining shortcomings, and the personal views expressed here are not necessarily condoned by him.

My colleague Ernst Hairer, of the Department of Mathematics at the University of Geneva, has used his program DOPRI for solving numerically the differential equations of chapters 12, 13 and 16; the stunning phase diagrams 12, 13 and 16 are his work. Ernst Hairer’s generosity led him also to write the program yielding the

Introduction to the first edition

xxi

initial values leading to equilibrium for any system of parameters characterizing the economy. I am in great debt to him.

Claudio Sfreddo let us use for our regressions in chapter 8 the data he had developed on a common basis for 16 OECD countries in his PhD thesis; we are very grateful to him.

My thanks go finally to those persons who read parts of the manuscript and offered corrections or very useful remarks. I would like to thank in particular Kenneth Arrow, Eunyi Chung, Jean-Marie Grether, Bjarne Sloth Jensen, Mingyun Joo, Rainer Klump, Patrick de Laubier, Hing-Man Leung, Edmond Malinvaud, Amin Nikoozadeh, Mario Piacentini, Mathias Thoenig, Brigitte Van Baalen, Juerg Weber, and Milad Zarin-Nejadan. Jon Bilam for Cambridge University Press did a marvellous job in revising the whole text, and I am extremely grateful to him. Dave Tyler prepared the index in a masterly way; my warmest thanks to him. I would also like to express my deep appreciation to Daniel Dunlavey, senior production editor at Cambridge University Press, who supervised the whole project with great expertise. Last but not least, I want to thank heartily Huong Nguyen for her beautiful, dedicated work at typing my manuscript. It was a joy working with her.