Quantum Spin Glasses, Annealing and Computation

Quantum annealing is a new-generation tool of information technology, which helps in solving combinatorial optimization problems with high precision, based on the concepts of quantum statistical physics.

This book focuses on the recent developments in quantum spin glasses, quantum annealing and quantum computations. It offers a detailed discussion on quantum statistical physics of spin glasses and its application in solving combinatorial optimization problems. Separate chapters on simulated annealing, quantum dynamics and classical spin models are provided for enhanced understanding. Notes on adiabatic quantum computers and quenching dynamics make it apt for the readers. This text will be useful for the students of quantum computation, quantum information, statistical physics and computer science.

Shu Tanaka is Assistant Professor at Waseda Institute for Advanced Study, Waseda University, Tokyo, Japan. He is also PRESTO Researcher, JST. He has worked in various fields of the theory of statistical physics, materials science, and quantum information theory. He has studied both of fundamental and application sides of quantum annealing. He has also edited three books of Kinki University Series on Quantum Computing. He got the 9th Award for the Encouragement of Young Physicists from the Physical Society of Japan (2015).

Ryo Tamura is Researcher at International Center for Materials Nanoarchitectonics (MANA) in National Institute for Materials Science (NIMS), Ibaraki, Japan. He has worked in the field of materials science, in particular, his research interests focus on magnetic materials including random spin systems and materials informatics.

Bikas K. Chakrabarti is Professor of Theoretical Condensed Matter Physics at Saha Institute of Nuclear Physics (SINP), Kolkata, India. His research interests include physics of fracture, quantum glasses and the interdisciplinary sciences of optimisation, brain modelling, and econophysics. Professor Chakrabarti is a recipient of the S. S. Bhatnagar Award (1997). He has also received the Outstanding Referee Award of the American Physical Society (2010). He is fellow of Indian National Science Academy & Indian Academy of Sciences.

Quantum Spin Glasses, Annealing and Computation

Shu Tanaka
Ryo Tamura
Bikas K. Chakrabarti

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, vic 3207, Australia
4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi - 110002, India
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107113190

First published 2017

Printed in India

A catalogue record for this publication is available from the British Library

ISBN 978-1-107-11319-0 Hardback

Additional resources for this publication at www.cambridge.org/9781107113190

This book is dedicated to the memory of Prof. Jun-ichi Inoue

The book contains three important notes contributed by Eliahu Cohen, Uma Divakaran, Sudip Mukherjee, Atanu Rajak and Boaz Tamir.

Cambridge University Press
978-1-107-11319-0 — Quantum Spin Glasses, Annealing and Computation
Bikas K. Chakrabarti , Jun-ichi Inoue , Ryo Tamura , Shu Tanaka
Frontmatter
More Information

Contents

Figures *xiii*
Tables *xxxi*
Preface *xxxiii*

1 Introduction

References 3

Part One Quantum Spin Glass, Annealing and Computation

2 Classical Spin Models: From Ferromagnetic Spin Systems to Spin Glasses

2.1 Ising Model 8
2.1.1 Phase transition and critical phenomena 8
2.1.2 Mean-field approximation and infinite-range model 10
2.1.3 Ising model on finite dimensional lattices 15
2.2 Basic Concept of Classical Spin Glass 22
2.2.1 Frustration and randomness 22
2.2.2 Spin glass order parameter 25
2.2.3 Replica method 26
2.3 Sherrington–Kirkpatrick Model 26
2.3.1 Free energy of the Sherrington–Kirkpatrick model 27
2.3.2 Replica symmetric solution 32
2.3.3 Replica symmetry breaking 36
2.4 Edwards–Anderson Model on the Finite Dimensional Lattices 42
2.A Several Spotlights on Classical Models 45
2.A.1 Potts model 46
2.A.2 XY model 48
2.A.3 Heisenberg model 50

References 51

3 Simulated Annealing

3.1 Relation Between Random Ising Models and Combinatorial Optimization Problems 60
3.2 Representative Combinatorial Optimization Problems 64
3.2.1 Computational complexity theory 64
3.2.2 Classification of combinatorial optimization problems 66
3.2.3 Examples of combinatorial optimization problems 67
3.3 Conventional Computation Methods Including Simulated Annealing 73
3.3.1 Monte Carlo methods 73
3.3.2 Mean-field methods 77
3.3.3 Preceding theoretical studies on simulated annealing 79
3.4 Convergence Theorem for Simulated Annealing 84
3.4.1 Inhomogeneous Markov chain 84
3.4.2 Ergodicity 85
3.4.3 Proof of the convergence theorem 86
References 97

4 Quantum Spin Glass

4.1 Ferromagnetic Ising Model in a Transverse Field 98
4.1.1 Semiclassical analysis and a simple mean-field approximation of the ferromagnetic Ising model in a transverse field 100
4.1.2 Phase transition in the finite-dimensional ferromagnetic Ising models in a transverse field 103
4.1.3 Hushimi-Temperely–Curie–Weiss model in a transverse field 110
4.2 Introduction of Quantum Spin Glass 115
4.3 Sherrington-Kirkpatrick Model in a Transverse Field 117
4.3.1 Mean-field calculations 118
4.3.2 Simulation results 129
4.4 Edwards–Anderson Model in a Transverse Field 137
4.5 Existence of Replica Symmetric Spin Glass Phase in the Random Quantum Ising Model 142
References 146

5 Quantum Dynamics

5.1 Landau–Zener Transition 151
5.2 Kibble–Zurek Mechanism 158
5.2.1 Basic concept of the Kibble–Zurek mechanism 159
5.2.2 Kibble–Zurek mechanism in the Ising chain with random ferromagnetic interactions 162
References 167

6 Quantum Annealing

6.1 General Concept of Quantum Annealing 170
6.2 Quantum Annealing by the Schrödinger Equation 174
6.3 Quantum Annealing by the Path-integral Monte Carlo Method 180
6.4 Quantum Annealing by the Green's Function Monte Carlo Method 185
6.5 Quantum Annealing by the Density Matrix Renormalization Group 191
6.6 Quantum Annealing by the Mean-field Approximation 195
6.7 Quantum Field Response to Magnetic Materials 201
6.8 Quantum Adiabatic Evolution 203
6.8.1 Adiabatic quantum computation 204
6.8.2 Quantum adiabatic theorem 205
6.8.3 Quantum adiabatic approximation 215
6.9 Convergence Theorem for Quantum Annealing 222
6.9.1 Convergence theorem for quantum annealing by the Schrödinger equation 222
6.9.2 Correspondence between classical and quantum systems 226
6.9.3 Convergence theorem for quantum annealing by the path-integral Monte Carlo method 231
6.9.4 Convergence theorem for quantum annealing by the Green's function Monte Carlo method 242
References 248

Part Two Additional Notes

7 Notes on Adiabatic Quantum Computers

7.1 Introduction 255
7.2 The Circuit Model 256
7.3 Adiabatic Computation 259
7.3.1 Simulated annealing and adiabatic computation 266
7.3.2 Different paths from an initial to a final eigenstate 267
7.3.3 Imaginary time and simulations 268
7.3.4 Complexity and universality 268
7.3.5 Additional methods 270
7.4 Quantum Annealing 270
7.4.1 Relation between simulated annealing, quantum annealing, and adiabatic computation 272
7.5 D-Wave Computer 274
7.5.1 Is the D-Wave a quantum computer? The future of quantum annealers 276
7.5.2 Universality 276
7.5.3 Coherence time of the SQUIDs 277

7.5.4 Scalability 278
7.5.5 Speed up 278
7.5.6 “Quantumness” 279
7.5.7 Does the computer exhibit entanglement? 281
7.5.8 Open questions and possible future routes 281

References 282

8 Quantum Information and Quenching Dynamics

8.1 Quantum Information Theoretic Measures: Indicator of a QCP 290
8.1.1 Quantum fidelity 290
8.1.2 Loschmidt echo 292
8.1.3 Entanglement 295
8.2 Dynamics in Quantum Manybody Systems 296
8.3 Fidelity Susceptibility and Loschmidt Echo for Generic Paths 296
8.3.1 Model and phase diagram 297
8.3.2 Fidelity susceptibility 299
8.3.3 Loschmidt echo 304
8.4 Loschmidt Echo: Effect of Gapless Phase 307
8.4.1 Model, phase diagram and anisotropic quantum critical point (AQCP) 307
8.4.2 Loschmidt echo for the Kitaev model 310
8.5 Effect of Double Local Quenches on Loschmidt Echo and Entanglement Entropy 313
8.5.1 Model 314
8.5.2 Semiclassical theory of quasiparticles 315
8.5.3 Loschmidt echo and entanglement entropy for a critical chain 316
8.5.4 Entanglement entropy: Ferromagnetic phase 321
8.5.5 QP^1 and QP^2: A comparative study 323
8.6 Topology in Condensed Matter Systems 324
8.6.1 Majorana fermions and condensed matter systems 326
8.6.2 Kitaev chain: Model and topological phases 327
8.7 One-dimensional Generalized Cluster Model 333
8.8 Sudden Quench: Dynamics of a Majorana Edge State 335
8.8.1 Quenching schemes and results 337
8.9 Slow Quench: Dynamics of a Majorana Edge State 340
8.9.1 Model, phase diagram and energy spectrum 341
8.9.2 Quenching dynamics of a Majorana edge state 343
8.10 Summary and Conclusion 347

References 350

9 A Brief Historical Note on the Studies of Quantum Spin Glasses, Annealing and Computation

9.1 Introduction 359
9.2 A Short History of the Development 360
9.3 Review of Some Recent Discussion Papers and Arxiv Preprints 367
9.4 Applications of QA 374
9.5 Summary and Discussions 375
References 379

Index 383

Figures

2.1 Temperature dependence of the magnetization per spin m of the Ising model with homogeneous ferromagnetic interactions by the mean-field approximation. We display the result of $h \to +0$ for $h = 0$. 12

2.2 Lattice structures of the infinite-range model given by Eq. (2.25) for $N = 4$ (left) and $N = 8$ (right). 14

2.3 Lattice structures of (a) chain (one-dimensional lattice), (b) square lattice (two-dimensional lattice), and (c) cubic lattice (three-dimensional lattice). 16

2.4 Temperature dependence of the exact solution for magnetization per spin of the Ising model on a chain under finite magnetic fields. 18

2.5 Temperature dependence of the exact solution of magnetization per spin of the Ising model on a square lattice when the magnetic field is zero. The dotted curve is the result from the mean-field approximation. The transition temperature from the exact solution is lower than that from the mean-field approximation. 19

2.6 Monte Carlo simulation results of the Ising model on a cubic lattice. The linear dimension is L. (a) Temperature dependence of the Binder ratio g of the magnetization. At $T_c/J \simeq 4.51$, the Binder ratios cross at the same point. The curves between points are a guide to the eye. (b) Finite-size scaling results of the Binder ratio of the magnetization using Eq. (2.59), where $T_c/J = 4.51$ and $\nu = 0.627$. 21

2.7 (a) Ground state spin configuration of the Ising model with homogeneous ferromagnetic interactions on a square lattice. The magnetic interaction between nearest-neighbor spins indicated by the solid line is the ferromagnetic interaction ($J_{ij} = +J$). (b) Ground state spin configuration of the Ising model with homogeneous antiferromagnetic interactions on a square lattice. The magnetic interaction between nearest-neighbor spins indicated by the dotted line is the antiferromagnetic interaction ($J_{ij} = -J$). (c) Ground state spin configuration of the $\pm J$ model. $J_{ij} = \pm J$ are randomly distributed. The plaquette marked by the star indicates the frustration plaquette and an energy loss exists at interactions indicated by the crosses. The energy is the same if the spins surrounded by the dashed squares are flipped. 23

2.8 Phase diagram of the SK model under the replica symmetric approximation. Depending on the temperature T/J and the mean value of interactions J_0/J, paramagnetic phase, ferromagnetic phase, and spin glass phase appear. 35

2.9 Phase diagram of the SK model including the replica symmetry breaking. In the mixed phase, the ferromagnetic order and the spin glass order coexist. The dashed line indicates the AT line. Below the AT line, the replica symmetric solution becomes unstable. 37

2.10 (a) Schematic of $q(x)$ defined in $1 \leq x \leq n$, that is, Eq. (2.131) is satisfied. (b) Schematic of $q(x)$ defined in $0 \leq x \leq 1$, that is, Eq. (2.133) is satisfied. (c) Schematic of $q(x)$ in the limit of $M \to \infty$. $q(x)$ becomes a continuous increasing function defined in $0 \leq x \leq 1$. 39

2.11 Schematic of the many-valley structure of the free energy landscape in the spin glass phase. 41

2.12 Temperature dependence of the correlation length ξ_L obtained by Monte Carlo simulations for the Edwards–Anderson model on a square lattice. The probability distribution of magnetic interactions is the Gaussian distribution with zero mean and standard deviation unity. The slope of this figure is the critical exponent ν (from Katzgraber et al. 2004). 43

2.13 Results obtained by Monte Carlo simulations of the Edwards–Anderson model on a cubic lattice. The probability distribution of magnetic interactions is the Gaussian distribution with zero mean and standard deviation unity. (a) Inverse temperature, $\beta = T^{-1}$, dependence of the Binder ratio of the spin glass order parameter. (b) Finite-size scaling result of the Binder ratio of the spin glass order parameter when $T_c = 0.931$ and $\nu = 2.67$ are used (from Katzgraber et al. 2006). 45

2.14 Schematic of the boundary between first-order and second-order phase transitions depending on the value of q and the spatial dimension d in the Potts model (Wu, 1982). 46

3.1 Typical snapshots of the ferromagnetic Ising model on a square lattice whose linear dimension is 128; these were obtained by Monte Carlo simulations. (Upper panel) Snapshots for rapid cooling case. (Lower panel) Snapshots for slow cooling case. 62

3.2 Schematics of the computational complexity for (a) P = NP and (b) P $\neq$ NP. 65

3.3 (a) Position of cities of a traveling salesman problem for $N = 6$. (b) A non-best solution. The lines denote the selected path. (c) The best solution where the length of path is the minimum value. 69

3.4 (a) Position of vertices of a vertex coloring problem for $N = 6$. (b) The answer for the three-coloring problem ($k = 3$). (c) The "best" solution for the two-coloring problem ($k = 2$). This solution does not satisfy the constraint: two adjacent vertices do not share the same color. 72

3.5 Two-level systems considered by Huse and Fisher (1986). 80

3.6 Energy landscape of the model considered by Shinomoto and Kabashima (1991). 83

4.1 (Left panel) A schematic of classical phase transition. The horizontal and vertical axes represent the temperature and the order parameter, respectively. (Right panel) A schematic of quantum phase transition. The horizontal and vertical axes represent the quantum field such as the transverse field and the order parameter, respectively. 99

4.2 $\Gamma/J(0)$-dependence of $\langle\hat{\sigma}^z\rangle$ (solid line) and $\langle\hat{\sigma}^x\rangle$ (dotted line). When $\Gamma \geq J(0)$, quantum paramagnetic state ($\langle\hat{\sigma}^x\rangle = 1$) appears. 102

4.3 Phase diagram of the ferromagnetic Ising model in a transverse field at the mean-field level. 103

4.4 (Left panel) Phase diagram of the ferromagnetic quantum Ising model in one dimension. The bold line indicates the ferromagnetic phase and the rest is the paramagnetic phase. (Right panel) Schematic phase diagram of the ferromagnetic quantum Ising model in $d = 2, 3$ dimensions. The bold curve indicates the boundary between the ferromagnetic and paramagnetic phases. 104

4.5 Schematic representations of the Suzuki–Trotter decomposition. 106

4.6 Specific heat C and the internal energy E as a function of temperature for a transverse Ising chain at $\Gamma/J = 1/2$. The diamond, solid, and open circles represent the quantum Monte Carlo data for $P = 4, 8$, and 12, respectively. The solid curves are exact solutions (from Nagai et al., 1986). 108

4.7 Specific heat C and the longitudinal susceptibility χ for the square lattice whose lattice size is 24×24 at $\Gamma/J = 1$. The data was obtained by quantum Monte Carlo simulations for $P = 24$. The solid curves are a guide to the eye (from Nagai et al., 1986). 109

4.8 Phase boundaries for the transverse Ising models on square ($d = 2$) and cubic ($d = 3$) lattices. The solid and broken curves represent results obtained by the series expansion (from Nagai et al., 1986). 110

4.9 Phase diagram of the Husimi–Temperely–Curie–Weiss model in a transverse field Γ for $h \to +0$. 115

Cambridge University Press
978-1-107-11319-0 — Quantum Spin Glasses, Annealing and Computation
Bikas K. Chakrabarti , Jun-ichi Inoue , Ryo Tamura , Shu Tanaka
Frontmatter
More Information

4.10 Transverse field, Γ, dependence of the longitudinal magnetization m_z of the Husimi–Temperely–Curie–Weiss model in a transverse field Γ when $T = 0$. The quantum phase transition occurs at $\Gamma_c/zJ = 1$. 116

4.11 Schematic phase diagram of the Ising model with random interactions in a transverse field Γ. O_{cl}, the spin glass phase, is characterized by the classical spin glass order. In the region indicated by O_q, there are possibilities of the existence of the replica symmetric spin glass phase due to the quantum fluctuation effect. 117

4.12 Phase diagram of the SK model in a transverse field under the single-spin approximation. 119

4.13 Phase diagram of the SK model in a transverse field under the replica symmetric approximation and static approximation. 129

4.14 (Left panel) Phase diagram of the SK model in a transverse field obtained by the replica symmetry breaking solution without using static approximation (from Goldschmidt and Lai, 1990). (Right panel) Phase diagram of the SK model in a transverse field obtained by the replica symmetric solution without using static approximation. "Perturbative" denotes the naive Landau expansion result which is the same as in Fig. 4.12. m stands for the number of modes expressing the dynamical nature; $m = 0$ is the result of static approximation. "Quantum" denotes the result of the improved Landau expansion in a quantum regime where the quantum fluctuation is dominant. The quantum regime is shown by the shaded portion (from Takahashi, 2007). 130

4.15 Monte Carlo simulation results of the SK model in a transverse field in a high-temperature region for $J = 1$. In these cases, the classical fluctuation is dominant. (a) Temperature, T, dependence of the Binder ratio g for $\Gamma = 0$. (b) Scaling plot of the Binder ratio g when $x_T = 0.31$ for $\Gamma = 0$. (c) Transverse field, Γ, dependence of g for $T = 0.65$. (d) Scaling plot of g when $x_\Gamma = 0.31$ and $T = 0.65$. (e) Γ-dependence of g for $T = 0.6$. (f) Scaling plot of g when $x_\Gamma = 0.31$ and $T = 0.6$ (from Mukherjee et al., 2015). 133

4.16 Monte Carlo simulation results of the SK model in a transverse field in a low-temperature region for $J = 1$. In these cases, the quantum fluctuation is dominant. (a) Transverse field, Γ, dependence of the Binder ratio g for $T = 0.30$. (b) Scaling plot of the Binder ratio g when $x_\Gamma = 0.50$ for $T = 0.30$. (c) Γ-dependence of g for $T = 0.25$. (d) Scaling plot of g when $x_\Gamma = 0.50$ for $T = 0.25$ (from Mukherjee et al., 2015). 134

4.17 Exact diagonalization results of the SK model in a transverse field at zero temperature for $J = 1$. (a) Transverse field, Γ, dependence of the Binder ratio

g. (b) Scaling plot of the Binder ratio g when $x_T = 0.51$ and $\Gamma_c = 1.62$ (from Mukherjee et al., 2015). 136

4.18 Schematic phase diagram of the SK model in a transverse field obtained by simulation results. The solid and dotted curves indicate the phase boundary dominated by the classical fluctuation and that by the quantum fluctuation, respectively. At $T/J \simeq 0.45$, crossover occurs. 137

4.19 L_τ-dependence of g for various K for the effective classical model corresponding to the EA model in a transverse field on the square lattice. The diamond, plus, square, cross, and triangular points indicate results for $L = 4$, 6, 8, 12, and 16, respectively. At $K^{-1} = 3.30$ (center figure), the maximum values at the peak become about the same value for various L. This result was obtained by Monte Carlo simulations (from Rieger and Young, 1994). 139

4.20 (a) Scaling plot of the Binder ratio g when the value of z is 1.5 for the effective classical model corresponding to the EA model in a transverse field on the square lattice. (b) First derivative of Binder ratio with respect to K at K_c : $g'(K = K_c)$ depending on L. The slope of the fitting line is $1/\nu = 1.0$. This result was obtained by Monte Carlo simulations (from Rieger and Young, 1994). 140

4.21 Scaling plot of the Binder ratio g when $K^{-1} = T = 4.3$ and $z = 1.3$ for the effective classical model corresponding to the EA model in a transverse field on the cubic lattice. This result was obtained by Monte Carlo simulations (from Guo et al., 1994). 141

4.22 K^{-1}-dependence of the Binder ratio g for the effective classical model corresponding to the EA model in a transverse field on the cubic lattice. Inset is the scaling plot of the Binder ratio g when $K^{-1} = T = 4.3$ and $1/\nu = 1.3$. This result was obtained by Monte Carlo simulations (from Guo et al., 1994). 142

4.23 Overlap distribution function $P_N(q)$ which is indicated by $W(q)$ in this figure as a function of q for various lattice sizes N at $\Gamma/J = 0.15$ and $T/J = 0.4$, which is located in the spin glass phase of the SK model in a transverse field. The dotted curve indicates the upper profiles of $P_N(q)$ (from Ray et al., 1989). 144

4.24 Overlap distribution function $P_N(q)$ which is indicated by $P(q)$ in this figure as a function of q for various lattice sizes N at $\Gamma/J = 0.4$ and $T/J = 0.4$, which is located in the spin glass phase of the SK model in a transverse field. The plus, cross, diamond, square, and circle points indicate results for $N = 20$, 40, 60, 80, and 100, respectively (from Lai and Goldschmidt, 1990). 145

5.1 Landau–Zener transition of a system whose Hamiltonian is given by $\mathscr{H} = -h(t)\sigma^z - \Gamma\sigma^x$, where $h(t) = -t/\tau$. (a) The instantaneous eigenenergies of the system. (b) Γ-dependence of the expectation value of σ^z as a function of $h(t)$ with $\tau = 100$. Here, the initial state is set to the ground state for $h = -5$. (c) τ-dependence of the expectation value of σ^z as a function of $h(t)$ with $\Gamma = 0.1$. Here, the initial state is set to the ground state for $h = -5$. 158

5.2 (a) Schematics of the temperature dependence of correlation length $\xi(g(t))$ in simulated annealing where the annealing speed τ_Q^{-1} ($\tau_{Q1} < \tau_{Q2} < \tau_{Q3}$). $T_i := T(t_s)$ for $i = 1, 2, 3$ denotes the temperature at which the system cannot reach the equilibrium state under simulated annealing whose annealing speed is τ_{Q_i}. The stopped correlation length is written as $\xi_i := \xi(g(t_s))$ for $i = 1, 2, 3$. Here, $\xi_1 < \xi_2 < \xi_3$. (b) Schematics of the temperature dependence of the density of the domain wall $n(t)$ in simulated annealing where the annealing speed τ_Q^{-1} is different. The stopped density of the domain wall is written as $n_i := n(t_s)$ for $i = 1, 2, 3$. Here, $n_1 > n_2 > n_3$. 161

6.1 Schematic of simulated annealing and quantum annealing. The purpose of these methods is to obtain the best solution at the position indicated by the star ($T = 0$, $\Gamma = 0$). 172

6.2 Schematic of the concept of quantum annealing to search for the ferromagnetic ground state. 172

6.3 Ground state spin configuration of the Ising model with homogeneous ferromagnetic interactions for $N = 8$. The magnetic field is applied for all spins, i.e., $h_i = 0.1J$ for all i to avoid the trivial degeneracy when the transverse field is absent. 177

6.4 Schematic of the schedules of annealing methods defined by Eqs. (6.21), (6.22), and (6.23). 178

6.5 Time dependence of $P_{\rm QA}(t)$, $P^{\rm st}_{\rm QA}(\Gamma(t))$, $P_{\rm SA}(t)$, and $P^{\rm st}_{\rm SA}(T(t))$ for the Ising model with homogeneous ferromagnetic interactions. (a) The schedule of annealing is $\Gamma(t)/J = T(t)/J = 3/\ln(t+1)$. (b) The schedule of annealing is $\Gamma(t)/J = T(t)/J = 3/\sqrt{t}$ (from Kadowaki and Nishimori, 1998). 179

6.6 Time dependence of $P_{\rm QA}(t)$, $P^{\rm st}_{\rm QA}(\Gamma(t))$, $P_{\rm SA}(t)$, and $P^{\rm st}_{\rm SA}(T(t))$ of an Ising model with random interactions. The schedule of annealing is $\Gamma(t)/J = T(t)/J = 3/\sqrt{t}$ (from Kadowaki and Nishimori, 1998). 180

6.7 Monte Carlo step, τ, dependence of residual energy $\varepsilon_{\rm res}(\tau)$ obtained by simulated annealing (classical annealing: CA) and quantum annealing for various temperature T and the Trotter number P when $PT = 1$ (from Martoňák et al., 2002). 182

6.8 Monte Carlo steps, τ, dependence of the excess length after annealing $\varepsilon_{\rm exc}$ obtained by simulated annealing (SA) and quantum annealing (QA). In order to compare the results by the simulation time, the result of QA multiplied by P is also shown. The dotted line indicates the value of $\varepsilon_{\rm exc} \simeq 1.57$ obtained by the Lin–Kernigham algorithm which is a standard local-search algorithm (from Martoňák et al., 2004). 183

6.9 (Left panel) Total annealing time (Monte Carlo step) dependence of the residual energy after simulated annealing (classical annealing, CA) and quantum annealing (QA). The dotted line indicates the Gardner energy for the 3-SAT problem, which is a lower bound of the energy of the metastable states. Inset is the Trotter number, P, dependence of the residual energy for the 3-SAT problem (Battaglia et al., 2004). (Right panel) Schedule of the temperature and the transverse field $J_\Gamma = \ln\coth(\beta\Gamma(t)/P)/2\beta$ in field cycling. (from Battaglia et al., 2005). 184

6.10 Monte Carlo step, τ, dependence of the residual energy obtained by quantum annealing based on the Green's function Monte Carlo method (GFMC) and the path-integral Monte Carlo method (PIMC-QA), and simulated annealing (classical annealing: CA) for the Ising model with random interactions. The results of PIMC-QA and CA are from Fig. 6.7. Two types of simulations for GFMC were performed using different trial wave function. When the trial wave function is constant, results of upper diamonds are obtained (GFMC $\psi_T = 1$). On the other hand, when the trial wave function is the Boltzmann type, results of lower diamonds are obtained (GFMC-QA). The time unit in GFMC is a step that a single-spin flip event is tried, while in PIMC-QA and CA, time unit is a step that the update of all spins is tried. Thus, in order to compare these results when the time unit is a single-spin flip, the results of PIMC-QA and CA multiplied by N are also shown (from Stella and Santoro, 2007). 190

6.11 Schematic of division of blocks in DMRG. 192

6.12 Residual energy, $E_{\rm res}$, dependence of τ when $J = 1.6$ obtained by the density matrix renormalization group for the random field Ising model. The dotted line is the guide to the eyes for logarithmic scaling (from Suzuki and Okada, 2007). 195

6.13 Residual energy depending on the annealing time τ when (a) $J = 2.0$ and (b) $J = 0.6$ for the random field Ising model on the square lattice by mean-field calculation based on the Bethe approximation. The obtained results of quantum annealing by transverse field defined by Eq. (6.83) (TF) and by transverse field and ferromagnetic interaction defined by Eq. (6.117) (FI) and simulated annealing (thermal) are compared. These results are averaged out over eighty configurations of random fields (from Suzuki et al., 2007). 201

6.14 Phase diagram of the magnetic materials $\mathrm{LiHo}_x\mathrm{Y}_{1-x}\mathrm{F}_4$. The horizontal axis corresponds to the concentration x of Ho^{3+} ions, and the vertical axis is the temperature. PM, FM, and SG denote the paramagnetic phase, ferromagnetic phase, and spin glass phase, respectively. The arrow shows the location of the decoupled cluster glass in which essentially isolated clusters with zero net moments appears at zero temperature. The coupling between these clusters increases with temperature; decoupled cluster glass does not appear at a finite temperature (from Reich et al., 1990). 202

6.15 Schematic phase diagram of the temperature T versus the magnetic field H_t corresponding to Γ of $LiHo_xY_{1-x}F_4$ for $x = 0.56$. The arrows indicate the quantum annealing (QA) process and the thermal annealing (simulated annealing: SA) process toward the point at $H_t = 7.2$ kOe under $T = 0.03$ K in the spin glass phase, respectively. 203

6.16 Time evolution of the coefficients of $\hat{\mathscr{H}}_0$ and $\hat{\mathscr{H}}_q$ in the adiabatic quantum computation. 204

7.1 Quantum annealing vs thermal annealing in a graph of energy as a function of configuration space. 256

7.2 The set of amplitudes following the first step of the Grover iteration in the 2-qubit case. 257

7.3 The set of amplitudes following the second step of the Grover iteration in the 2-qubit case. 258

7.4 8 qubit cell. 275

7.5 The vertex 0 in G is mapped into two such vertices to satisfy the physical requirements of C_n, that is, no more than 4 couplings to each physical qubit. 275

8.1 The phase diagram of the one-dimensional three-spin interacting TFIM with different paths along which the MCP (point A) is approached. Three different critical lines are shown satisfying the relations $h = 1$, $h = J_x - 1$ and $h = -J_x - 1$, whereas, Paths I, II, III and IV are denoted by lines with arrows. A shaded region also has been shown in the figure where quasicritical points exist (from Rajak and Divakaran, 2014). 298

8.2 The plot shows χ_F as a function of h at $J_x = 2, J_3 = -1$. Here the system size is $L = 100$. The first peak that occurs at $h = -3$ is an Ising critical point which shows linear scaling with L, and the second peak occuring at $h = 1$ corresponds to the MCP where L^5 scaling is found. The inset figure shows that the FS oscillates close to the MCP, indicating the presence of quasicritical points there (from Rajak and Divakaran, 2014). 301

8.3 The scaling of χ_F when the MCP is approaced along four different paths. The plots shown in (a), (b) and (c) are for Paths I, II and III resulting in $\chi_F \propto L^5$ that shows the effect of quasicritical points; whereas it is linear in L for Path IV as shown in (d) due to the absence of quasicritical points along this path or line (from Rajak and Divakaran, 2014). 302

8.4 Variation of the LE as a function of h. The sharp dips in the LE are seen at the Ising critical points ($h = -2, 0$) as well as at anisotropic critical point at $h = 1$ (from Rajak and Divakaran, 2014). 305

8.5 Variation of LE as a function of scaled time t/L^{α} to highlight the scaling of time period with system size L, with α being the scaling exponent ($T \propto L^{\alpha}$) as obtained in the text for various types of critical points, i.e., (a) the anisotropic critical point where $T \propto L$, (b) multicritical point where $T \propto L^2$ and (c) quasicritical point with $T \propto L^3$. In (d), we present the almost linear variation of $\ln \mathscr{L}$ with t^2 for small times with $L = 100$ at the various critical points confirming the general small time behavior given by $\mathscr{L} \sim e^{-\Gamma t^2}$ (from Rajak and Divakaran, 2014). 306

8.6 The figure shows a schematic representation of the Kitaev model on a honeycomb lattice with the interactions denoted by J_1, J_2 and J_3; $\vec{M}_1 = \frac{\sqrt{3}}{2}\hat{\imath} - \frac{3}{2}\hat{\jmath}$ and $\vec{M}_2 = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath}$ are the spanning vectors of the lattice (from Sharma and Rajak, 2012). 307

8.7 The phase diagram of the Kitaev model in which all the points satisfy the relation $J_1 + J_2 + J_3 = 4$. The gapless phase of the model is given by the inner equilateral triangle satisfying the relations $J_1 \leq J_2 + J_3$, $J_2 \leq J_3 + J_1$ and $J_3 \leq J_1 + J_2$. The study of LE is carried out along Paths I, II and III where J_3 is varied (from Sharma and Rajak, 2012). 309

8.8 The plots show variation of LE with the parameter J_3 (J_3 is changed Path I). The LE shows a sharp dip at $J_3 = 2 - \delta$ (point A) and following a small but finite revival throughout the gapless phase, it further decays at point B, $J_3 = 0$ (see Fig. (8.7)). Here, $N_x = N_y = 200$, $\delta = 0.01$ and $t = 10$. Inset (a) shows LE as a function of J_3 when the change in J_3 is carried out along Path II ($J_1 = J_2 + 1$) with $N_x = N_y = 200$, $\delta = 0.01$ and $t = 10$ exhibiting a sharp dip at point P ($J_3 = 2 - \delta$) and again there is a sharp increase at point Q ($J_3 = 1 - \delta$). Inset (b) shows the dip in LE at $J_3 = 2 - \delta$ when J_3 is changed along the line $J_1 + J_3 = 4$ (Path III). For this case, $N = 400$, $\delta = 0.01$ and $t = 10$ (from Sharma and Rajak, 2012). 311

8.9 The LE as a function of time t at the AQCP A (see Fig. 8.7) with $J_1 = J_2 = 1$, $\delta = 0.01$ and $J_3 = 2 - \delta$, considering $N_x(= 100)$ as fixed and N_y as changing confirms the scaling relations discussed in the text. The inset figure shows that the time period of the collapse and revival does not depend on N_x (from Sharma and Rajak, 2012). 312

8.10 The variation of LE with time for different values of δ when J-quench and h-quench are performed at $L_1 = L/2$ and $L' = L/3$ respectively. The h-quenching here is made by changing its value from 1 to $1 + \delta$ at L'. In the case of only J-quenching (i.e., $\delta = 0$ case), the LE marks a peak at $t_3 = L/v_{\max} = 150$ and $T = 2t_3$. Here, $v_{\max} = 2$ and $L = 300$. By applying both the local quenches simultaneously at time $t = 0$, we find a small peak at $t' = 50$ and a relatively stronger peak at $t'' = 200$. It can also be noted that there are small fluctuations close to $t_1 = 100$ which is more clear for the $\delta = 1$ curve (from Rajak and Divakaran, 2016). 317

8.11 Main: The time evolution of the LE for two local quenches having $L_1 = 100, L_A = 100$ and $L = 300$ and different L'. The first peak in the LE is seen at time $t' = 2(L_1 - L')/v_{\max}$; $t' = 25$ and 50 for $L' = L/4$ and $L/6$ respectively. The other time scales are given by $t_3 = 100$, $t_4 = 200$ and t''. It can be noted that $t'' = 225$ and 250 for $L' = 75$ and 50, respectively. Inset: The LE as a function of time t for only h-quench at $L/3$ with $L = 300$. Time scales $t_1 = 100$ and $t_2 = 200$ both are observed in this case. It is easy to find that the magnitude of decay in this case is very small (from Rajak and Divakaran, 2016). 318

8.12 The plot shows time evolution of the entanglement entropy having $L_1 = L_A = 100$ and different values of L'. For $L' = L/4$, the departure from the only J-quench case is seen at $t'/2 = 12.5$, whereas at $t' = 25$, QP_L^2 get in the system B after reflection at L' in which QP_R^2, the other partner, is already present. The EE keeps on decreasing till QP_R^2 enters system A at time $t = L_2 = 200$. A dip is also seen at $t = t'' = 225$ where QP_L^2 and QP_R^2 interchanges their systems. In a similar fashion, we can make an argument for the time evolution of EE when $L' = 3L/4$. The deviation from the case of J-quench alone starts at $t = 62.5$. There is a sharp decay in EE at $t = 100$ when QP_L^2 enters system B. On the other hand, QP_R^2 goes into system A at $t = 125$ that results in an increase in the EE since its other partner remains in system B. The normal rise at $t = 200$ which is clearly present for the single quenching (J-quenching) case can also be found here. This may be an artefact of getting a transmitted component of the QP wavepacket at L' which indicates that the reflection of the QPs at L' is not perfect, as already discussed in Sec. 8.5.4. We can also observe the time scales $t'' = 225$ and $T = 300$ (from Rajak and Divakaran, 2016). 320

8.13 (a) Variation of the EE as a function of time following single and double quenches when the total spin chain is in the ferromagnetic region at $h = 0.5$. Here, we consider $L_1 = 100$, $L_A = 100$ and $L = 300$ and different L'. For $L' = L/4$, the EE deviates from the only J- quenching case, starting at $t = t'/2$ with $t' = 50$ and $v_{\max} = 2h = 1$. For time $t > 50$, both the quasiparticles QP_L^2 and QP_R^2 exist in subsystem B that leads to the decrease in the EE continuing up to $t_4 = 400$. Again, for $L' = L/5$, we find $t' = 80$ and observe the expected behavior. The explanation of the absence of t'' is given in the text. The plot (b) is the same with $h = 0.99$. The observed time scales are given by $t' = 25.25$, $t'/2$, $t_3(\sim 101)$ $t_4(\sim 202)$ and $t''(\sim 227)$ calculated with $v_{\max} = 1.98$ (from Rajak and Divakaran, 2016). 322

8.14 The plot shows f_q as a function of q for a critical chain with $L_1 = 100$, $L' = 60$ and $L = 300$. The dashed line represents the case of the h-quench alone which is clearly smaller than the J-quench alone (the dotted line) by an order of magnitude. The solid line is for the double quench, which almost overlaps with the J-quenching case. A similar type of behavior can also be observed for a spin chain residing in the ferromagnetic region (from Rajak and Divakaran, 2016). 324

8.15 Phase diagram of the p-wave superconducting model in one dimension (see Eq. (8.52)). In this diagram, Phases I and II are topologically non-trivial, whereas, Phase III is topologically trivial. Sudden quenching in this system is performed along Paths A and B (see Sec. 8.8). 329

8.16 Schematic representation of the Majorana chain with the Hamiltonian as defined in Eq. (8.57), considered a virtual ladder. It may be thought that the middle of each vertical bond here represents a fermionic site j which supports two Majorana fermions a_j and b_j represented by black and gray filled circles respectively with their intra-site interaction as μ. In this representation, other two couplings are symbolized by diagonal links. 330

8.17 Schematic representation of the Hamiltonian in Eq. (8.57) with different special conditions as defined in the text. (a) $\mu \neq 0$, $w = \Delta = 0$, (b) $\mu = 0$, $w = \Delta$ and (c) $\mu = 0$, $w = -\Delta$. In (a), the pair of Majoranas at each site is connected to each other by the strength μ resulting in a topologically trivial phase. (b) In this regime, there are two isolated Majorana modes a_1 and b_N at the left end and right end respectively. (c) This regime also represents a topologically non-trivial phase with two isolated Majorana modes b_1 and a_N. 331

8.18 Two Majorana edge states are localized at two ends of a 100-site open Majorana chain in Phase I ($\Delta = 0.1$ and $\mu = 0.0$) with an exponential decay into the bulk of the chain. j labels the Majorana sites $1, 2, \ldots 200$ (from Rajak and Dutta, 2014). 332

8.19 (a)–(c) Schematic representation of three terms in Eq. (8.63) using the language of Majorana fermions where (a), (b), and (c) describe the first, second, and third terms respectively. The unpaired Majorana modes enclosed in dotted lines exist at two ends of the chain. (d) The plot shows the phase diagram of the generalized cluster model of Eq. (8.63) at zero temperature for $J^{XZX} > 0$. The thick solid curves are the phase boundaries where the energy gap vanishes at a particular k. The different phases like ferromagnetic, antiferromagnetic, paramagnetic, cluster and dual cluster are represented by F, AF, P, C and C* respectively. The superscripts in F and AF denote the direction of the order. (e) These phases are determined by string order parameters. The string order parameters O_{XZX}, O_{XX} and O_{YY}

have been calculated along the thick dotted line $J^{YZY}/J^{XZX} = 0.5$ (see Fig. 8.19(d)) using the iTEBD (from Ohta et al., 2015). 335

8.20 The diagram for the energy levels of the Majorana chain of system size $N = 100$ as a function of $\xi = \Delta/w$ with $\mu = 0$ and $w = 1$ considering (a) open boundary condition and (b) periodic boundary condition, respectively. One can observe that two zero energy Majorana edge modes exist in case (a) but not in case (b). It is mentioned that the energy spectrum is scaled by a constant factor $1/4$ in Eq. (8.65) compared to Eq. (8.54) (from Rajak and Dutta, 2014). 337

8.21 Majorana survival probability of a Majorana edge state for various quench protocols along Path A and probability of Majorana at certain time t after quench. (a) There is a sudden decay in the MSP due to a quenching from Phase I ($\Delta = 0.1$) to Phase II ($\Delta = -0.1$) and after that, it remains minimum without any revival. (b) On the other hand, following a quench from Phase I ($\Delta = 0.1$) to the QCP ($\Delta = 0.0$), MSP exhibits nearly perfect oscillation with a time period scaling linearly with the system size N. (c) Probability of the Majorana end mode at time $t = 100$ for quenching at the QCP with the Majorana site j. At this time, the probability of the Majorana mode is maximum, close to the center of the chain (from Rajak and Dutta, 2014). 338

8.22 A sudden quench is performed within Phase I from ($\Delta = 0.2$ and $\mu = 0.0$) to ($\Delta = 0.1$ and $\mu = 0.0$) and the MSP becomes (a) a rapidly oscillating function of time t having nearly 0.8 mean value and (b) probability of the Majorana end mode at $t = 100$, with the Majorana sites j following the quench (from Rajak and Dutta, 2014). 340

8.23 Phase diagram of the system with Hamiltonian in Eq. (8.52) for various phases of complex hopping term (a) $\phi = 0$, (b) $\phi = \pi/4$, (c) $\phi = 2\pi/5$ and (d) $\phi = \pi/2$. In this plot, I and II represent two different topological phases, whereas III is the topologically trivial phase. The path $\mu = 0$ is represented by the vertical arrow along which quenching is performed (from Rajak et al., 2014). 341

8.24 The energy levels of the Hamiltonian in Eq. (8.52) as a function of parameter $\xi = \frac{\Delta}{w_0}$ (where $w_0 = 1$) for (a) periodic and (b) open boundary conditions with $N = 100$ and $\phi = \pi/10$. The inverted energy levels inside the gapless phase of the system is indicated by the red color (from Rajak et al., 2014). 343

8.25 (a) Variation of the probability of defect (P_{def}) with τ for different values of ϕ exhibiting dip at different values of $\tau \geq \tau_c$ where the value of τ_c increases as ϕ decreases. (b) The probability of Majorana (P_m) exhibits a peak precisely at those values of τ where P_{def} shows dips. Here, $N = 100$ (from Rajak et al., 2014). 345

8.26 (a) P_{def}, P_{neg} and P_m as a function of τ with $\phi = \pi/10$ exhibit that all of them add up to unity for any τ. (b) The plot shows that $\ln(\tau_c)$ varies almost linearly with ln(sin ϕ) having slope $(= -0.9)$ nearly equal to -1 (from Rajak et al., 2014). 346

8.27 (a) Variation of P_{m} as a function of τ for different system sizes N with $\phi = \pi/5$. P_{m} exhibits peak at different values of $\tau \geq \tau_c$. (b) The figure shows a plot between $\ln \tau_c$ and $\ln N$ for ϕ with slope $(= 0.94)$ nearly equal to 1 suggesting $\tau_c \sim N$ (from Rajak et al., 2014). 347

8.28 Variation of P_{def} as a function of τ determining overlaps between the time-evolved Majorana state at final time and different number of positive energy states (near zero energy) at the final parameter values. Let us consider the case with $\phi = \pi/10$ and $N = 100$, where the number of inverted energy levels in both positive and negative sides of the zero energy is given by 18 (see Ref. (Rajak et al., 2014)). The plot indicates that for $\tau \leq \tau_c$ ($\Delta t < \Delta t_{\mathrm{th}}$), the initial edge Majorana mixes with all the bulk energy levels. For $\tau \geq \tau_c$, on the other hand, the P_{def} determined by using only the inverted positive bulk energy levels nearly coincides with the exact P_{def} procured considering all positive energy levels. This clearly verifies that quenched Majorana state interacts only with the inverted energy levels for a passage time $\Delta t \geq \Delta t_{\mathrm{th}}$ (from Rajak et al., 2014). 348

9.1 Title and abstract from the first published paper proposing the idea that quantum tunneling across the free energy barriers in the Sherrington–Kirkpatrick spin glass model can lead to an efficient way of searching for its ground state(s). It was argued that "quantum tunneling between the classical 'trap' "states", separated by infinite (but narrow) barriers in the free energy surface, is possible, as quantum tunneling probability is proportional to the barrier area which is finite." They suggested that any amount of transverse field would lead to the collapse of the overlap distribution to a delta function. It may be noted that computationally hard problems can often be mapped into such long-range spin glass models; the advantage of quantum tunneling in such quantum spin glass models has lead ultimately to the development of the quantum annealer. A related reference is Chakrabarti (1981). (Permission to use title and abstract from the paper is given by American Physical Society) 360

9.2 Title and abstract from the first paper reporting on the experimental studies of an Ising spin glass sample under the influence of a tunable transverse field. The observed nature of the tunneling induced phase diagram compared well with that predicted by Ray et al. (1989). (Permission to use title and abstract from the paper is given by American Physical Society) 361

9.3 Title and abstract of the first published paper demonstrating the search of the ground state of a Lennard–Jones system using 'quantum annealing' (term appearing for the first time in paper title). (Permission to use title and abstract from the paper is given by Elsevier) 361

9.4 Title and abstract from a paper demonstrating the clear advantage of quantum annealing in Ising models with frustrating interactions. It was found that in case of quantum annealing not only there was a higher probability of the initial state to converge in the ground state of such models but also the convergence time was comparatively less than that of simulated annealing. A related reference is Chakrabarti et al. (1996). (Permission to use title and abstract from the paper is given by American Physical Society) 362

9.5 Title and abstract of the first published experimental paper reporting on the advantage of quantum annealing in finding the ground state of disordered magnets. Such experimental observations have put quantum annealing on firm physical ground. Some related references are Wu et al. (1991), Wu et al. (1993), Bitko et al. (1996), Kadowaki and Nishimori (1998). (Permission to use title and abstract from the paper is given by The American Association for the Advancement of Science) 363

9.6 Title and abstract of a paper reporting on the zero temperature quantum adiabatic algorithm for NP-hard problems. A related reference here is Kadowaki and Nishimori (1998). (Permission to use title and abstract from the paper is given by The American Association for the Advancement of Science) 364

9.7 Title and abstract of a paper on the application of quantum annealing in estimating the remaining fraction of undesired solutions in some optimization searches in Ising spin glasses. They also indicated the relative fastness of quantum annealing with respect to simulated annealing. Some related references here are Wu et al. (1993), Finnila et al. (1994), Kadowaki and Nishimori (1998), Brooke et al. (1999), Brooke et al. (2001) and Farhi et al. (2001). (Permission to use title and abstract from the paper is given by The American Association for the Advancement of Science) 365

9.8 Two important early reviews helped the subsequent development of quantum annealing significantly. Title and abstract for the first review on adiabatic quantum computation and annealing, proclaiming that the idea of quantum tunneling through the infinitely high energy barriers in long-range frustrated spin glasses was introduced in Ray et al. (1989). Other related references here are Finnila et al. (1994), Chakrabarti et al. (1996), Kadowaki and Nishimori (1998), Brooke et al. (1999) and Das et al. (2005). (Permission to use title and abstract from the paper is given by Institute of Physics) 366

9.9 Title and abstract from the second (early) review on quantum annealing and quantum computation. Authors summarized that following the indication of Ray et al. (1989), both theoretical and experimental successful studies on QA made this technique extremely useful in solving hard optimization problems, in binary to analog quantum computers. Some related references here are Finnila et al. (1994), Kadowaki and Nishimori (1998), Brooke et al. (1999), Farhi et al. (2001) and Santoro et al. (2002). (Permission to use title and abstract from the paper is given by American Physical Society) 367

9.10 Title and abstract from the review discussing the detailed mathematical structure and theorems related to QA. Some related references here are Finnila et al. (1994), Kadowaki and Nishimori (1998), Santoro et al. (2002), Santoro and Tosatti (2006), Das et al. (2005), Das and Chakrabarti (2005) and Das and Chakrabarti (2008). (Permission to use title and abstract from the paper is given by American Institute of Physics) 368

9.11 Title and abstract from a paper extending and clarifying the quantum annealing method used by Brooke et al. (1999) for quantum glasses. Authors argued that at low enough temperatures, and “in cases where barriers to relaxation are tall and narrow, quantum mechanics can enhance the ability to traverse the free energy surface (Ray et al. 1989)”. Other related references cited are Wu et al. (1991), Wu et al. (1993). (Permission to use title and abstract from the paper is given by American Physical Society) 369

9.12 Title and abstract of the first paper by the D-Wave group giving the basic architecture of their quantum annealing processor. Some related references here are Finnila et al. (1994), Kadowaki and Nishimori (1998), Brooke et al. (1999) and Farhi et al. (2001). (Permission to use title and abstract from the paper is given by Nature Publishing Group) 370

9.13 Title, abstract and some excerpts from a paper by scientists from the University of Southern California, University of California, ETH Zurich and Microsoft Research. They argued in favour of the quantum nature of the D-Wave quantum annealer by comparing its performance with that of a classical one. In the introduction, the authors had commented “The phenomena of quantum tunneling suggests that it can be more efficient to explore state space quantum mechanically in a quantum annealer (Ray et al. 1989, Finnila et al. 1994, Kadowaki and Nishimori 1998)”. Some other related references here are Brooke et al. (1999), Farhi et al. (2001) and Johnson et al. (2011). (Permission to use title and abstract from the paper is given by Nature Publishing Group) 371

9.14 Title and abstract from a paper where authors critically investigate the claim of the probable quantum speed-up over classical annealing. Their investigations suggest that the previously observed (Santoro et al., 2002; Martoňák et al., 2002) advantage of QA compared to SA in searching for the ground state of 2D spin glasses was due to the discretization of time in the quantum Monte Carlo algorithm. They have not found any advantage in continuous time limit. Other related cited papers are Ray et al. (1989), Finnila et al. (1994), Kadowaki and Nishimori (1998), Farhi et al. (2001) and Das and Chakrabarti (2008). (Permission to use title and abstract from the paper is given by The American Association for the Advancement of Science) 372

9.15 Title and abstract from a paper discussing the out of equilibrium quantum Monte Carlo study of spin glasses on 3-regular graphs. Authors observe the inability of quantum algorithm regarding fastness relative to SA in bringing down the system to glassy state. Some related references here are Ray et al. (1989), Finnila et al. (1994), Kadowaki and Nishimori (1998), Brooke et al. (1999), Farhi et al. (2001), Santoro et al. (2002), Das and Chakrabarti (2008) and Boixo et al. (2014). (Permission to use title and abstract from the paper is given by American Physical Society) 373

9.16 Title and abstract from a paper where authors scrutinize the role of decoherence in quantum computation as well as quantum annealing. They indicate that decoherence does not affect adiabatic quantum computation. The related references here are Ray et al. (1989), Kadowaki and Nishimori (1998), Farhi et al. (2001), Das and Chakrabarti (2008), Boixo et al. (2013) and Boixo et al. (2014). (Permission to use title and abstract from the paper is given by American Physical Society) 374

9.17 Title and abstract from a paper reporting on an interesting implementation of singular-value decomposition by QA in cases of big data and image analysis. The related references are Ray et al. (1989), Finnila et al. (1994), Kadowaki and Nishimori (1998), Farhi et al. (2001), Das and Chakrabarti (2005), Das and Chakrabarti (2008) and Suzuki et al. (2013). (Permission to use title and abstract from the paper is given by American Physical Society) 375

9.18 Title and abstract from the paper suggesting a generalized method for the reduction of the dimension of the Hilbert space of quantum systems. The related references are Ray et al. (1989), Finnila et al. (1994), Kadowaki and Nishimori (1998), Farhi et al. (2001) and Perdomo-Ortiz et al. (2012). (Permission to use title and abstract from the paper is given by American Physical Society) 376

9.19 Title and abstract from the paper reporting on experimental evidence of the success of QA using quantum tunneling across free energy barriers. The related references are Ray et al. (1989), Finnila et al. (1994), Kadowaki and

Nishimori (1998), Brooke et al. (1999), Santoro et al. (2002), Johnson et al. (2011) and Boixo et al. (2013). 377

9.20 Title and abstract from a paper reporting on the failure of the quantum annealer in context of speed enhancement with respect to SA. Some related references here are Finnila et al. (1994), Kadowaki and Nishimori (1998), Brooke et al. (1999), Santoro et al. (2002), Das and Chakrabarti (2008), Boixo et al. (2014) and Mukherjee and Chakrabarti (2015). (Permission to use title and abstract from the paper is given by American Physical Society) 378

9.21 Title, abstract and excerpts from the first major paper supporting the claim of D-Wave quantum computer used in searching for low energy conformations of the lattice protein model, reported by the Harvard University group. The authors commented in the introductory section "Harnessing quantum-mechanical effects to speed up the solving of classical optimization problems is at the heart of quantum annealing algorithms (Finnila et al., 1994; Kadowaki and Nishimori, 1998; Farhi et al., 2001; Santoro and Tosatti, 2006; Das and Chakrabarti, 2008; Ray et al., 1989)". Some other relevant references here are Amara et al. (1993), Brooke et al. (1999), Farhi et al. (2001) and Johnson et al. (2011). (Permission to use title and abstract from the paper is given by Nature Publishing Group) 379

Tables

2.1 Critical exponents of the Ising model obtained by the mean-field approximation. 13

2.2 Critical exponents of the Ising model on two-dimensional lattices. 20

2.3 Critical exponents of the Ising model on three-dimensional lattices. 20

2.4 Critical exponents of the Potts model with homogeneous ferromagnetic interactions depending on the value of q and the spatial dimension (Wu, 1982). Note that critical exponents in the two-state Potts model are equivalent to that in the Ising model (see Table 2.2 and Table 2.3). 47

2.5 Critical exponents of the XY model with homogeneous ferromagnetic interactions on three-dimensional lattices obtained by Monte Carlo simulations (Campostrini et al. 2001). 49

2.6 Critical exponents of the Heisenberg model with homogeneous ferromagnetic interactions on three-dimensional lattices obtained by Monte Carlo simulations (Holm and Janke, 1993). 50

Cambridge University Press
978-1-107-11319-0 — Quantum Spin Glasses, Annealing and Computation
Bikas K. Chakrabarti , Jun-ichi Inoue , Ryo Tamura , Shu Tanaka
Frontmatter
More Information

Preface

This book intends to introduce the readers to the developments in the researches on phase transition of quantum spin glasses, their dynamics near the phase boundary and applications in the efforts to solve multi-variable optimization problems by using quantum annealing. In view of the recent successes of both theoretical and experimental studies, major efforts were undertaken in employing these ideas in developing some prototype quantum computers (e.g., by D-wave Systems Inc.) and these have led to a revolution in quantum technologies.

The ideas developed in solving the dynamics of frustrated random systems like spin glasses, in particular of the celebrated Sherrington–Kirkpatrick model (SK model) (1975), have led to the understanding of the intrinsic nature of the problems involved in searches for the least cost solutions in multi-variable optimization problems. In particular, the pioneering idea of simulated or classical annealing technique by Kirkpatrick, Gelatt, and Vecchi (1983) had already led to major breakthroughs. It has also led to some crucial concepts regarding how the hardness of such optimization problems come about through their mapping to the ruggedness of the cost function landscape of the SK model. The idea that quantum fluctuations in the SK model can lead to some escape routes by tunneling through such macroscopically tall but thin barriers (Ray, Chakrabarti, and Chakrabarti, 1989) those which are difficult to scale using classical fluctuations, have led to some important clues. With this and some more developments, the quantum annealing technique was finally launched through a landmark paper by Kadowaki and Nishimori in 1998. Since then, as mentioned earlier, a revolution has taken place through a surge of outstanding papers both in theory and in technological applications, leading finally to the birth of this new age of quantum technologies.

This book intends to present and review these developments in a step-by-step manner, mainly from the point of view of theoretical statistical physicists. We hope the book will also be useful to physicists in general and to computer scientists as well. As one can easily see, the subject is growing at a tremendous rate today, and many more materials will soon be needed to supplement our knowledge on quantum annealing. We believe, however, the materials discussed in the book will prove indispensable for young researches and Ph. D course students who are eager to get into this exciting field of research! Indeed, we have added three notes by other experts (B. Tamir and E. Cohen, A. Rajak and U. Divakaran, and S. Mukherjee) to provide some complementary ideas and historical accounts for the benefit of the readers.

This book is dedicated to the loving memory of Professor Jun-ichi Inoue with whom we shared many ideas and developed part of the studies described here. Indeed, we had an early plan to write this book together with him. His untimely death has robbed us of that opportunity. We still hope, he would be delighted to see this book and its contents.

We are grateful to the Cambridge University Press, in particular to M. Choudhary, R. Dey, and D. Majumdar, for their immense patience and constant encouragements. We do hope the book will be useful and enjoyable to the readers.

Shu Tanaka, Tokyo
Ryo Tamura, Tsukuba
Bikas K. Chakrabarti, Kolkata
August 2016