Lectures on Quantum Mechanics

Second Edition

Nobel Laureate Steven Weinberg combines exceptional physical insight with his gift for clear exposition, to provide a concise introduction to modern quantum mechanics, in this fully updated second edition of his successful textbook. Now including six brand new sections covering key topics such as the rigid rotator and quantum key distribution, as well as major additions to existing topics throughout, this revised edition is ideally suited to a one-year graduate course or as a reference for researchers. Beginning with a review of the history of quantum mechanics and an account of classic solutions of the Schrödinger equation, before quantum mechanics is developed in a modern Hilbert space approach, Weinberg uses his remarkable expertise to elucidate topics such as Bloch waves and band structure, the Wigner–Eckart theorem, magic numbers, isospin symmetry, and general scattering theory. Problems are included at the ends of chapters, with solutions available for instructors at www.cambridge.org/9781107111660.

STEVEN WEINBERG is a member of the Physics and Astronomy Departments at the University of Texas at Austin. His research has covered a broad range of topics in quantum field theory, elementary particle physics, and cosmology, and he has been honored with numerous awards, including the Nobel Prize in Physics, the National Medal of Science, and the Heinemann Prize in Mathematical Physics. He is a member of the US National Academy of Sciences, Britain's Royal Society, and other academies in the USA and abroad. The American Philosophical Society awarded him the Benjamin Franklin medal, with a citation that said he is "considered by many to be the preeminent theoretical physicist alive in the world today." His books for physicists include *Gravitation and Cosmology*, the three-volume work *The Quantum Theory of Fields*, and, most recently, *Cosmology*. Educated at Cornell, Copenhagen, and Princeton, he also holds honorary degrees from sixteen other universities. He taught at Columbia, Berkeley, M.I.T., and Harvard, where he was Higgins Professor of Physics, before coming to Texas in 1982.

"Steven Weinberg, a Nobel Laureate in physics, has written an exceptionally clear and coherent graduate-level textbook on modern quantum mechanics. This book presents the physical and mathematical formulations of the theory in a concise and rigorous manner. The equations are all explained step-by-step, and every term is defined. He presents a fresh, integrated approach to teaching this subject with an emphasis on symmetry principles. Weinberg demonstrates his finesse as an excellent teacher and author."

Barry R. Masters, *Optics and Photonics News*

"...*Lectures on Quantum Mechanics* must be considered among the very best books on the subject for those who have had a good undergraduate introduction. The integration of clearly explained formalism with cogent physical examples is masterful, and the depth of knowledge and insight that Weinberg shares with readers is compelling."

Mark Srednicki, *Physics Today*

"Perhaps what distinguishes this book from the competition is its logical coherence and depth, and the care with which it has been crafted. Hardly a word is misplaced and Weinberg's deep understanding of the subject matter means that he leaves no stone unturned: we are asked to accept very little on faith ... it is for the reader to follow Weinberg in discovering the joys of quantum mechanics through a deeper level of understanding: I loved it!"

Jeff Forshaw, *CERN Courier*

"An instant classic ... clear, beautifully structured and replete with insights. This confirms [Weinberg's] reputation as not only one of the greatest theoreticians of the past 50 years, but also one of the most lucid expositors. Pure joy."

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For Louise, Elizabeth, and Gabrielle

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Preface

Preface to First Edition

The development of quantum mechanics in the 1920s was the greatest advance in physical science since the work of Isaac Newton. It was not easy; the ideas of quantum mechanics present a profound departure from ordinary human intuition. Quantum mechanics has won acceptance through its success. It is essential to modern atomic, molecular, nuclear, and elementary particle physics, and to a great deal of chemistry and condensed matter physics as well.

There are many fine books on quantum mechanics, including those by Dirac and Schiff from which I learned the subject a long time ago. Still, when I have taught the subject as a one-year graduate course, I found that none of these books quite fit what I wanted to cover. For one thing, I like to give a much greater emphasis than usual to principles of symmetry, including their role in motivating commutation rules. (With this approach the canonical formalism is not needed for most purposes, so a systematic treatment of this formalism is delayed until Chapter 9.) Also, I cover some modern topics that of course could not have been treated in the books of long ago, including numerous examples from elementary particle physics, alternatives to the Copenhagen interpretation, and a brief (very brief) introduction to the theory and experimental tests of entanglement and its application in quantum computation. In addition, I go into some topics that are often omitted in books on quantum mechanics: Bloch waves, time-reversal invariance, the Wigner–Eckart theorem, magic numbers, isotopic spin symmetry, "in" and "out" states, the "in–in" formalism, the Berry phase, Dirac's theory of constrained canonical systems, Levinson's theorem, the general optical theorem, the general theory of resonant scattering, applications of functional analysis, photoionization, Landau levels, multipole radiation, etc.

The chapters of the book are divided into sections, which on average approximately represent a single seventy-five minute lecture. The material of this book just about fits into a one-year course, which means that much else has had to be skipped. Every book on quantum mechanics represents an exercise in selectivity – I can't say that my selections are better than those of other authors, but at least they worked well for me when I taught the course.

There is one topic I was not sorry to skip: the relativistic wave equation of Dirac. It seems to me that the way this is usually presented in books on quantum mechanics is profoundly misleading. Dirac thought that his equation is a relativistic generalization of the non-relativistic time-dependent Schrödinger equation that governs the probability amplitude for a point particle in an external electromagnetic field. For some time after, it was considered to be a good thing that Dirac's approach works only for particles of spin one half, in agreement with the known spin of the electron, and that it entails negative-energy states, states that when empty can be identified with the electron's antiparticle. Today we know that there are particles like the $W^{\pm}$ that are every bit as elementary as the electron, and that have distinct antiparticles, and yet have spin one, not spin one half. The right way to combine relativity and quantum mechanics is through the quantum theory of fields, in which the Dirac wave function appears as the matrix element of a quantum field between a one-particle state and the vacuum, and not as a probability amplitude.

I have tried in this book to avoid an overlap with the treatment of the quantum theory of fields that I presented in earlier volumes.[1] Aside from the quantization of the electromagnetic field in Chapter 11, the present book does not go into relativistic quantum mechanics. But there are some topics that were included in *The Quantum Theory of Fields* because they generally are not included in courses on quantum mechanics, and I think they should be. These subjects are included here, especially in Chapter 8 on general scattering theory, despite some overlap with my earlier volumes.

The viewpoint of this book is that physical states are represented by vectors in Hilbert space, with the wave functions of Schrödinger just the scalar products of these states with basis states of definite position. This is essentially the approach of Dirac's "transformation theory." I do not use Dirac's bra–ket notation, because for some purposes it is awkward, but in Section 3.1 I explain how it is related to the notation used in this book. In any notation, the Hilbert space approach may seem to the beginner to be rather abstract, so to give the reader a greater sense of the physical significance of this formalism I go back to its historic roots. Chapter 1 is a review of the development of quantum mechanics from the Planck black-body formula to the matrix and wave mechanics of Heisenberg and Schrödinger and Born's probabilistic interpretation. In Chapter 2 the Schrödinger wave equation is used to solve the classic bound state problems of the hydrogen atom and harmonic oscillator. The Hilbert-space formalism is introduced in Chapter 3, and used from then on.

[1] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, 1995; 1996; 2000).

Addendum for the Second Edition

Since the publication of the first edition, I have come to think that several additional topics needed to be included in this book. I have therefore added six new sections: Section 4.9 on the rigid rotator; Section 5.9 on van der Waals forces; Section 6.8 on Rabi oscillations and Ramsey interferometers; Section 6.9 on open systems, including a derivation of the Lindblad equation; Section 8.9 on time reversal of scattering processes, including a proof of the Watson–Fermi theorem; and Section 11.8 on quantum key distribution. There have also been many additions within the sections of the first edition, including discussions of the universality of black-body radiation in Section 1.1, lasers in Section 1.2, unentangled systems in Section 3.3, the groups $O(3)$ and $SO(3)$ in Section 4.1, $3j$ symbols and the addition theorem for spherical harmonics in Section 4.3, the application of the eikonal approximation to scattering by long-range forces in Section 7.10, and error-correcting codes in Section 12.3. I have also taken the opportunity to correct many minor errors, as well as a major error in the formulation of degenerate perturbation theory in Sections 5.1 and 5.4.

In Section 3.7 of the first edition I reviewed various interpretations of quantum mechanics, and explained why none of them seem to me entirely satisfactory. I have now reorganized and expanded this discussion, with no change in its conclusion.

* * * * *

I am grateful to Raphael Flauger and Joel Meyers, who as graduate students assisted me when I taught courses on quantum mechanics at the University of Texas, and suggested numerous changes and corrections to the lecture notes on which the first edition of this book was based. I am also indebted to Robert Griffiths, James Hartle, Allan Macdonald, and John Preskill, who gave me advice on various specific topics that proved helpful in preparing the first edition, and to Scott Aaronson, Jeremy Bernstein, Jacques Distler, Ed Fry, Christopher Fuchs, James Hartle, Jay Lawrence, David Mermin, Sonia Paban, Philip Pearle, and Mark Raizen who helped with the coverage of various topics in the second edition. Thanks are due to many readers who pointed out errors in the first edition, especially Andrea Bernasconi, Lu Quanhui, Mark Weitzman, and Yu Shi. Cumrun Vafa used the first half of the first edition as a textbook for a one-term graduate course on quantum mechanics that he gave at Harvard, and was able to make many valuable suggestions of points that should be included or better explained. Of course, only I am responsible for any errors that may remain in this book. Thanks are also due to Terry Riley, Abel Ephraim, and Josh Perlman for finding countless books and articles, and

to Jan Duffy for her helps of many sorts. I am grateful to Lindsay Barnes and Roisin Munnelly of Cambridge University Press for helping to ready this book for publication, to Dr. Steven Holt for his careful and sensitive copy editing, and especially to my editor, Simon Capelin, for his encouragement and good advice.

STEVEN WEINBERG

Notation

Latin indices i, j, k, and so on generally run over the three spatial coordinate labels, usually taken as 1, 2, 3.

The summation convention is not used; repeated indices are summed only where explicitly indicated.

Spatial three-vectors are indicated by symbols in boldface. In particular, $\boldsymbol{\nabla}$ is the gradient operator.

∇^2 is the Laplacian $\sum_i \partial^2/\partial x^i\, \partial x^i$.

The three-dimensional 'Levi-Civita tensor' ϵ_{ijk} is defined as the totally antisymmetric quantity with $\epsilon_{123} = +1$. That is,

$$\epsilon_{ijk} \equiv \begin{cases} +1, & ijk = 123,\ 231,\ 312, \\ -1, & ijk = 132,\ 213,\ 321, \\ 0, & \text{otherwise.} \end{cases}$$

The Kronecker delta is

$$\delta_{nm} = \begin{cases} 1, & n = m, \\ 0, & n \neq m. \end{cases}$$

A hat over any vector indicates the corresponding unit vector: Thus, $\hat{\mathbf{v}} \equiv \mathbf{v}/|\mathbf{v}|$.

A dot over any quantity denotes the time-derivative of that quantity.

The step function $\theta(s)$ has the value $+1$ for $s > 0$ and 0 for $s < 0$.

The complex conjugate, transpose, and Hermitian adjoint of a matrix A are denoted A^*, A^{T}, and $A^\dagger = A^{*\mathrm{T}}$, respectively. The Hermitian adjoint of an operator O is denoted $O^\dagger$. $+$H.c. or $+$c.c. at the end of an equation indicates the addition of the Hermitian adjoint or complex conjugate of the foregoing terms.

Where it is necessary to distinguish operators and their eigenvalues, upper case letters are used for operators and lower case letters for their eigenvalues. This convention is not always used where the distinction between operators and eigenvalues is obvious from the context.

Factors of the speed of light c, the Boltzmann constant k_{B}, and Planck's constant h or $\hbar \equiv h/2\pi$ are shown explicitly.

Unrationalized electrostatic units are used for electromagnetic fields and electric charges and currents, so that $e_1 e_2/r$ is the Coulomb potential of a pair of charges e_1 and e_2 separated by a distance r. Throughout, $-e$ is the unrationalized charge of the electron, so that the fine structure constant is $\alpha \equiv e^2/\hbar c \simeq 1/137$.

Numbers in parenthesis at the end of quoted numerical data give the uncertainty in the last digits of the quoted figure. Where not otherwise indicated, experimental data are taken from K. Nakamura *et al.* (Particle Data Group), "Review of Particle Properties," *J. Phys. G* **37**, 075021 (2010).