

## The Logic in Philosophy of Science

Major figures of twentieth-century philosophy were enthralled by the revolution in formal logic, and many of their arguments are based on novel mathematical discoveries. Hilary Putnam claimed that the Löwenheim–Skølem theorem refutes the existence of an objective, observer-independent world; Bas van Fraassen claimed that arguments against empiricism in philosophy of science are ineffective against a semantic approach to scientific theories; W. v. O. Quine claimed that the distinction between analytic and synthetic truths is trivialized by the fact that any theory can be reduced to one in which all truths are analytic. This book dissects these and other arguments through in-depth investigation of the mathematical facts undergirding them. It presents a systematic, mathematically rigorous account of the key notions arising from such debates, including theory, equivalence, translation, reduction, and model. The result is a far-reaching reconceptualization of the role of formal methods in answering philosophical questions.

**Hans Halvorson** is Stuart Professor of Philosophy at Princeton University, New Jersey. He has written extensively on philosophical issues in physics and the other sciences, on mathematical logic, and on the relationship between science and religion.

Cambridge University Press  
978-1-107-11099-1 — The Logic in Philosophy of Science  
Hans Halvorson  
Frontmatter  
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Princeton University



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## CAMBRIDGE UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom  
One Liberty Plaza, 20th Floor, New York, NY 10006, USA  
477 Williamstown Road, Port Melbourne, VIC 3207, Australia  
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India  
79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)  
Information on this title: [www.cambridge.org/9781107110991](http://www.cambridge.org/9781107110991)  
DOI: 10.1017/9781316275603

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First published 2019

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

*A catalogue record for this publication is available from the British Library.*

*Library of Congress Cataloging-in-Publication Data*

Names: Halvorson, Hans, author.

Title: The logic in philosophy of science / Hans Halvorson (Princeton University, New Jersey).

Description: Cambridge ; New York, NY : Cambridge University Press, 2019. |

Includes bibliographical references and index.

Identifiers: LCCN 2018061724 | ISBN 9781107110991 (hardback : alk. paper) |

ISBN 9781107527744 (pbk. : alk. paper)

Subjects: LCSH: Science–Philosophy.

Classification: LCC Q175 .H2475 2019 | DDC 501–dc23

LC record available at <https://lcn.loc.gov/2018061724>

ISBN 978-1-107-11099-1 Hardback

ISBN 978-1-107-52774-4 Paperback

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## Preface

The twentieth century's most interesting philosophers were enthralled by the revolution in mathematical logic, and they accordingly clothed many of their arguments in a formal garb. For example, Hilary Putnam claimed that the Löwenheim-Skølem theorem reduces metaphysical realism to absurdity; Bas van Fraassen claimed that arguments against empiricism presuppose the syntactic view of theories; and W. v. O. Quine claimed that Carnap's notion of an "external question" falls apart because every many-sorted theory is equivalent to a single-sorted theory. These are only a few of the many arguments of twentieth-century philosophy that hinge upon some or other metalogical theorem.

Lack of understanding of the logical theorems can be a huge obstacle to assessing these philosophers' arguments, and this book is my attempt to help remove that obstacle. However, my ideal reader is not the casual tourist of twentieth-century philosophy who wants the minimal amount of logic needed to get the big picture. My ideal reader is the (aspiring) logician-philosopher who wants to strip these arguments down to their logical nuts and bolts.

Although my motivation for writing this book wasn't to get across some particular philosophical point, a few such points emerged along the way. First, the distinction between realism and antirealism really boils down to one's attitude toward theoretical equivalence. Realists are people with a conservative notion of equivalence, and antirealists are people with a liberal notion of equivalence. Second, and relatedly, to give a philosophical account of a relation between theories (e.g., equivalence and reducibility) is tantamount to recommending certain norms of inquiry. For example, if you say that two theories  $T$  and  $T'$  are equivalent, then you mean (among other things) that any reason for accepting  $T$  is also a reason for accepting  $T'$ . Hence, you won't bother trying to design an experiment that would test  $T$  against  $T'$ . Similarly, if you think that  $T$  and  $T'$  are equivalent, then you'll consider as confused anyone who argues about which of the two is better. In short, to adopt a view on relations between theories is to adopt certain rules about how to use those theories.

I should explain one glaring omission from this book: modal logic. I didn't leave out modal logic because I'm a Quinean extensionalist. To the contrary, I've come to think that the metatheory of first-order logic, and of scientific theories more generally, is chock full of intensional concepts. For example, the models of a scientific theory represent the nomologically possible worlds according to the theory. Furthermore, a scientific theory comes equipped with a notion of "natural property" (in the sense of David Lewis), and these natural properties determine a notion of similarity between possible worlds, which

in turn licenses certain counterfactual inferences. So, while my goal is to theorize about the extensional logic that forms the backbone of the sciences, I believe that doing so calls for the use of intensional concepts.

A final note on how to read this book: Chapters 1–3 are introductory but are not strictly prerequisite for the subsequent chapters. Chapters 1 and 3 treat the metatheory of propositional logic, teaching some Boolean algebra and topology along the way. In Chapter 3, we go through the proof of the Stone duality theorem, because it exemplifies the duality between syntax and semantics that informs the remaining chapters. Chapter 2 covers the basics of both category and set theory in one go, and it's the most technically demanding (and least philosophical) chapter of the book. You don't have to know categorical set theory in order to benefit from the other chapters, it would be enough to know some set theory (e.g., Halmos' *Naive Set Theory*) and to flip back occasionally to look up category-theoretic concepts.

*Acknowledgements:* Thanks to Bas van Fraassen for the inspiration to pursue philosophy of science both as a science and as an art.

The idea behind this book arose during a year I spent in Utrecht studying category theory. I thank the Mellon New Directions Fellowship for financing that year. Thanks to my Dutch hosts (Klaas Landsman, Ieke Moerdijk, and Jaap van Oosten) for their warm hospitality.

When I returned home, I rediscovered that it's difficult to do two (or fifty) things at once. The project might have foundered, had it not been for the theorem-proving wizardry of Thomas Barrett, Neil Dewar, Dimitris Tsementzis, and Evan Washington. I also found my philosophical views shaped and sharpened by conversations with several students and colleagues, especially John Burgess, Ellie Cohen, Robbie Hirsch, Laurenz Hudetz, Michaela McSweeney, Alex Meehan, Gideon Rosen, Elliot Salinger, David Schroeren, and Jim Weatherall. I probably left somebody out, and I'm sorry about that. For comments and corrections on earlier versions of the manuscript, I thank Thomas Barrett, Gordon Belot, Neil Dewar, Harvey Lederman, Dimitris Tsementzis, Jim Weatherall and Isaac Wilhelm.

Finally, thank you to Hilary and Sophie at CUP for their initial belief in the project and for persevering with me to the end.