MODERN OPHTHALMIC OPTICS

This book is a comprehensive account of the most recent developments in modern ophthalmic optics. It makes use of powerful matrix formalism to describe curvature and power, providing a unified view of the optical and geometrical properties of lenses. This unified approach is applicable to the design and properties of not only spectacle lenses, but also contact and intraocular lenses (IOL). The newest developments in lens design, manufacturing, and testing are discussed, with an emphasis on the description of free-form technology, which has surpassed traditional manufacturing methods and allows digital lenses to be specifically designed with the unique requirements of the user. Other important topics covered include modern lens materials, up-to-date lens measuring techniques, contact and intraocular lenses, progressive power lenses, low vision aids, ocular protection, and coatings. Providing a broad overview of recent developments in the field, it is ideal for researchers, manufacturers, and practitioners involved in ophthalmic optics.

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Foreword

A few years back I discovered a book by Theodore Obrig titled *Modern Ophthalmic Lenses and Optical Glass*. Published in 1935, its mouldering pages depicted the state of the spectacle lens art eight decades ago. I needed the book to make a point in a patent litigation case. Although its teachings comprised a useful historical perspective, they no longer provided guidance for spectacle lens designers in the present day. That same patent case caused me to seek a comparable text that could be used to define the state of the art today, but I could not find one. Patiently I assembled a dossier with hundreds of documents including patent applications, journal articles, advertisements and technical manuals. I gathered books on differential geometry, lens design and ophthalmic dispensing. It was a tedious task.

That is why I was so delighted to learn about the publication of this book. It will be a tremendous resource for both students and experts in ophthalmic optics. I have spent many pleasant hours in conversation and correspondence with Jose Alonso. My dossier included works by him and his coauthors and they demonstrate a complete mastery of the most relevant and *modern* subject matter. To their great credit, they have shared much of that mastery and it is all right here, in a single reference with blessedly consistent notation.

The preface rightly emphasizes the importance and utility of matrices for representation and calculation. Yet there is so much more. Let's start at the beginning: the breadth of information about ophthalmic materials is outstanding. Of course, the text teaches the fundamentals of refractive error, but the rigor of presentation is remarkable. One of my most fervent wishes is that our industry will learn to their methods to perform statistical analysis of dioptric power data. This could prevent many unfortunate errors, for example in reports of ophthalmic clinical trials. The rigorous and complete treatment of the lens-eye system and its analysis by wavefront or rays could have saved me years of work, had it been published sooner. With patience and concentration, we also can learn here how to design, fabricate, measure and evaluate lens surfaces including progressives. Speaking of progressive lenses, I was very happy to see the work of my former colleagues Scott Fisher and David Pope given rightful credit; I was even happier to read the discussion of Julie Preston's dissertation. Those of us privileged to work in R&D at SOLA were proud to support her research.

I encourage readers to pay careful attention to the content in the Appendices. The mathematical methods are powerful yet easy to implement in code or even spreadsheets. Learning them will open doors to careers and advancement in research, education and industry. Other books aim to teach the eyecare professional how to recommend, provide and dispense ophthalmic lenses. Such a clinical emphasis is necessary for practising that which is already known. But the concepts you can learn herein will enable you to make further discoveries that may lead us to the next phase of modernity.

Michael Alan Morris O.D.
Santa Rosa, California

Preface

This book attempts to be a comprehensive account of the most relevant topics in modern ophthalmic optics. It stands out from other books on the subject in the profuse use of the unifying and powerful matrix description of curvature to offer a unified view of the properties of ophthalmic compensation elements, from classic single-vision spherical lenses to the modern, customized, free-form progressive lenses. Throughout the book we discuss the newest developments in the paraxial properties of lenses, lens design, and manufacturing and testing, with special mention of free-form technology, which has surpassed traditional manufacturing methods. Additionally, we cover other important subjects such as lens materials, contact and intraocular lenses, low vision aids, and ocular protection and coatings.

Matrix methods are especially well suited for describing astigmatic optical systems, and visual optics is more *about* astigmatic powers and vergences than spherical ones. But matrix methods are not just a set of rules to get tidier formulas. Curvature is a tensor, and its natural representation in a Cartesian coordinate system is a matrix. When we abstractly think of the velocity of an object in 3D space we do not think about the individual polar or cartesian components; we rather think of the velocity as a whole vector, a single concept expressing speed and direction at the same time. Similarly, we should think of curvature as a single concept; it just happens to have three components. This kind of thinking proves to be an extremely powerful method to understand, work, and develop astigmatic optics. Along the last thirty years, matrix methods have been commonplace in many research papers in optometry, and visual and ophthalmic optics. However, a compilation of these techniques and at least the most representative results that have been achieved with them was lacking. For 25 years we have been teaching ophthalmic optics in both undergraduate and graduate courses at the University Complutense of Madrid. In 2008 we changed the standard mathematical approach and started using a matrix formalism as the *fabric* the subject was constructed upon. We initially feared that students could get jammed with the new tools, but experience showed otherwise. Not only were the matrix tools a tidier, more uniform, and easier to remember way to write down the geometrical and optical relationships, but they did not interfere with the understanding of the basic phenomena. We would rather say the unification of curvature and power that the matrix formalism makes so evident boosted this understanding. The notes we wrote back then for these courses transformed into the seed material for this book.

The book is primarily aimed at eye care professionals (ECP), researchers, and graduate students in the field that wish to expand their knowledge in ophthalmics optics. We think it will also be useful to optical engineers involved in the design of optometric and ophthalmic instruments and lenses. To get to such a wide audience we have kept a practical approach: the book is not clinical, but we provide hints and references on how clinicians can benefit from the formalism. Neither is it too technical: We have sacrificed the most abstract foundations of the formalism (for example, we avoid discussing symplecticity and the Hamiltonian origin of the matrix transfer formalism) to provide a problem-solving approach. Similarly, most of the contents are kept within the paraxial approximation. Deeper incursions in non-paraxial methods could have been of interest to some optical engineers, but they would have required a substantial raise on the mathematical complexity, making the book too harsh for a large community of vision professionals, which we preferred could enjoy the beauty and power of the matrix methods in visual and ophthalmic optics. Still, matrix paraxial and pseudo-paraxial approaches provide an excellent starting point to lens design, as we exemplify with the calculation of contact and intraocular lenses and with a new surface model for progressive surfaces. With some trimming of the most advanced material, the book can serve as a textbook for undergraduate courses in ophthalmic optics. Having this possible application in mind, we decided to include topics not directly related with the tensor nature of curvature, such as optical materials, lens manufacturing and measurement, frames, and protection filters. In this way the book has become more self-contained with respect to the standard curriculum in ophthalmic optics.

The book starts with a description of the most common optical materials used in the ophthalmic lens industry. Using a classic approach, the study is divided into optical glass and optical polymers, presenting their most relevant optical, mechanical, and chemical properties. We also include a simple representation technique that may help the ECP with the selection of ophthalmic materials.

The second chapter is devoted to the study of the curvature and other geometrical properties of the surfaces typically used in ophthalmic optics. The description is a bit more complete than the usual introduction offered in traditional textbooks; in return, it provides the interested reader with the tools and expressions for accurate thickness calculation and lens representation. The description of surfaces will also be useful for those attempting to proceed with exact ray-tracing techniques. Central to the book, in this chapter we establish the relation between curvature and the Hessian matrix.

Chapter 3 is the link between the description of surfaces – physical and wavefronts – and the ophthalmic lens theory and practice. In this chapter we describe the duality between wavefronts and rays. By identifying the vergence with the tensor curvature of wavefronts, and the rays with the vectors perpendicular to the same wavefronts, we provide our own view of the bridge between the ray picture widely developed by W. F. Harris, and the wavefront-based picture described, for example, by E. Acosta and R. Blendowske.

Once we have the key ingredients ready, we present a first description of ophthalmic lenses. Their geometrical properties follow from the matrix curvature of their surfaces, and the back vertex power follows from the propagation of matrix vergences through the lens. In this chapter we mainly deal with the back vertex power, the one we are interested in, though we also make a small incursion into the generalization for astigmatic systems of the traditional equivalent power. We also provide here matrix and scalar expressions and algorithms to compute lenses.

In Chapter 5 we model the lens-eye system to understand the way a compensating lens interacts with the eye. We define and describe refractive error in terms of second-order eye aberrations, and provide the links between the different ways to describe power: spherocylindrical prescription, power matrix, power vectors, and second order Zernike polynomials. To study prismatic effect and magnification with the greatest accuracy and generality, we introduce the 4×4 transfer matrix formalism using the tools developed in Chapter 3. We present a novel interpretation of the relation between Remole's dynamic magnification and Keating's magnification matrix. According to the problem-solving philosophy mentioned before, we avoid the introduction of the standard 5×5 augmented matrix formalism and study decentered lenses using the standard 4×4 transfer matrices and Harris' methods for managing decentered and tilted optical systems. In the end, the 5×5 augmented matrix formalism does not introduce new insight, it is just a tidier arrangement that will be useful for systems with many decentered or tilted surfaces (as an eye implanted with an IOL, for example) but it is not that necessary for a single decentered ophthalmic lens.

After describing the interaction between the ophthalmic lens and the eye within the paraxial approximation, we discuss the aberrations that mainly affect ophthalmic lenses and how they determine their design. First, we outline the classical third order theory of oblique astigmatism and mean power error leading to the Tscherning theory of ophthalmic lens design. We then describe how modern lenses are designed using lens-eye models beyond the scope of the classical Tscherning theory and show some examples of what free form technology allows.

The study of contact and intraocular lenses (IOL), even at the most basic level, requires separate books, as their optical properties amount to just a small fraction of the complex, nonoptical interaction they have with the eye. However, we wanted to include a small chapter, the seventh, just to show the reader how the matrix formalism presented in the previous chapters can be readily applied to the paraxial calculation of complex bitoric contact lenses and toric IOLs. Of course, we do not even mention other important topics on this type of lenses such as band design, oxygen permeability and material biocompatibility, contact lens adaptation, haptic design, and any medical or clinical issues.

Chapter 8 is devoted to multifocal lenses. We start with a brief summary of presbyopia, lens effectiveness when accommodation is in play, and the computation of the ranges of clear vision. Then we study bifocals, where once again we use the matrix formalism to obtain the basic power relations and to compute prismatic effects and image jump. Our approach to progressive power lenses (PPL) is somewhat unusual. Actual design and evaluation of PPLs requires massive amounts of numerical computation not adequate for a standard course in ophthalmic lenses. Analytical models that could allow the nonspecialized designer to understand, compute, and test their basic properties were lacking, and dissertations on progressive lenses have been limited to historically guided enumerations of PPL designs. In most cases, the only source of information about those designs comes from the designer, using a qualitative and in many cases vague language. Even worse, in recent years the advance of PPL seems to come from the adding of particular "technologies" applied here and there that are supposed to enhance the lens performance, but no model or theory is provided that could support why or when the new lens design behaves better. In fact, a complete theory or model does not exist yet, but we put forward a model for progressive surfaces that will help the reader to understand the nature of progressive variation of curvature, its restrictions, and their consequences for lens performance. It is well known that PPL design is about balancing pros and cons of each design feature: we show how to put numbers into these balances, at least those for which a simple analytical model can be laid out.

The next chapter is dedicated to low vision aids and high powered lenses describing the practical and dispensing issues related with them. Two main types of low vision aid, magnifiers and telescopes, are discussed.

Chapters 2–9 deal with the geometrical and refractive properties of ophthalmic lenses. The remaining two chapters address measurement, manufacturing, and coatings. In Chapter 10 we explain the general principles of lens manufacturing with special focus on free-form technology. We also describe different techniques for lens measurement beyond the classical focimeter, introducing the main technologies used by electronic lensmeters, lens mappers, and profilometers. We include a discussion on quality control for ophthalmic lenses, describing current ISO standards and presenting new methods for quality control of progressive power lenses.

The final chapter of the book is about the control of lens transmittance, either to filter out all or part of the light spectrum, or to enhance transmittance. We first include a study of the possible damages derived from the interaction between electromagnetic radiation and the ocular tissues, then we describe the different technologies used to control the transmittance and reflectance of spectacle lenses, the associated manufacturing processes, and some guidelines for filter prescription.

The topic on frames, frame adaptation, and lens centering has been consigned to the first appendix. On the one hand, full understanding of the rules used for lens centering requires a similarly good understanding of the optical properties of lenses. On the other hand, knowledge of frame coordinate and dimensioning systems lets us understand the constraints imposed by the frame in lens alignment. According to this, the first three sections of the appendix can be read at any time, probably best before starting Chapter 5. The fourth section can also be read at any time if the centering rules are considered axiomatically. To fully understand the reasons for these rules, Chapters 5, 6, and 8 should be first studied.

Appendices B and C are mathematical complements on matrix algebra and geometry that introduce and briefly describe the tools needed in the book. Appendix E contains a brief description of Seidel aberrations and their relation to Zernike polynomials. Finally, the last appendix contains Abelés' theory of multilayers, a supplement to those looking for a deeper understanding of the workings of interferential coatings.

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Symbols and Acronyms

ACD	Anterior chamber depth
BFL	Back focal length
BFP	Back focal power
BVP	Back vertex power
C	Cylindrical component of astigmatic power
CTF	Contrast transfer function
D	Diameter, optical density
DOE	Diffractive optical element
DOF	Depth of field
DRP	Distance reference point
EFL	Effective focal length
FFL	Front focal length
FFP	Front focal power
FSL	Frame symmetry line
g	Shape factor (scalar)
$\mathbb{H}$	Hessian matrix of a surface
$\mathbb{G}$	Shape factor (matrix)
H, K	Mean and Gaussian curvatures/powers
HCL	Horizontal center line
HOA	High order aberrations
IOL	Intraocular lens
MTF	Modulation transfer function
NPD	Naso-pupillary distance
NRP	Near reference point
O_R	Remote point
O_P	Near point
OTF	Optical transfer function
$\mathbf{p}$	Prism (vector)
$\mathbb{P}$	Power matrix, general (refractive power, lens power)
P	Scalar power, general (refractive power, lens power)

P_B	Refractive power of the base curve of a toric surface
PPL	Progressive power lens
P_T	Refractive power of the cross curve of a toric surface
PRP	Prism reference point
P_S	Refractive power of a spherical surface (for spherocylindrical lenses)
PSF	Point spread function
P_V, P_E, P_{TL}	Back vertex power, equivalent power, thin lens power (scalar)
$\mathbb{P}_V, \mathbb{P}_E, \mathbb{P}_{TL}$	Back vertex power, equivalent power, thin lens power (matrix)
P_x, P_y, P_t	Components of the power matrix
$P_{1,2}$	Refractive powers of the two surfaces of a lens
P_-, P_+	Minimum and maximum principal powers
R	Radius (general), reflectance
$\mathbb{R}$, R	Refractive error (matrix/scalar)
r	Radial coordinate, radius of the circular contour of a round lens, radius of cap
RGP	Rigid gas permeable
S	Spherical component of astigmatic power
t	Thickness
t_e	Edge thickness
t_0	Center thickness
T, T_v	Transmittance, luminous (visible) transmittance
V	Vergence (scalar)
$\mathbb{V}$	Vergence (matrix)
$X_{1,2}$	Any magnitude referred to the front (1) or back (2) surface
λ	Wavelength of light, length along ray
n	Refractive index
κ	Scalar curvature
$\kappa_B\,\kappa_T$	Curvatures of the base and cross curves of toric surfaces
ν	Abbe number, constringence
p	Prism (modulus)
Φ_w	Light flux (radiant)
$[\ ]_\otimes$	Curvature/power matrix in the reference system of its main meridians