

## THE STRUCTURE AND DYNAMICS OF CITIES

With over half of the world's population now living in urban areas, the ability to model and understand the structure and dynamics of cities is becoming increasingly valuable. Combining new data with tools and concepts from statistical physics and urban economics, this book presents a modern and interdisciplinary perspective on cities and urban systems. Both empirical observations and theoretical approaches are critically reviewed, with particular emphasis placed on derivations of classical models and results, along with analysis of their limits and validity. Key aspects of cities are thoroughly analyzed, including mobility patterns, the impact of multimodality, the coupling between different transportation modes, the evolution of infrastructure networks, spatial and social organization, and interactions between cities. Drawing upon knowledge and methods from areas of mathematics, physics, economics, and geography, the resulting quantitative description of cities will be of interest to all those studying and researching how to model these complex systems.

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# THE STRUCTURE AND DYNAMICS OF CITIES

Urban Data Analysis and Theoretical Modeling

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*To Esther and Rebecca*

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## Preface

Most of the world's people are now living in cities and urbanization is expected to keep increasing in the near future. The resulting challenges are complex, difficult to handle, and range from increasing dependence on energy, to the emergence of socio-spatial inequalities, to serious environmental and sustainability issues. Understanding and modeling the structure and evolution of cities is then more important than ever as policy makers are actively looking for new paradigms in urban and transport planning.

The recent advances obtained in the understanding of cities have generated increased attention to the potential implication of new theoretical models in agreement with data. Questions such as urban sprawl, effects of congestion, dominant mechanisms governing the spatial distribution of activities and residences, and the effect of new transportation infrastructures are fundamental questions that we need to understand if we want a harmonious development of cities in the future, from both social and economic points of view.

Cities were for a long time the subject of numerous studies in a large number of fields. Discussion of the ideal city can be traced back at least to the Renaissance, and more recently scientists have tried to describe quantitatively the formation and evolution of cities. Regional science and then quantitative geography addressed various problems such as the spatial organization of cities, the impact of infrastructures, and transport. It is remarkable to note that as early as the 1970s quantitative geographers realized the crucial importance of networks in these systems, and produced visionary studies about networks, their evolution, and the complexity of cities (Haggett et al. 1977).

These studies were further developed mathematically by economists who discussed the interplay between space and economic aspects in cities. Many important models find their origin in the seminal paper of Von Thunen and describe isolated, monocentric cities in terms of utility maximization subject to budget constraints. These models allowed spatial economics to get a grasp of

the relations between space, income, and transportation; for example. Japanese economists Fujita and Ogawa discussed the impact of agglomeration effects between firms in a general model that deals with the location choice for individuals and companies. However, as is often the case in the physics of complex systems, writing complicated equations that are essentially impossible to solve does not lead to much progress in our understanding. In fact, at the heart of many economic processes such as location choice or job search lies the problem of how individuals make decisions. Describing this process is a rather formidable task and utility optimization or other optimal strategies are interesting attempts to accomplish it. Many theoretical papers and books discuss these various problems, but in most of these approaches there is one fundamental flaw: the relation between these theoretical models and the reality.

This is where a recent game changer enters the arena: huge amounts of data on every possible aspect of cities have suddenly (at the human scale) become available. In the hope of uncovering underlying laws governing the dynamics and the evolution of these systems, researchers can now begin the systematic analysis of many different cities. Today we are making the first transversal studies to try to uncover universal behavior, independent from the history, culture, or geography of these systems. This trend is reinforced by ideas such as the self-organization of complex systems. The view that a city can be described as a complex, self-organized system is comparatively recent and can be traced back to studies done by geographers such as Pumain in France and Batty in the UK, but had also been rapidly understood and accepted by leading economists such as Krugman.

In addition to geographers and economists, statistical physicists have recently become interested in cities. Indeed, the availability of data, together with the large size of cities, the large number of their constituents, and the variety of processes, are all ingredients that are both very attractive and challenging for a statistical physicist. Attractive, because statistical physics has a long history of understanding and characterizing emergent macroscopic phenomena in terms of the dynamical evolution of the basic constituents of the system. Also, the recent large-scale analysis of different cities has provided evidence for common emergent properties that go beyond specific features of these systems and thus lead to the possibility of proposing general models. Challenging, because individuals and institutions are not atoms, and because there are so many aspects to cities that it might be extremely difficult to disentangle the different processes and to reach a clear, minimal model in agreement with data.

We have thus witnessed in the last few years an explosion of new studies on cities with the help of new data and new tools to analyze them. Various models and aspects have recently been discussed from this perspective, revisiting urban

economics in the light of statistical physics and usually informed by new datasets. Many results have been obtained and the purpose of this book is an attempt to provide some kind of unified view of some of these new models, their variants, and their impact on our understanding of cities. In doing so, we will make a special effort to define, for each model or phenomenon studied, the appropriate language used in the field, and to offer the reader a mapping between languages and techniques used in different disciplines.

We will discuss many aspects of cities and for each one we will essentially focus on new approaches that are not necessarily mainstream, but that are connected to data and that propose a new perspective on a phenomenon. Each chapter is as much as possible self-contained and addresses a specific phenomenon that takes place in cities. For the sake of clarity and readability we have tried to be as modular as possible, in order to allow the reader interested in just one phenomenon to focus essentially on the corresponding chapter.

The chapters are sequenced according to a rough measure of increasing complexity. In the first chapter, we discuss general features of cities and try to bring new perspectives whenever possible. We start with a general discussion of the definition of cities and what is currently believed about their origins. We then discuss important properties of cities – some of them known for a long time, such as the Zipf law for population – and their various spatial and temporal scales. In particular, we will also discuss the order of magnitude of areas, densities of cities, the number of cities, and the size of the largest city in each country. We end this first chapter with naïve scaling that relates various quantities among them. These elementary arguments are interesting as they provide some benchmarking for discussing empirical observations.

In the second chapter, we propose a discussion about methods to study cities. We discuss how statistical physics could help – from both a methodological and a conceptual point of view – in understanding and modeling cities, with important ideas about parameters and minimal models. We also develop a critical discussion of urban economics, and in particular of problems generated by the equilibrium assumption, and the existence of interactions. We will also discuss the important problem related to the choice of utility in these models. We end this chapter with a discussion about data, their sources and scales, and by discussing the necessity of interdisciplinarity in urban studies, including its current limitations that need to be overcome.

Chapters 3 to 8 are devoted to more specific subjects. In Chapters 3 and 4 we discuss the spatial structure of cities and transportation infrastructures, naturally opening the way to Chapters 5 and 6, which are devoted to mobility. Chapter 7 concerns the complex subject of the socioeconomic properties of cities, and in

Chapter 8 we advance a discussion of systems of cities and their hierarchical structure.

More precisely, in Chapter 3 we discuss the spatial organization of cities, and we will insist on their polycentric structure. We start with the problem of the optimal location of public facilities and retail stores, followed by a discussion on the empirical measures of polycentricity. We then turn to the theoretical side of polycentricity and review classical approaches such as the Fujita–Ogawa model and the “edge city” model proposed by Krugman. In particular, we show that these models are unable at this stage to predict correctly quantitative features such as the number of activity centers versus population. In order to understand this, we revisit the Fujita–Ogawa model and simplify it, obtaining results in agreement with empirical observations. This model, which describes in a simplified way where individuals are living and working, allows us to make predictions about mobility-related quantities such as CO<sub>2</sub> emissions or total commuting distance.

In Chapter 4, we discuss the structure and dynamics of transportation infrastructures. These objects evolve on very long time scales and only recently could we gather extensive data about their dynamics. We start this chapter by discussing the road and street network, which plays a central part in the morphodynamics of cities. In particular, its structure is intimately related to the distribution of activities and population. We then discuss the structure of subway networks and their time evolution. We also propose simple arguments that allow us to connect their network properties with economic aspects of their environment.

Mobility is the main subject of both Chapters 5 and 6. It is a crucial aspect of cities and is intimately related to the spatial organization of the city, its infrastructure, and the location of residences and activities. We have divided the various studies and discussions of this subject into two (related) chapters. In Chapter 5, we discuss patterns of mobility, described essentially by the origin–destination matrix, while in Chapter 6, we focus on mobility from the point of view of multimodality, described here in the framework of multilayer networks. More precisely, in Chapter 5 we discuss empirical results about the origin–destination matrix, obtained with the help of new data sources. We discuss how we can extract useful mesoscopic information from large, detailed datasets. In a second part, we will present the current theoretical understanding about mobility, starting from old approaches such as the gravity model to new theoretical discussions based on simple stochastic processes. In Chapter 6, we present another aspect of mobility, namely multimodality: there is, in fact, a growing trend in large cities to use different transportation modes. These different transportation networks are coupled to each other, and the recently developed framework of multilayer networks allows a new perspective on these systems. Within this framework, we discuss the effect of the coupling between the road and subway

networks. We will also consider an information perspective on these networks and show that it will be increasingly difficult to find our way around large cities.

In Chapter 7, we discuss socioeconomic features of cities. This provides an opportunity to introduce classical models of urban economics such as the Alonso–Muth–Mills (AMM) model, the archetype of this type of approach. We believe that it is important to know and understand these models in order to test them empirically and also to be able to improve and to elaborate on them. Since much of this literature has been written by and for economists, we believe that a simple explanation of the main results for non-economists ought to be helpful and valuable for anyone interested in cities and their economics. In particular, we will go through all the derivations in these models in simple language accessible to scientists not trained in economics. These models, despite their many issues, constitute good starting points for other approaches that could be more elaborate, at least from the point of view of ingredients or mechanisms. In addition to the classical AMM model, we also discuss the Beckmann model that proposes a framework for understanding the interplay between the social network structure and the spatial organization of cities, a very real problem, thanks to the availability of social data. We also recount a recent discussion of the Schelling model for segregation and the problem of individual versus global optimization. We end this chapter with a discussion on how various macroscopic parameters describing specific features of cities vary with population. This allows us to introduce scaling properties for these systems, and we review here the main results and discussions of this topic.

In Chapter 8, we discuss models of systems of cities. The main purpose of these models is to explain the hierarchy of cities. This hierarchy is observed in the distribution of population as reflected by the Zipf law. We discuss in detail the mainstream models by Gibrat and Gabaix and also the model of diffusion with noise. This model – also called stochastic diffusion – is another regularization of the Gibrat model and likely represents a promising direction of research for understanding the coupling between cities. It is also a nice example of how modern approaches from statistical physics can be helpful for understanding complex systems such as cities. The hierarchy of cities is also reflected in their spatial organization, as discussed almost a century ago by Christaller in his central place theory. We present Okabe and Sadahiro’s review of this famous approach, showing that fluctuations in a null random model could actually explain most of the observations made by Christaller. This is an interesting example as it makes the connection between a classical approach in regional science and properties of disordered systems. In particular, it highlights the importance of considering null models when discussing empirical data.

A final chapter is devoted to the outlook for the field, with discussions on testing qualitative ideas about cities and their future, and concluding thoughts about the possibility of a science of cities.

Of course these chapters cannot cover the whole variety of new studies about cities, and represent a small share of all city-related topics. Important studies in geography and economics are certainly missing, but what we have attempted here is to put together new approaches combining new tools and data in order to understand quantitatively as many aspects of cities as possible. This book thus inevitably covers a relatively subjective selection of topics. Owing to these personal biases, space limitations, and the sheer volume of current literature, important topics have been omitted. In this respect, I apologize in advance to those colleagues who feel that their work is not well represented here. Incomplete and imperfect as it is, I hope however that this book will be helpful to scientists interested in quantitative approaches to urban systems, a fascinating subject.

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