QUANTUM GRAVITY AND THE FUNCTIONALRE NORMALIZATION GROUP

The Road towards Asymptotic Safety

During the past two decades the gravitational Asymptotic Safety scenario has undergone a major transition from an exotic possibility to a serious contender as a realistic theory of quantum gravity. It aims at a mathematically consistent quantum description of the gravitational interaction and the geometry of spacetime within the realm of quantum field theory, keeping its predictive power at the highest energies. This volume provides a self-contained pedagogical introduction to Asymptotic Safety and introduces the functional renormalization group techniques used in its investigation, along with the requisite computational techniques. The foundational chapters are followed by an accessible summary of the results obtained thus far. It is the first detailed exposition of Asymptotic Safety, providing a unique introduction to quantum gravity. The text assumes no previous familiarity with the renormalization group and thus serves as an important resource for both practicing researchers and graduate students entering this maturing field.

Martin Reuter is a professor of Theoretical Physics at the Johannes Gutenberg University Mainz. He previously worked at the European Laboratory for Particle Physics, CERN, in Geneva, the DESY laboratory in Hamburg, and the Leibniz University of Hanover. In the 1990s, he initiated the exploration of quantum gravity and the Asymptotic Safety scenario using the functional renormalization group and subsequently played a key role in developing the program into its present form.

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CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India

79 Anson Road, #06-04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning, and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781107107328
DOI: 10.1017/9781316227596

First published 2019

Printed and bound in Great Britain by Clays Ltd, Elcograf S.p.A.

A catalogue record for this publication is available from the British Library.

ISBN 978-1-107-10732-8 Hardback

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Preface

This book grew out of a series of lectures the authors have given at various universities and summer schools. Its intention is to provide an easily accessible, pedagogical account of the basic conceptual ideas and methods underlying the asymptotic safety approach to quantum gravity. Knowledge of General Relativity and quantum field theory at a master-course level should suffice to follow the exposition. The necessary technical background is developed from the beginning and no previous familiarity with functional renormalization group methods is assumed. As much as possible we try to supplement formal derivations by intuitive arguments. Our hope is that the book provides a valuable resource for graduate students and researchers, enabling them to follow the cutting-edge research in this field and placing it into the broad context.

It is impossible to thank all people here who directly or indirectly contributed to our work on quantum gravity and to this book ultimately. However, M. R. is particularly grateful to Christof Wetterich for the crucial and highly inspiring collaborations during the early days of the functional renormalization group and more than three decades of creative exchange and to Roberto Percacci for sharing the dream about asymptotic safety right from the start and for his decisive work toward making it a reality. It is also a pleasure to thank Alfio Bonanno for an enjoyable collaboration from very early on in an effort to understand the phenomenological consequences of asymptotic safety. Special heartfelt thanks go to Ennio Gozzi and Walter Dittrich for their support and guidance across all of physics, and well beyond.

F. S. thanks Jan Ambjørn, Renate Loll, and the quantum gravity group at Nijmegen for the continuous exchange of ideas and countless discussions, which have significantly influenced his view on quantum gravity in the last few years. A loving thank you goes to his wife Tetyana and daughter Sophia: without their continuous support and patience this exposition would not have been the same.

Much of the work reported in this book would not have seen the light of the day without the enthusiasm and ingenuity of the students and young researchers who collaborated with us on the asymptotic safety program: N. Alkofer, D. Becker, D. Benedetti, J. Biemans, A. Codello, A. Contillo, J.-E. Daum, M. Demmel, G. D'Odorico, J.-W. Goossens, K. Groh, U. Harst, W. B. Houthoff, A. Kurov, O. Lauscher, P. Machado, E. Manrique, A. Nink, C. Pagani, A. Platania,

S. Rechenberger, G. M. Schollmeyer, M. Schutten, J.-M. Schwindt, E. Tuiran, F. Versteegen, H. Weyer, and O. Zanusso.

We are particularly indebted to Gregor M. Schollmeyer for his invaluable input and untiring care in typesetting, shaping, and improving this monograph. We are also grateful to Peter van Dongen for suggesting numerous improvements, and Robin Ooijer and Arthur Vereijken for their help in proofreading the (almost) final manuscript.

List of Abbreviations

ADM	Arnowitt–Deser–Misner
BRST	Becchi–Rouet–Stora–Tyutin
CDT	Causal Dynamical Triangulation
CREH	conformally reduced Einstein–Hilbert
EAA	Effective Average Action
EH	Einstein-Hilbert
FRG	Functional Renormalization Group
FRGE	Functional Renormalization Group Equation
GFP	Gaussian fixed point
GR	General Relativity
IR	infrared
LHS	left-hand side
LPA	local potential approximation
LQG	Loop Quantum Gravity
LSZ	Lehmann–Symanzik–Zimmermann
NGFP	non-Gaussian fixed point
ODE	ordinary differential equation
QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
QEG	Quantum Einstein Gravity
RG	renormalization group
RHS	right-hand side
UV	ultraviolet