PART I

New Logics in the Functioning of Legal Orders
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Logics of Argumentation and the Law

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Introduction

In his Legal Traditions of the World (Glenn 2010), Patrick Glenn observes that the world contains many different legal traditions, often inconsistent with each other, and that even a single tradition can contain different sub-traditions that may be inconsistent with each other. Moreover, he notes that these traditions may interact with each other in complex ways. In chapter 10, Glenn raises the question of how to account for this from the perspective of formal logic. In chapter 14 of his The Cosmopolitan State (Glenn 2013) he writes that new logics may be needed that are multivalent, paraconsistent or non-monotonic and do not adhere to the classic rules of non-contradiction and the excluded middle. In this chapter I will explore the use of one such new kind of logic, namely, logics of argumentation. It will turn out that such logics offer what Patrick Glenn is asking for without giving up classical two-valued logic. Instead, classical logic is in argumentation logics embedded in a larger formal framework, and it is this larger framework that has the desired nonstandard behavior. Thus argumentation logics provide a way to cope with inconsistent legal traditions without having to give up two-valued logic, a way that is moreover arguably close to the way lawyers think since notions like argument, counterargument and rebuttal are natural them.

Introductory textbooks to logic often portray logically valid inference as ‘foolproof’ reasoning: an argument is deductively valid if the truth of its premises guarantees the truth of its conclusion. In other words, if one accepts all premises of a deductively valid argument, then one also has to accept its conclusion, no matter what. However, we all construct arguments from time to time that are not foolproof in this sense but that merely make their conclusion plausible when their premises are true. For example, if we are told that John and Mary are married and that John

1 Parts of this chapter are adapted from Prakken and Sartor (2009).
lives in Amsterdam, we conclude that Mary will live in Amsterdam as well, since we know that usually married people live where their spouses live. Sometimes such arguments are overturned by counterarguments. For example, if we are told that Mary lives in Rome to work at the foreign offices of her company for two years, we have to retract our previous conclusion that she lives in Amsterdam. However, as long as such counterarguments are not available, we are happy to live with the conclusions of our fallible arguments. The question is: are we then reasoning fallaciously or is there still logic in our reasoning?

The answer to this question has been given in more than thirty years of research in Artificial Intelligence (AI) on so-called logics for defeasible reasoning, partly inspired by earlier developments in philosophy and argumentation theory. At first sight it might be thought that patterns of defeasible reasoning are a matter of applying probability theory. However, many patterns of defeasible reasoning cannot be analysed in a probabilistic way. In the legal domain this is particularly clear: while reasoning about the facts can (at least in principle) still be regarded as probabilistic, reasoning about normative issues clearly is of a different nature. Moreover, even in matters of evidence reliable numbers are usually not available so that the reasoning has to be qualitative.

In this chapter an account is sketched of legal reasoning that respects that arguments can be fallible for various reasons. In short, the account is that reasoning consists of constructing arguments, attacking these arguments with counterarguments, and adjudicating between conflicting arguments on grounds that are appropriate to the conflict at hand. Just as in deductive reasoning, arguments must instantiate inference schemes (now called 'argument schemes') but only some of these schemes capture fool-proof reasoning: in our account deductive logic turns out to be the special case of argument schemes that can only be attacked on their premises.

This chapter is organised as follows. In Section 2 the notions of argument, counterargument and the relations of attack and defeat between conflicting arguments are introduced. These ingredients are combined in Section 3 into the idea of dialectical argument evaluation, which completes the general architecture of an argumentation logic. Then in Section 4 the distinction between deductive and defeasible arguments is introduced, and in Section 5 some stereotypical patterns of defeasible arguments are presented, together with stereotypical ways to attack them. Section 6 delves deeper into the formalization of argumentation logics; this section is primarily meant for more formally
interested readers with some background in formal logic. Section 7 then concludes this chapter.

This chapter is intended to be a tutorial on argumentation logics and their relevance for legal reasoning. For this reason I will be sparse with references. A more formal introduction to argumentation logics with references to the literature can be found in Prakken (2011). The use of argumentation logics for modelling legal reasoning is reviewed in Prakken and Sartor (2015).

2 Arguments and Counterarguments

As just said, we assume that any argument instantiates some argument scheme. (More generally, arguments chain instantiations of argument schemes into trees, since the conclusion of one argument can be a premise of another.) Argument schemes are inference rules: they have a set of premises and a conclusion. What are the ‘valid’ argument schemes of defeasible reasoning? Much can be said on this and we will do so later on in Sections 4 and 5, but at least the deductively valid inference schemes of standard logic will be among them. In this section we examine how deductive arguments can be the subject of attack.

Consider the following example. According to section 3:32 of the Dutch civil code (natural) persons have the capacity to perform legal acts (this means, for instance, that they can engage in contracts or sell their property), unless the law provides otherwise. Suppose John argues that he has legal capacity since he is a person and the law does not provide otherwise. Then in standard propositional logic we can write this argument as follows:

Argument A:

Somebody is a person \& \neg The law provides otherwise \rightarrow S/he has legal capacity
John is a person
\neg The law provides otherwise
Therefore, John has legal capacity

(Here \& stands for ‘and’, \neg for ‘it is not the case that’ and \rightarrow for ‘if . . . then’.)

This argument is deductively valid, since it instantiates the deductively valid argument scheme of modus ponens:

Modus Ponens Scheme:

\[ P \rightarrow Q \]
\[ P \]
Therefore, Q
(where \( P \) and \( Q \) can be any statement). This scheme is deductively valid: it is impossible to accept all its premises but still deny its conclusion, since the truth of its premises guarantees the truth of its conclusion.

Now does the deductive validity of argument A mean that we have to accept its conclusion? Of course not: any first lesson in logic includes the advice: if you don’t like the conclusion of a deductive argument, then challenge its premises. According to section 1:234 of the Dutch civil code, minors have the capacity to perform legal acts if and only if they have consent from their legal representative. Now suppose John’s father claims that John is in fact a minor and does not have such consent. Then the following deductive argument against the premise ‘\(~\text{The law provides otherwise}\)’ can be constructed, in two steps. First application of 1:234 results in the conclusion that John does not have the capacity to perform legal acts:

**Argument B:**
Somebody is a minor → (S/he has consent ↔ S/he has legal capacity)
John is a minor
\(~\) John has consent
Therefore, \(~\) John has legal capacity

(Here ↔ stands for ‘if and only if’). The double arrow expresses that when a person is a minor, then having consent is not only a sufficient but also a necessary condition for having the capacity to perform legal acts. So, since John is a minor but does not have such consent, he does not have legal capacity. This conclusion can then be used to attack the third premise of argument A:

**Argument B (continued):**
\(~\) John has legal capacity
\(~\) John has legal capacity → The law provides otherwise
Therefore, the law provides otherwise

Now we must choose whether to accept the premise ‘\(~\) The law provides otherwise’ of argument A or whether to give it up and accept the conclusion of counterargument B. Clearly the phrase “unless the law provides otherwise” of section 3:32 of the Dutch civil code is meant to express that any place where the law expresses otherwise is an exception to section 3:32. Since argument B is based on such a statutory exception, we must therefore give up the premise of A and accept the counter-argument. In this case we say that argument B not just *attacks* but also *defeats* argument A.
However, not all attacks are a matter of statutory exceptions. In our example, John might attack his father’s argument B by saying that he does have consent of his legal representative since his mother consented and she is his legal representative. This gives rise to an argument attacking the third premise of argument B (before the continuation):

**Argument C:**

Somebody’s mother consented → S/he has consent  
John’s mother consented  
Therefore, John has consent

This time we have a genuine conflict, namely, between John’s father’s claim that John acted without consent of his legal representative (the third premise of argument B) and John’s claim that he acted with consent of his legal representative (the conclusion of argument C). Now note that if one accepts all premises of argument C, then one must also accept its conclusion, since argument C instantiates the deductively valid scheme of modus ponens. And if one accepts argument C’s conclusion, one must, of course, reject the third premise of argument B. In the latter case we say that argument C not only attacks but also defeats argument B. Let us assume that the latter is indeed the case.

In sum then, what we have so far is that all three arguments are deductively valid but that argument A is defeated by argument B on its third premise while argument B is in turn defeated by argument C on its third premise. This implies that it is rational to accept the conclusions of arguments A and C: even though A is defeated by B, it is defended by C, which defeats A’s only defeater.

This leads to a very important insight. In order to determine what to believe or accept in the face of a body of conflicting arguments it does not suffice to make a choice between two arguments that directly conflict with each other. We must also look at how arguments can be defended by other arguments. In our example this is quite simple: it is intuitively obvious that C defends A so, since C is not attacked by any argument, both argument A and argument C (and their conclusions) are acceptable. However, we can easily imagine more complex examples where our intuitions fall short. For instance, another argument D could be constructed such that C and D defeat each other, then an argument E could be constructed that defeats D but is defeated by A, and so on: which arguments can now be accepted and which should be rejected? Here we cannot rely on intuitions but need a calculus, or an argumentation logic. Its input will be a collection of arguments plus an assessment of which...
arguments defeat each other, while its output will be an assessment of the dialectical status of these arguments in terms of three classes (three and not two since some conflicts cannot be resolved). Intuitively, the justified arguments are those that survive all conflicts with their attackers and so can be accepted, the overruled arguments are those that are attacked by a justified argument and so must be rejected; and the defensible arguments are those that are involved in conflicts that cannot be resolved. Furthermore, a statement is justified if it has a justified argument, it is overruled if all arguments for it are overruled, and it is defensible if it has a defensible argument but no justified arguments. In terms more familiar to lawyers, if a claim is justified, then a rational adjudicator is convinced that the claim is true; if it is overruled, such an adjudicator is convinced that the claim is false; while if it is defensible, s/he is neither convinced that it is true nor that it is false.

Before an argumentation logic can be presented, a subtlety concerning the defeat relation between arguments must be explained. Above we made a distinction between attack and defeat. An argument A defeats an argument B if A attacks B and is not inferior to B (according to the appropriate criteria for comparing arguments). This definition allows that two arguments defeat each other, namely, if neither argument is inferior or superior to the other. In such cases we say that the two arguments weakly defeat each other; otherwise (if one argument is superior to the other) we say that one argument strictly defeats the other. Suppose in our example that further evidence was given by both John and his father on whether John’s mother consented. John presents a testimony by his brother that his mother consented, while his father presents a testimony by John’s mother that she never consented:

Argument C (continued)
John’s brother says that John’s mother consented
What a witness says is usually true
Therefore, John’s mother consented

Argument D
John’s mother says that she never consented
What a witness says is usually true
Therefore, ¬ John’s mother consented

(In Section 4 we return to the question of whether these arguments can be reconstructed as deductively valid and how it can be that they attack each other on their conclusions instead of on their premises.) Suppose that the
court cannot find a reason why one witness testimony is stronger than the other: then it must conclude that both conflicting arguments (weakly) defeat each other.

3 Logic of Argumentation

Let us now discuss what an argumentation logic looks like. Just as in deductive logic, there is no single universally accepted one and there is an ongoing debate in AI about what is a good argumentation logic. However, we need not go into the details of this debate, since it turns out that there is a simple and intuitive definition that suffices for most applications. The idea is to regard an attempt to prove an argument justified as a *debate* between a proponent and opponent of the argument.$^2$ The proponent starts with the argument that he wants to prove justified and then the turn shifts to the opponent, who must provide all its defeating counterarguments. It does not matter whether they weakly or strictly defeat their target, since the opponent’s task is to interfere with the proponent’s attempt to prove his argument justified. For each of these defeating arguments, the proponent must then construct one strict defeater (it has to be a strict defeater since the proponent must prove his argument justified). This process is repeated as long as it takes: at each of her turns, the opponent constructs all mutual and all strict defeaters of the proponent’s previous arguments, while at each of his turns, the proponent constructs a strict defeater for each of the opponent’s previous arguments, and so on. The idea is that our initial argument is justified if proponent can eventually make opponent run out of moves in every of opponent’s lines of attack.

This process can be visualised as follows (note that this figure does *not* model the above example but is a new, abstract example).

Note that if an argument is justified, this does not mean that the proponent will in fact win the game: he could make the wrong choice at some point. All that it means is that the proponent will win if he plays optimally. In terms of game theory, an argument is justified if the proponent has a so-called *winning strategy* in a game that starts with

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$^2$ The proponent and opponent should not be seen as real human beings; they are a metaphor for the dialectical nature of the reasoning process, where both pros and cons are considered. Such a dialectical reasoning process can just as well take place in the mind of a single reasoner.
the argument. In fact, there is a simple way to verify whether the proponent has a winning strategy. The idea is to label all arguments in the tree as *in* or *out* according to the following definitions:

1. An argument is *in* if and only if all its defeating counterarguments are *out*
2. An argument is *out* if and only if it has a defeating counterargument that is *in*

In the figures *in* is coloured as grey and *out* as white. It is easy to see that because of (1) all leaves of the tree are trivially *in*, since they have no counterarguments. Then we can work our way upwards to determine the colour of all other arguments, ultimately arriving at a colour of the initial argument. If it is grey, i.e., *in*, then we know that the proponent has a winning strategy for it, namely by choosing a grey argument at each point he has to choose. If, on the other hand, the initial argument is
white, i.e., out, then it is the opponent who has a winning strategy, which can be found in the same way. So in the above figure the opponent has a winning strategy, which she can follow by choosing argument O1b at her first turn.

Suppose now that new information becomes available that gives rise to a strictly defeating counterargument P3b against O2c. Then the situation is as in Figure 1.3.

Now argument P1 is in so now it is the proponent who has a winning strategy, viz. choosing P2b instead of P2a when confronted by O1a. This illustrates that when new information becomes available from which new arguments can be constructed, the dialectical status of arguments may change.

It should be noted that each argument appearing as a box in these trees has an internal structure. In the simplest case it just has a set of premises and a conclusion, but when the argument combines several inferences, it

![Figure 1.2: An extended dialectical tree.](image-url)